THE PROCESS OF COULOMB DISSOCIATION OF WEAKLY BOUND RELATIVISTIC NUCLEI AND HYPERNUCLEI WITHIN THE TWO-CLUSTER MODEL

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1. Introduction

The process of dissociation of nuclei in the Coulomb field of fast charged particles has been discussed repeatedly (see, for example, [1-3]). In the recent years the interest to this electromagnetic process was revived in connection with the creation of beams of relativistic nuclei and the problem of identification of relativistic hypernuclei [4,5]. In the paper [6] the process of excitation and disintegration of relativistic nuclei and hypernuclei was investigated using the direct analogy with the problem of ionization and excitation of atoms at the propagation of relativistic charged particles through matter [7]. In this work (see also [8]) we will discuss the application of the results of [6] to the weakly bound deuteron-like systems consisting of two compact clusters (charged and neutral), the distance between which essentially exceeds the sizes of clusters themselves and the target nucleus radius R. The sharp increase of the cross-section of Coulomb dissociation at the decrease of binding energies allows one to determine experimentally these binding energies in the case of weakly bound nuclei and hypernuclei, studying their disintegration at ultrarelativistic velocities. We will investigate also the role of the finite sizes of the target nucleus.

2. Excitation and disintegration of relativistic nuclei in the Coulomb field of a point-like charge

It was shown in the paper [6] that the total cross-section of excitation and dissociation of a relativistic nucleus in the field of an immovable Coulomb centre with the charge Ze can be presented in the following form:

$$\sigma = \frac{4\pi(Z\alpha)^2}{v^2}\int\left(\mathbf{q}_\perp^2+\frac{\epsilon_{n0}}{\gamma^2v^2}\right)^{-2}\sum_{n\neq 0}\overline{\left|\langle n|\mathbf{j}|0\rangle\left(\frac{\mathbf{q}_\perp}{\epsilon_{n0}}-\frac{\mathbf{v}}{\gamma^2v^2}\right)\right|^2}\,d(\mathbf{q}_\perp^2)\,, \qquad (1)$$

Here $\hbar = c = 1$, $\alpha = e^2 = 1/137$ is the electromagnetic constant, $\mathbf{q}_\perp$ is the transverse momentum transferred to the nucleus, $v = |\mathbf{v}|$ is the velocity of the projectile nucleus in the rest frame of the Coulomb centre (i.e. in the *laboratory* frame), $\langle n|\mathbf{j}|0\rangle$ is the vector of the current of transition from the ground state $|0\rangle$ of the projectile nucleus to the excited state of the continuous or discrete spectrum $|n\rangle$, ϵ_{n0} is the excitation energy. The summation in Eq.(1) is performed over all quantum numbers of final states including spin and angular variables, and the upper bar denotes the averaging over polarizations of the initial ground state of the projectile nucleus, which is assumed to be unpolarized [1] .

[1]At the first glance, the formula (1) for the cross-section of Coulomb dissociation corresponds to the one-photon exchange, i.e. to the approximation $Z\alpha \ll 1$. However, the analysis shows that even at large

At small transverse and longitudinal momenta transferred to the projectile nucleus ($|\mathbf{q}_\perp| \ll 1/R_{\rm pr}$, $q_{||} = \epsilon_{n0}/v \ll 1/R_{\rm pr}$, where $R_{\rm pr}$ is the radius of the projectile nucleus) the transition current is expressed directly through the matrix element of the dipole moment:

$$\langle n|\mathbf{j}|0\rangle = -i\epsilon_{n0}\langle n|\sum_p \mathbf{r}_p|0\rangle\,. \tag{2}$$

In Eq.(2) the summation is performed over the coordinates of all the protons in the projectile nucleus. In accordance with the rule of multiplication of matrices, taking into account the equality $\langle 0|\sum_p \mathbf{r}_p|0\rangle = 0$, that arises due to the space parity conservation, the following relation holds:

$$\sum_{n\neq 0} |\langle n|\sum_p \mathbf{r}_p|0\rangle|^2 = \langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle\,. \tag{3}$$

Finally, dividing the integration range into two intervals corresponding to very small and larger $\mathbf{q}_\perp^2$, the following formula for the cross-section of the Coulomb dissociation emerges, after all transformations [6]:

$$\sigma = \frac{4\pi(Z\alpha)^2}{3v^2}\langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle\left[\ln\left(\frac{\gamma^2 v^2}{\epsilon_{\rm bin}^2\langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle}\right) - 2A + B - v^2\right]. \tag{4}$$

Here $\epsilon_{\rm bin}$ is the binding energy of the projectile nucleus, the constant

$$A = \ln\frac{\epsilon}{\epsilon_{\rm bin}} = \frac{\sum_{n\neq 0}\langle n|\sum_p \mathbf{r}_p|0\rangle|^2 \ln(\epsilon_{n0}/\epsilon_{\rm bin})}{\langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle} \tag{5}$$

involves the dependence of the minimal momentum transfer, at the transition to excited states $|n\rangle$, upon the excitation energy $\epsilon_{n0} > \epsilon_{\rm bin}$ ($\epsilon = \epsilon_{\rm bin}e^A \ll 1/R_{\rm pr}$); the constant B describes the contribution of *comparatively large* transfers of transverse momentum. Calculations lead to the expressions:

$$B = -3\int_0^\infty \ln y \frac{d}{dy}\left(\frac{G(y)}{y}\right)dy\,, \qquad y = \mathbf{q}_\perp^2\langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle\,. \tag{6}$$

Taking into account the completeness condition,

$$G(y) = \sum_{n\neq 0} |\,\langle n|\sum_p \exp(-i\mathbf{q}_\perp\mathbf{r}_p)|0\rangle\,|^2 =$$

$$= \langle 0|\,|\sum_p \exp(-i\mathbf{q}_\perp\mathbf{r}_p)\,|^2\,|0\rangle - |\langle 0|\sum_p \exp(-i\mathbf{q}_\perp\mathbf{r}_p)\,|0\rangle\,|^2\,. \tag{7}$$

At $y \ll 1$ the function $G(y) \approx y/3$, at $y \gg 1$ the function $G(y) \to z$, where z is the number of protons in the projectile nucleus. For the projectile nucleus with the unity charge we have: $G(y) = 1 - F^2(y)$, where

$$F(y) \equiv F(\mathbf{q}_\perp^2) = \langle 0|\,\exp(-i\mathbf{q}_\perp\mathbf{r}_p)\,|0\rangle$$

is the electromagnetic formfactor of the ground state.

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values of Z, when $Z\alpha \sim 1$, the corrections to this formula still remain small. That is connected with the fact that the considered result can be justified in the framework of the impulse approach with the amplitude of the Coulomb scattering; in doing so, the exact amplitude of the Coulomb scattering in the region of small transferred momenta, where the *main* contribution into σ is provided, differs from the amplitude obtained within the Born approximation only by phase.

3. Contribution of finite sizes of the target nucleus

Taking into account that the target nucleus is not point-like, one should subtract the correction term ΔB from the constant B in Eq. (4):

$$\Delta B = 3\int_0^\infty \frac{G(y)(1-H(y))}{y^2}\,dy\,. \tag{8}$$

Here $G(y)$ is determined by Eq. (7) as before, and

$$H(y) = \frac{1}{Z^2}|\,\langle 0'|\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}'_p)\,|0'\rangle\,|^2 \tag{9}$$

is the square of the electromagnetic formfactor of the ground state of the target nucleus $|0'\rangle$; in doing so, $\mathbf{r}'_p$ is the coordinate of a proton in the target nucleus. For the uniform distribution of protons over the volume of the target nucleus

$$H(y) = 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2\,, \tag{10}$$

where

$$x = \sqrt{y}\frac{R_{\rm tag}}{\sqrt{\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle}}\,, \tag{11}$$

$R_{\rm tag}$ is the radius of the target nucleus. It is clear that $H(0)=1$.

Let us note that Eq. (8) describes the influence of finite sizes of the target nucleus on the process of Coulomb dissociation of the projectile nucleus *without* the excitation and breakup of the target nucleus. When the quantum state of the target nucleus is not fixed and transitions into all excited states are taken into account, the function $H(y)$ is replaced by the expression [6]

$$\widetilde{H}(y) = \frac{1}{Z^2}\langle 0'|\,|(\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}'_p)|^2\,|0'\rangle\,. \tag{9a}$$

It is easy to show that at the uniform distribution of the charge Z the functions $\widetilde{H}(y)$ and $H(y)$ are connected by the relation

$$1-\widetilde{H}(y) = [1-H(y)]\frac{Z-1}{Z}\,; \tag{9b}$$

thus, in the case of heavy target nuclei the corrections ΔB and $\widetilde{\Delta B} = [1-(1/Z)]\Delta B$ practically coincide.

It is clear from Eq.(4) that in the case of relativistic nuclei with small binding energies the principal contribution into the cross-section of the Coulomb dissociation is conditioned by the logarithmic term being proportional to $\ln[\gamma^2 v^2/(\epsilon_{\rm bin}^2\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle)]$. At ultrarelativistic energies of the projectile nucleus ($v\to 1$, $\gamma \gg 1$) the cross-section of the Coulomb dissociation of this nucleus, irrespective of the relation between radii of the projectile and target nuclei, has the structure:

$$\sigma = a\ln\gamma + b, \quad a = \frac{8\pi(Z\alpha)^2}{3}\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle\,. \tag{12}$$

4. Weakly bound systems. Two-cluster model

Now we will consider the Coulomb disintegration of the "friable", deuteron-like nuclei consisting of two clusters (charged and neutral), the average distance between which is significantly larger than the radius of the force action and the sizes of the clusters themselves. In this case the excited bound states are absent, and the normalized wave function of the ground state, corresponding to the zero orbital momentum, has the following form:

$$\phi_0(r) = \frac{1}{\sqrt{2\pi\rho}} \frac{\exp(-r/\rho)}{r} , \tag{13}$$

where $r = |\mathbf{r}|$ is the distance between the clusters,

$$\rho = \left(2\frac{m_1 m_2}{M}\epsilon_{\rm bin}\right)^{-1/2} . \tag{14}$$

Here m_1 and m_2 are the masses of the charged and neutral clusters, respectively, $M = m_1+m_2$ is the mass of the deuteron-like nucleus, $\epsilon_0 = (-\epsilon_{\rm bin})$ is the energy of its bound state. In the given case the quantity $\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle$ can be explicitly determined:

$$\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle \approx 4\pi z^2 \left(\frac{m_2}{M}\right)^2 \int_0^\infty \phi_0^2(r) r^4 dr =$$

$$= z^2 \left(\frac{m_2}{M}\right)^2 \frac{\rho^2}{2} = \frac{1}{4} z^2 \frac{m_2}{m_1 M \epsilon_{\rm bin}} , \tag{15}$$

where z is the number of protons in the charged cluster.

Using Eqs. (7) and (13), we obtain the analytical expression for the function $G(y)$:

$$G(y) = z^2 \left(1 - \left|\int \phi_0^2(r) \exp\left(-i\mathbf{q}_\perp \mathbf{r}\frac{m_2}{M}\right) d^3\mathbf{r}\right|^2\right) =$$

$$= z^2 \left(1 - \frac{2z^2}{y}\left[\arctan\left(\sqrt{\frac{y}{2z^2}}\right)\right]^2\right) . \tag{16}$$

Here, according to Eq. (15),

$$y = \frac{1}{4}\mathbf{q}_\perp^2 z^2 \frac{m_2}{m_1 M \epsilon_{\rm bin}} . \tag{17}$$

According to Eq.(14), at very small binding energies the effective radius ρ of the projectile nucleus considerably exceeds the target radius R ($\rho \gg R$). In so doing, we may, in the first approximation, take $H(y) = 1$, considering the target nucleus as a point-like Coulomb centre. Substituting Eq.(16) into Eq.(6), we obtain the following value for the constant B:

$$B = -3z^2 \int_0^\infty \ln y \frac{d}{dy}\left(\frac{1}{y} - \frac{2z^2}{y^2}\left[\arctan\left(\sqrt{\frac{y}{2z^2}}\right)\right]^2\right) dy = \ln 2z^2 + C , \tag{18}$$

where [6]

$$C = 6\int_0^\infty \ln u \left(\frac{1}{u^3} + \frac{\arctan u}{u^4(1+u^2)} - \frac{2}{u^5}(\arctan u)^2\right) \approx 0.316 . \tag{19}$$

Meantime, taking into account that at small binding energies all excited states belong to the continuous spectrum, for a two-cluster system with the wave function $\phi_0(r)$ (see Eq. (13)) the constant A, determined by Eq. (5), is given by the integral:

$$A = \frac{16}{\pi}\int_0^\infty \frac{t^{3/2}}{(1+t)^4}\ln(1+t)dt \approx 1.218\,. \qquad (20)$$

Finally, Eq. (4) leads to the following result for the total cross-section of Coulomb disintegration of the *weakly bound two-cluster nuclei*, taking into account also the correction term $\Delta B(Z)$ connected with the finite size of the target nucleus:

$$\sigma = \frac{\pi}{3}(Z\alpha)^2 z^2 \frac{m_2}{v^2 M m_1 \epsilon_{\rm bin}}\left[\ln\left(\frac{8\gamma^2 v^2 M m_1}{m_2 \epsilon_{\rm bin}}\right) - (2A - C) - v^2 - \Delta B(Z)\right]. \qquad (21)$$

Here $2A - C \approx 2.12$; the quantity $\Delta B(Z)$ in Eq.(21) is determined according to Eq.(8).

It is well seen from Eq.(21) that in the limit of very small binding energies $\epsilon_{\rm bin}$ the cross-section of Coulomb disintegration increases inversely proportionally to $\epsilon_{\rm bin}$; in so doing, the logarithmic term inside the square brackets essentially exceeds the other ones. Measuring experimentally the Coulomb dissociation cross-section σ for weakly bound nuclei and hypernuclei, one can determine, in principle, the value of the binding energy $\epsilon_{\rm bin}$ for these nuclei.

5. Projectile nuclei with small binding energies: corrections due to the finite radius of the target nucleus

Due to the correction term $\Delta B(Z)$, the cross-section of Coulomb dissociation σ decreases as compared with the case of the point-like target ($\Delta\sigma < 0$). Substituting Eqs. (16) and (10) into Eq.(8), we obtain the quantity $\Delta B(Z)$ in the case of two-cluster projectile nuclei as the integral

$$\Delta B(Z) = \int_0^\infty \frac{s^2 - (\arctan s)^2}{s^5}\left[1 - 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2\right] ds\,, \qquad (22)$$

where, in accordance with Eqs. (11) and (17),

$$s = \frac{1}{z}\sqrt{\frac{y}{2}}, \quad x = \chi s, \quad \chi = 2\frac{R_{\rm tag}}{\rho}\frac{M}{m_2} = 2\sqrt{2}\,R_{\rm tag}\sqrt{\frac{M m_1}{m_2}\epsilon_{\rm bin}}\,. \qquad (23)$$

It should be emphasized that, owing to the dependence of the quantity ΔB upon the radius of the target nucleus ($R_{\rm tag} = 1.1\,A^{1/3}$, where A is the total number of nucleons in the nucleus; for heavy nuclei $R_{\rm tag} \approx 1.5\,Z^{1/3}$) the total cross-section of Coulomb dissociation σ gets a certain deviation from the pure dependence $\sim Z^2$. However, according to our calculations, the dependence of the total cross-section of Coulomb disintegration of weakly bound nuclei and hypernuclei upon Z *cannot be presented* as $\sim Z^{2-\delta}$ with a small *constant* δ, contrary to the statements in the papers [9,10].

The dependence of the correction term ΔB on the parameter χ is presented in Fig.1 and Fig.2. The corresponding correction to the Coulomb dissociation cross-section is

$$\Delta\sigma = -\frac{\pi}{3}(Z\alpha)^2 z^2 \frac{m_2}{v^2 M m_1 \epsilon_{\rm bin}}\Delta B(Z)\,, \qquad (24)$$

As shown by the analysis, the quantity $\Delta B(Z)$ tends to zero with the decrease of the ratio R_{tag}/ρ as $\Delta B \sim \chi^2 \ln \chi$. Let us note that, for a given projectile nucleus, at charges of target nuclei in the interval $Z = 50 \div 100$ the dependence of ΔB on χ is almost linear ($\Delta B \approx b_{\text{pr}} \chi$). As a result, the quantities ΔB, $\Delta\sigma$ are proportional, respectively, to the factors $Z^{1/3}$, $Z^{7/3}$: $\Delta B \sim Z^{1/3}$, $\Delta\sigma \sim Z^{7/3}$. In so doing, the "effective" charge, determining the cross-section of the Coulomb dissociation, is $Z_{\text{eff}} = Z(1 - a_{\text{pr}} Z^{1/3})^{1/2}$. The coefficients b_{pr} and a_{pr} depend on the concrete projectile nucleus.

6. Calculations of the cross-section of Coulomb dissociation for the hypernuclei $^3H_\Lambda$ and $^6He_\Lambda$

1. Let us consider the process of Coulomb dissociation of the hypernucleus $^3H_\Lambda$ into the deuteron and the Λ-particle:

$$^3H_\Lambda + Z \to d + \Lambda + Z$$

($z = 1$, $M = M(^3H_\Lambda) = 2993.6$ MeV/c^2: $m_1 = m_d = 1878$ MeV/c^2; $m_2 = m_\Lambda = 1115.7$ MeV/c^2). According to the experimental data, the binding energy of the Λ hyperon in the hypernucleus $^3H_\Lambda$ is $\epsilon_{\text{bin}}^{(\Lambda)} = (0.01 \pm 0.07)$ MeV [11], or $\epsilon_{\text{bin}}^{(\Lambda)} = (0.15 \pm 0.07)$ MeV [12]. Taking $\gamma = 6$ and $\epsilon_{\text{bin}}^{(\Lambda)} \approx 0.08$ MeV, we obtain:

$$\chi \approx 0.43 Z^{1/3}; \quad \Delta B \approx 0.63\chi; \quad Z_{\text{eff}}^2 = Z^2(1 - 0.02 Z^{1/3}).$$

In particular, for the tin target ($Z = 50$):
$\sigma_0 = 1.88$ barn; $\Delta\sigma = -0.14$ barn; $\sigma = 1.74$ barn.
For the uranium target ($Z = 92$):
$\sigma_0 = 6.38$ barn; $\Delta\sigma = -0.58$ barn; $\sigma = 5.8$ barn.

Here σ_0 is the cross-section calculated for the point-like Coulomb centre, $\Delta\sigma$ is the correction to the cross-section due to the finite radius of the target nucleus.

2. Now let us consider the process of Coulomb dissociation of the hypernucleus $^6He_\Lambda$ into $^5He_\Lambda$ and the neutron:

$$^6He_\Lambda + Z \to {}^5He_\Lambda + n + Z$$

($z = 2$, $M = M(^6He_\Lambda) = 5.78$ GeV/c^2, $m_1 = m(^5He_\Lambda) = 4.84$ GeV/c^2, $m_2 = m_n = 939$ MeV/c^2).
The data on the binding energies of the Λ-hyperon in the hypernuclei $^6He_\Lambda$ and $^5He_\Lambda$ [12] and on the masses of their nucleon bases (ordinary nuclei ^5He and ^4He [13]) lead to the following estimation of the binding energy of the *neutron* in the hypernucleus $^6He_\Lambda$: $\epsilon_{\text{bin}}^{(n)} = (0.23 \pm 0.13)$ MeV. Taking $\gamma = 6$ and $\epsilon_{\text{bin}}^{(n)} \approx 0.15$ MeV, we have:

$$\chi \approx 1.43 Z^{1/3}; \quad \Delta B \approx 0.5\chi: \quad Z_{\text{eff}}^2 \approx Z^2(1 - 0.05 Z^{1/3}).$$

In the case of the tin target ($Z = 50$): $\sigma_0 = 0.74$ barn; $\Delta\sigma = -0.14$ barn; $\sigma = 0.6$ barn.
For the uranium target ($Z = 92$): $\sigma_0 = 2.50$ barn; $\Delta\sigma = -0.54$ barn; $\sigma = 1.96$ barn.

So, for the considered cases (especially – for $^6He_\Lambda$) the correction $\Delta\sigma$ to the Coulomb dissociation cross-section, emerging due to the finite size of the target, proves to be rather essential.

7. Summary

1. The process of Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei has been studied within the two-cluster "deuteron-like" model. Explicit expressions for the total effective cross-section of Coulomb disintegration, taking into account the corrections conditioned by the finite size of the target nucleus, are obtained.

2. The experimental measurement of the Coulomb dissociation cross-section for weakly bound nuclei and hypernuclei enables one to determine the value of the binding energy for these systems.

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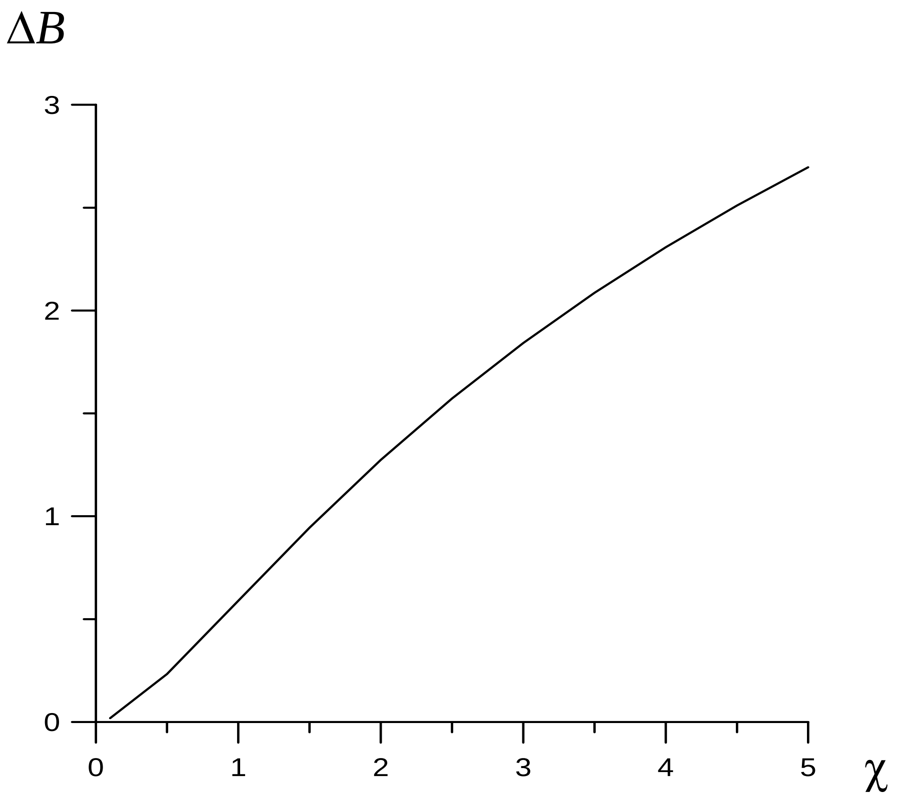

Fig. 1. Dependence of the correction term ΔB (22) upon the parameter χ (23) .

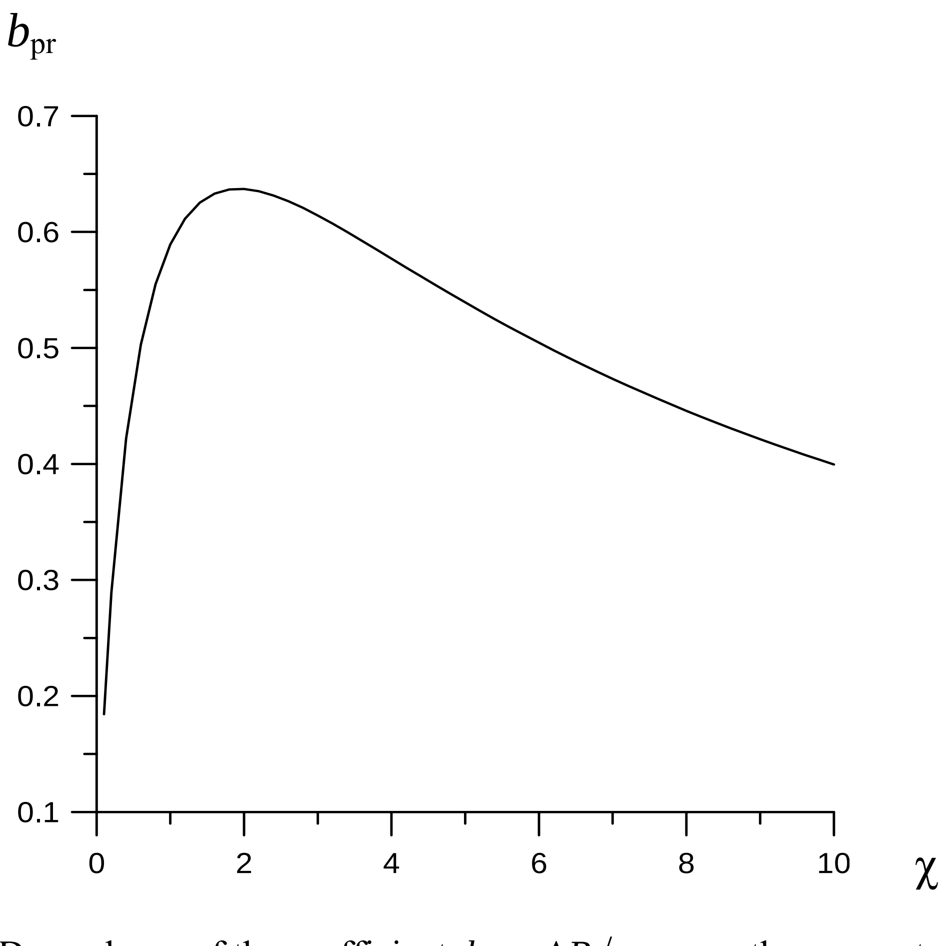

Fig. 2. Dependence of the coefficient $b_{pr} = \Delta B / \chi$ upon the parameter χ .