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Precision PEP-II optics measurement with an SVD-enhanced Least-square fitting

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Abstract

A Singular value decomposition (SVD)-enhanced Least-square fitting technique is discussed. By automatic identifying, ordering, and selecting dominant SVD modes of the derivative matrix that responds to the variations of the variables, the converging process of the Least Square fitting is significantly enhanced. Thus the fitting speed can be fast enough for a fairly large system. This technique has been successfully applied to precision PEP-II optics measurement in which we determine all quadrupole strengths (both normal and skew components) and sextupole feed-downs as well as all BPM gains and BPM cross-plane couplings through Least-Square fitting of the phase advances and the Local Green's functions as well as the coupling ellipses among BPMs. The local Green's functions are specified by 4 local transfer matrix components R12, R34, R32, R14. These measurable quantities (the Green's functions, the phase advances and the coupling ellipse tilt angles and axis ratios) are obtained by analyzing turn-by-turn Beam Position Monitor (BPM) data with a high-resolution model-independent analysis (MIA). Once all of the quadrupoles and sextupole feed-downs are determined, we obtain a computer virtual accelerator which matches the real accelerator in linear optics. Thus, beta functions, linear coupling parameters, and interaction point (IP) optics characteristics can be measured and displayed.

Key words: Optics measurement, Least Square fitting, Electron-positron storage rings
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1 INTRODUCTION

For a system with sufficient known constraints and specific quantities that can be accurately measured, one may be able to build a computer model that

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can simulate the system. One needs to identify efficient variables as inputs to the model that can generates response outputs that are to be identified to be equal to their corresponding system-measured specific quantities. This requires training process that re-identifies new efficient variables and eliminates degenerate or unnecessary (low efficient) variables through Least-Square fitting of those well-chosen responses from the model to those corresponding measured quantities. In many occasions, this training process may also involve identifying new responses with measurable corresponding quantities from the system. To interpret the above in short, one can have a simple mathematical formulae,

$$\vec{Y}(\vec{X}) = \vec{Y}_m, \tag{1}$$

where all variables are represented by an array (a vector) $\vec{X}$; responses and their corresponding measurable quantities are represented by an array $\vec{Y}$ and an array $\vec{Y}_m$ respectively. Note that $\vec{Y}$ is the response to the $\vec{X}$ and therefore is a vector function of $\vec{X}$ as is explicitly shown in the equation. Also note that the array length of $\vec{Y}$ and $\vec{Y}_m$ must be the same and must be larger than the array length of $\vec{X}$ to avoid any degeneracy. Indeed, the array length of $\vec{Y}$ is preferred to be significantly larger than the the array length of $\vec{X}$ to make sure an over determined Least-Square fitting for adding on accuracy and most of the time adding on convergence, too. The Least-Square fitting is to update $\vec{X}$ through iteration such that the residual of $\vec{Y}(\vec{X}) - \vec{Y}_m$ converge to a minimum that is sufficiently small.

2 The SVD-enhanced Least-Square fitting

To perform Least-Square fitting, let's first denote the iteratively updated or the initially reasonably guessed variable values to be $\vec{X}_o$ and Let $\vec{X} = \vec{X}_o + \vec{x}$. Then, after performing Taylor expansion of $\vec{Y}(\vec{x}_o + \vec{x})$, Eq. (1) can be written as

$$Y(\vec{X}) = Y(\vec{X}_o + \vec{x}) = \vec{Y}(\vec{X}_o) + M\vec{x} + \vec{\eta}(\vec{x}) = \vec{Y}_m, \tag{2}$$

where the Taylor expanded nonlinear term $\vec{\eta}(\vec{x})$ is much smaller than the linear term $M\vec{x}$ and finally becomes negligible once $\vec{X}_o$ is getting close to $\vec{X}$ (and so $\vec{x}$ is very small) through convergent Least-Square fitting iterations. Thus, for fitting iteration pupose, Eq. (2) can be written as

$$M\vec{x} = Y_m - Y(\vec{X}_o) = \vec{b}, \tag{3}$$

where $\vec{b}$ is the residual after a given fitting iteration. The task is to take a limited fitting iterations to achieve a small enough residual $\vec{b}$.

If one were to consider the regular Least-Square fitting, then each iterative equations would be simply

$$\vec{x} = (M^T(\vec{X}_o)M(\vec{X}_o))^{-1}M^T(\vec{X}_o)\vec{b},$$

where $\vec{X}_o$ and $\vec{b}$ are updated from the last iteration by taking $\vec{X}_o = \vec{X}_o + \vec{x}$ and $\vec{b} = Y_m - Y(\vec{X}_o)$. However, such regular Least-Square fitting cannot take care of degeneracies that ultimately cause the iteration to diverge. To overcome the degeneracy effect, we consider an SVD-enhanced Least-Square fitting by identifying and eliminating those degeneracy modes in the iteration so as to always get a convergent iteration process. In practice, by identifying the dominant SVD modes, we actually select those efficiently convergent modes for an optimized converging iterations.

Let us perform a singular value decomposition for the derivative matrix M as follows,

$$M = U\Lambda V^T, \tag{4}$$

where Λ is the singular value diagonal matrix with singular values, $\vec{\lambda}$, given in an order from a large to a small magnitude. Then the Least-Square fitting solution becomes

$$\vec{x} = V\Lambda^{-1}U^T\vec{b} = \sum_{i=1}^{n} \frac{1}{\lambda_i}(\vec{V}_i\vec{U}_i^T)\vec{b}. \tag{5}$$

Since the larger singular modes of M^TM, which is proportional to λ_i^2, are more efficient, the SVD modes are re-arranged in the order of the magnitudes of $\lambda_i^3\vec{U}_i^T\vec{b}$. The first $k < n$ modes are automatically tested for efficiency and then chosen for each iteration or sub-iteration, i.e.

$$\vec{x} = \sum_{j=1}^{k} \frac{1}{\lambda_j}(\vec{V}_j\vec{U}_j^T)\vec{b}.$$

We have successfully applied such an SVD-enhanced Least-Square fitting to the study of PEP-II optics measurement.

3 Application to PEP-II optics measurement

With the above SVD-enhanced Least Square fitting technique applied for PEP-II optics studies, we consider all quadrupole strengths and sextupole feed-downs in the lattice model as well as all BPM gains and BPM cross-plane couplings as variables, i.e., the $\vec{X}$. $\vec{Y}(\vec{X})$, the to-be-calculated responses from the lattice model, are the Local Green's functions (1), the phase advances (2) as well as the eigen-mode coupling ellipses tilt angles and axis ratios among BPMs (3). $\vec{Y}_m$, the corresponding measurable quantities, are derived from orbit measurement using a model-independent analysis (MIA) (4). Once the lattice

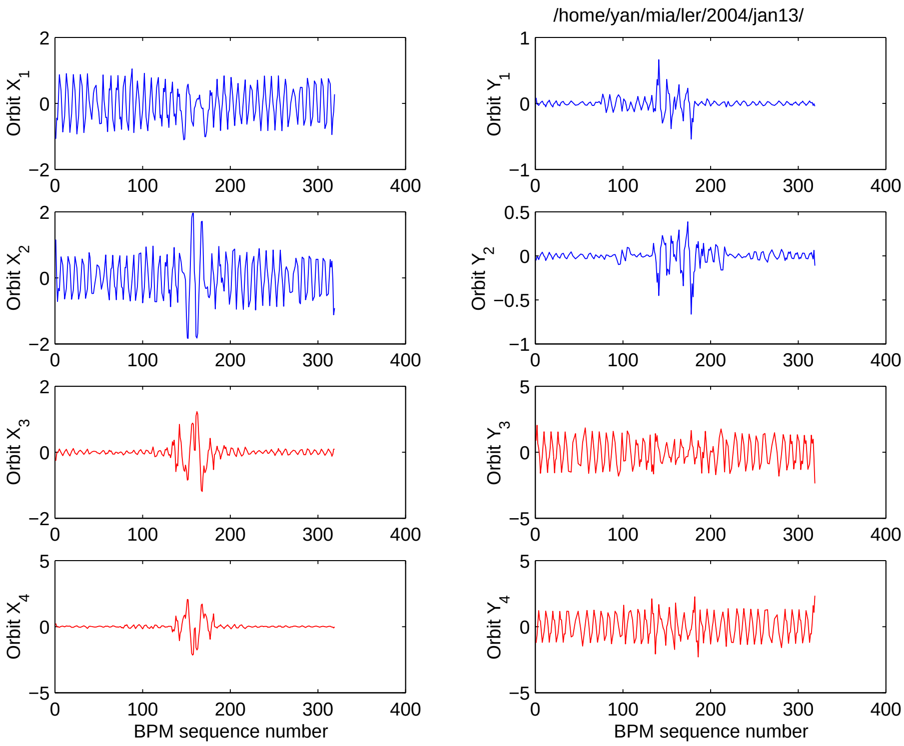

Fig. 1. *Four independent orbits extracted from PEP-II LER BPM buffer data taken on January 13, 2004. The first two orbits (x1, y1) and (x2, y2) are extracted from beam orbit excitation at the horizontal tune while the other two orbits (x3, y3) and (x4, y4) are from excitation at the vertical tune.*

model is fitted to the orbit measurement, we call this lattice model the computer virtual accelerator which matches the real accelerator in linear optics. To obtain the $\vec{Y}_m$, i.e., the Local Green's functions represented by the matrix components, $R_{12}, R_{14}, R_{32}, R_{34}$, the phase advances, and the eigen-mode coupling ellipses tilt angles and axis rations among BPM locations, we make two resonance excitations at the horizontal and vertical tunes respectively to obtain two pairs of Fourier conjugate (Cosine-like and Sine-like) orbits, one pair (two orbits) for the horizontal-tune resonance excitation and the other pair for the vertical-tune resonance excitation as shown in Figure 1 for a typical sample from PEP-II Low-Energy Ring (LER). The Y_m are derived from these 4 independent linear orbits. Since the linear optics is determined by 4 independent linear orbits, we have a complete set of constraints for the SVD-enhanced Least-Square fitting to determine the computer virtual accelerator. Figure 2 shows a typical set of PEP-II linear optics measured with this SVD-enhanced Least-Square fitting, where the beta functions of the PEP-II LER for the whole ring are calculated from the fitted virtual accelerator comparing with the original ideal lattice model. The beta beating shown are subsequently corrected by first correcting the virtual accelerator and then applied to the real accelerator (5). Figure 2 also shows the beta function and the waist shifts at IP. The other IP optics characteristics, i.e., the linear coupling parameters

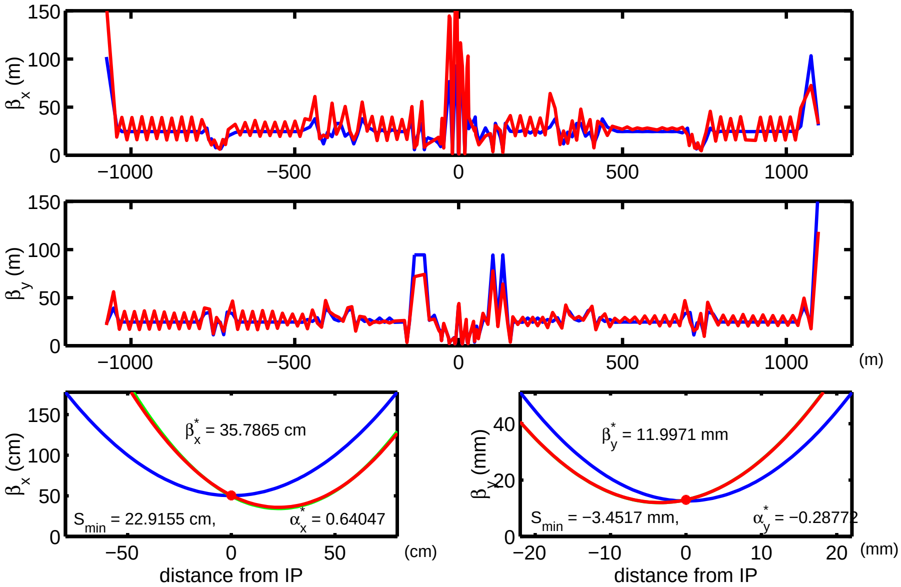

Fig. 2. *Typical plots for comparing measured beta functions from the virtual accelerator (red color) to those of the designed lattice (blue color)for PEP-II LER on January 13, 2004. The top two plots show the beta functions for the whole ring and then the bottom plots for beta functions at IP, which show the β^*'s and the waists.*

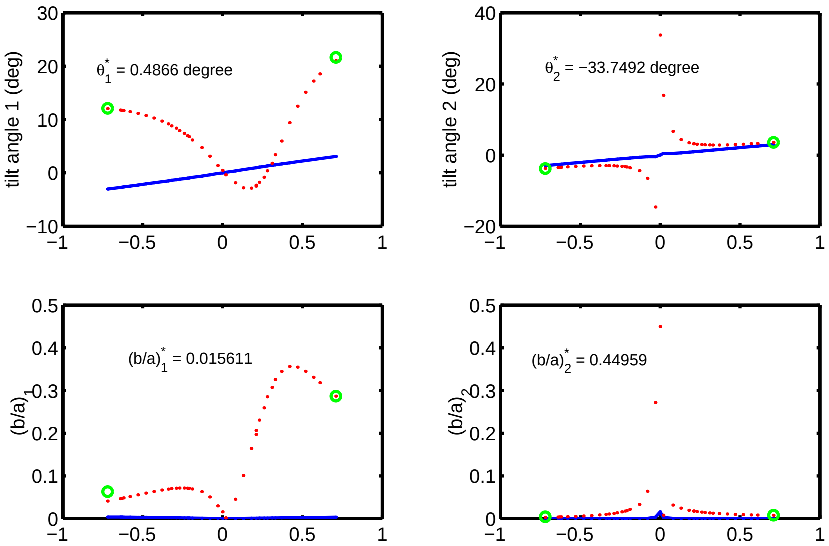

Fig. 3. *Typical plots to show IP linear coupling characteristics. The eigen-mode ellipses' tilt angles (top plots) and axis ratios (bottom plots) are compared between measurement (red) and the design lattice (blue).*

(eigen-mode ellipse tilt angles and axis ratios at IP) are shown in figure 3. These IP optics measurements provide valuable information about geometrical process of the e+-e- collisions that helps subsequent adjustment of the IP optics. The linear coupling parameters, the eigen-mode ellipses' tilt angles and axis ratios at double-view BPM locations around the whole ring are shown in figure 4.

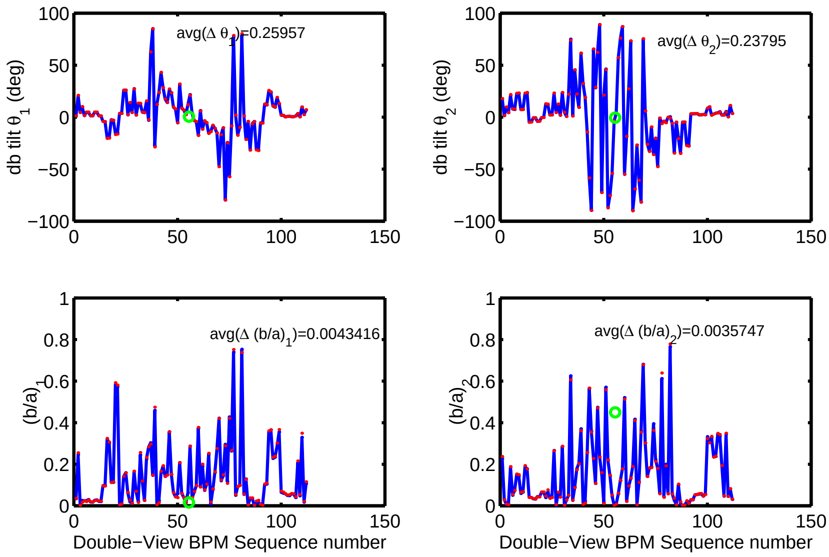

Fig. 4. *Typical plots to show linear coupling characteristics for the whole ring. The eigen-mode ellipses' tilt angles (top plots) and axis ratios (bottom plots) are compared between measurement (red) and the design lattice (blue) at all double-view BPM locations.*

4 Summary

We have developed a mature SVD-enhanced Least-Square fitting that has been successfully applied in the PEP-II linear optics studies as proved by improvement of the PEP-II optics (5). The success basically comes from three key points: (a) the SVD-enhances Least-Square fitting can avoid degeneracies and has a fairly fast convergent rate allowing for application to a fair large system such as the PEP-II ring as a whole; (b) The PEP-II ring has a reasonable amount of BPMs allowing for extracting sufficient physical quantities for fitting; and (c) the linear Green's functions between any two BPMs can provide even more fitting constraints that add significantly on the convergence.

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References

[1] J. Irwin, and Y.T. Yan, "Beamline model verification using model-independent analysis" SLAC-PUB-8515 (2000), in EPAC2000 Conference Proceedings; Y. Cai, J. Irwin, M. Sullivan, and Y.T. Yan, "Application

of Model-Independent Analysis to PEP-II rings." in PAC2001 Conference Proceedings.

[2] Y.T. Yan, Y. Cai, J. Irwin, and M. Sullivan, "Linear Optics Verification and Correction for the PEP-II with Model-Independent Analysis," SLAC-PUB-9368 (2002), in Proceedings of the 23rd Advanced Beam Dynamics Workshop on High Luminosity e+e- Colliders (2002).

[3] Y.T. Yan and Y. Cai, "Precision measurement of coupling ellipses parameters in a storage ring" SLAC-PUB-10371 (2004).

[4] J. Irwin, C.X. Wang, Y.T. Yan, et. al., "Model-Independent Beam Dynamics Analysis," Phys. Rev. Lett. **82**, 1684 (1999).

[5] Y.T. Yan, etc., "PEP-II beta beat fixes with MIA", SLAC-PUB-10369, in Proceedings of the 30th Advanced Beam Dynamics Workshop on High Luminosity e+e- Colliders (2003);