

Research Article

Dmitri L. Khokhlov*

Model of the Galaxy with Hot Dark Matter

<https://doi.org/10.1515/astro-2018-0034>

Received Oct 10, 2017; accepted Jul 26, 2018

Abstract: The model of the galaxy is considered as a structure of the baryonic matter embedded into the hot dark matter. The dark matter is supposed to come into being from the decaying matter after the epoch of structure formation. The galaxy is divided into two regions. In the inner region, the baryonic matter predominates over the hot dark matter while in the outer region, the hot dark matter predominates over the baryonic matter. The motion of the test particle is bounded in the inner region (elliptic orbit) and unbounded in the outer region (parabolic orbit). Observational constraints on the proposed model are considered from the rotation curves of the galaxies: Milky Way, M33, NGC 2366 and IC 2574.

Keywords: galaxies: kinematics and dynamics, galaxies: structure, dark matter

1 Introduction

Measurements by Planck and WMAP demonstrate that dark matter (DM) contributes nearly 85% to the matter density in the universe (Ade et al. 2016). The strongest evidence for DM comes from astrophysical observations in which the mass of the objects (galaxies, galaxy clusters) cannot be explained by the mass of the visible matter thus implying the presence of the DM (Trimble 1987). Rotation curves of spiral galaxies are the best probe for DM on galactic scales, *e.g.* (Battaner & Florido 2000) and references therein. Alternative explanation of the rotation curves of spiral galaxies is provided for by the modified Newtonian gravity (MOND) (Famaey & McGuagh 2012).

The investigation on the nature of DM has shown that large part of it should be non-baryonic (Trimble 1987). DM can be divided into three types: cold (CDM), warm (WDM) and hot (HDM). The current models assign DM a key role in the structure formation. Observational constraints favour the models of structure formation with CDM and discard those with HDM. Now, Λ CDM is generally considered the standard model of cosmology (Ostriker & Steinhardt 1995). The currently preferred CDM candidates are weakly interacting massive particles (WIMPs). WIMPs have not been identified in direct and indirect detection experiments (Bertone 2010). On large scales, the Λ CDM model is consistent with observations (Reyes et al. 2010). See, however, the discussion of the problems of the Λ CDM model (López-Corredoira 2017) and references therein. To this end, the model of the universe alternative to the Λ CDM

model can be found in Khokhlov (2011a,b, 2013). The Λ CDM model encounters the problems on galactic scales, *e.g.* (Weinberg et al. 2015) and references therein. Several solutions of the problems have been proposed in baryonic physics: gas cooling, star formation, and associated feedback, as well as in DM physics: WDM, self-interacting DM, *e.g.* (Weinberg et al. 2015) and references therein. DM contributions to the dynamics can be neglected in extended theories of gravity, *e.g.* (Stabile & Capozziello 2014).

The problems of the Λ CDM model on galactic scales were discussed by Kroupa (2012, 2015) and references therein. The validity of the Λ CDM model is challenged by two independent arguments. The first, the dual dwarf galaxy theorem claiming that there exist two types of dwarf galaxies: primordial dwarf galaxies which would be DM dominated and tidal dwarf galaxies which would have formed without a DM halo. The observational data show the same dynamical M/L values and morphological properties of both types of dwarf galaxies that falsifies the Λ CDM model. A consistency test for this conclusion comes from the significantly anisotropic distributions of satellite galaxies which orbit in the same direction around their hosting galaxies in disk-like structures which cannot be derived from DM models. The second, the action of dynamical friction due to expansive and massive DM halos must be evident in the galaxy population. The evidence for dynamical friction is poor or even absent. Kroupa (2012, 2015) came to the conclusion that DM particles do not exist, and effective gravitational physics on galactic scales and beyond ought to be non-Newtonian/non-Einsteinian as it is in the MOND framework. It is worth noting that MOND (Famaey & McGuagh 2012) allows to explain the missing mass problem in galaxies without DM but not in

Corresponding Author: Dmitri L. Khokhlov: Sumy State University (Retired), Ukraine; Email: dlkhokhl@rambler.ru

galaxy clusters. To this end, the dark dynamical effects of modified gravity theory were explored in a series of works (Bhatti et al. 2018) and references therein.

Outer disk kinematics of massive star-forming galaxies at $0.6 \leq z \leq 2.6$ from the KMOS^{3D} and SINS/zC-SINF surveys have been explored by Lang et al. (2017). The average rotation curve, extending out to ~ 4 effective radii, exhibits a significant drop in rotation velocity beyond the turnover. This outer fall-off strikingly deviates from the flat or mildly rising rotation curves of local spiral galaxies of similar masses. The results are in agreement with recent studies (Genzel et al. 2017) demonstrating that star-forming disks at high redshift are strongly baryon dominated within the disk scale. The falling rotation curve can be explained by a high mass fraction of baryons relative to the total dark matter halo in combination with a sizeable level of pressure support in the outer disk (Lang et al. 2017). The falling rotation curves of high redshift galaxies question both Λ CDM and MOND.

Demiański & Doroshkevich (2017) considered a phenomenological model of the complex DM consisting of the CDM and HDM fractions. They calculated the correlation function of the matter density for the standard Λ CDM power spectrum and the combined spectra with the dominant HDM fraction, 0.1 CDM plus 0.9 HDM. The observational data of DM dominated objects in the range of virial masses $10^6 - 10^{15} m_\odot$ favour the model with the dominant HDM fraction.

Khokhlov (2015) suggested that DM may arise in the decay of the protons falling onto the gravastar. The gravastar (Mazur & Mottola 2004; Chapline 2003) is an alternative to black hole which contains a rigid surface instead of event horizon. The decay of the proton falling onto the gravastar was considered by Chapline (2003) within the $SU(5)$ theory of grand unification (Georgi & Glashow 1974), with the dominant mode of proton decay $p \rightarrow e^+ \pi^0$ where the end products are positron and photons. Barberi & Chapline (2012) explained an excess of 511 keV radiation from positron annihilation from the centre of the Galaxy (Prantzos et al. 2011) by the decay of the protons under their falling onto Sgr A* while interpreting Sgr A* as a gravastar. This explanation was shown (Khokhlov 2014) to be consistent with the accretion rate onto Sgr A* but to predict too high luminosity of Sgr A* in comparison with the observed one; see also the further investigation of the problem (Khokhlov 2017). Therefore, the mode of the proton decay, $p \rightarrow e^+ \pi^0$, is ruled out. One should consider another mode of the proton decay. The decay of proton at the Planck scale into positron and hypothetical Planck neutrinos, $p \rightarrow e^+ 4\nu_{Pl}$, was proposed in Khokhlov (2011c). Planck neutrino can be interpreted as a HDM particle.

So, the Λ CDM model encounters the problems on galactic scales. This raises the question of an alternative to the Λ CDM model. In the Λ CDM model, the CDM exists starting the early stages of the universe and plays a key role in the structure formation. The observational data (Lang et al. 2017; Genzel et al. 2017) show the baryon dominated high redshift galaxies. It is reasonable to assume that only baryonic matter (BM) existed in the early universe while DM comes into being after the epoch of structure formation. Thus, only BM took part in the structure formation. In this case, there is no restrictions on the type of DM from the structure formation. The observational data on the density perturbations (Demiański & Doroshkevich 2017) give support to the model with the dominant HDM fraction. Assume that the HDM particles emerge from the decaying matter after the epoch of structure formation. The HDM particles may arise in the decay of the protons falling onto the gravastar (Khokhlov 2015). In the present paper, we shall consider the model of the galaxy with HDM to avoid the problems of the Λ CDM model on galactic scales.

2 The Model of the Galaxy

Consider the model of the galaxy with HDM. Assume that the HDM particles emerge from the decaying matter after the epoch of structure formation. In this case, only BM takes part in the structure formation thus there is no restrictions on the type of DM from the structure formation. Assume that the proton decays at the Planck scale into positron and four hypothetical Planck neutrinos, $p \rightarrow e^+ 4\nu_{Pl}$, as it was proposed in Khokhlov (2011c). The process may go effectively for the protons falling onto the gravastar. Planck neutrino can be interpreted as a HDM particle. It was defined as a massless particle propagating with the speed of light (Khokhlov 2011c).

Suppose that the HDM consists of Planck neutrinos. We shall consider the HDM as a fluid of uniformly distributed massless particles within the framework of the Jeans model (Peebles 1980). The Jeans length for massless particles is of order of the universe horizon. Therefore, within the universe horizon, the HDM does not form a structure due to gravity. The BM forms the structure of the galaxy which is embedded into the fluid of the HDM particles.

Consider the BM structure of the galaxy. At the radius r from the centre of BM mass, the gravitational potential is defined by the BM mass restricted within the radius r as

$$\Phi = \frac{Gm_b(< r)}{r} \quad (1)$$

where G is the Newton constant, $m_b(< r)$ is the BM mass within the radius r . The HDM mass (energy) restricted within the radius r gives an addition to the gravitational potential

$$\delta\Phi = \frac{Gm_{hdm}(< r)}{r} \quad (2)$$

where $m_{hdm}(< r)$ is the HDM mass within the radius r . The circular velocity of the test particle at the radius r is given by

$$v_c^2 = \Phi + \delta\Phi. \quad (3)$$

The radial velocity due to the HDM pressure is given by

$$v_r^2 = 2\delta\Phi. \quad (4)$$

The expression for the energy of the test particle of the unity mass takes the form

$$E = \frac{1}{2}v_r^2 + \frac{1}{2}v_c^2 - \Phi - \delta\Phi. \quad (5)$$

For energies $E < 0$, the test particle moves along the elliptic orbit (Landau & Lifshitz 1960).

Suppose that the HDM density is constant with radius, $\rho_{hdm} = \text{const}$. The HDM mass grows with radius as $m_{hdm}(< r) \propto r^3$. Suppose that, in the inner region of the galaxy, the BM predominates over the HDM while, in the outer region of the galaxy, the HDM predominates over the BM. At some radius r_0 , the HDM mass is equal to the BM mass, $m_{hdm}(< r_0) = m_b(< r_0)$. This means that the HDM perturbation of the gravitational potential is equal to the gravitational potential, $\delta\Phi = \Phi$, and the radial velocity of the test particle is equal to its circular velocity, $v_c = v_r$. The energy of the test particle is equal zero, $E = 0$, that defines the parabolic orbit of the test particle (Landau & Lifshitz 1960). As such, the HDM perturbation of the gravitational potential makes the test particle unbound. We come to the model of the galaxy, with the different behaviour of the test particle in the inner and outer regions. In the inner region $r < r_0$, the motion of the test particle is finite while, in the outer region $r \geq r_0$, infinite.

At $r > r_0$, the HDM mass is more than the BM mass, $m_{hdm}(< r) > m_b(< r)$. However, the maximum HDM perturbation of the gravitational potential is equal to the BM gravitational potential, $\delta\Phi = \Phi$. Hence, the enclosed dynamical mass is twice the enclosed baryonic mass, $m_{dyn}(< r) = 2m_b(< r)$. One can define the parabolic orbit from the minimum radius $r_{min} \geq r_0$ to infinity. The circular velocity of the test particle at the radius $r_{min} \geq r_0$ is given by Eq. (3) wherein the HDM perturbation is not defined by the total HDM mass within the radius r_{min} but taken equal to the BM gravitational potential, $\delta\Phi(r_{min}) = \Phi(r_{min})$.

Let the test particle move along the parabolic orbit from r_0 to infinity. At r_0 , the circular velocity of the test particle is equal to the radial velocity, $v_c = v_r$. At $r > r_0$, the

radial velocity decreases with radius as $v_r \propto r^{-0.5}$, and the circular velocity decreases with radius as $v_c \propto r^{-1}$. Hence, the radial velocity predominates over the circular velocity. At large distances $r \gg r_0$, the circular velocity is negligible, and the radial velocity of the test particle can be cast as

$$v_r^2 \approx 2(\Phi + \delta\Phi) = 4\Phi. \quad (6)$$

One can derive the enclosed dynamical mass in the far outer region $r \gg r_0$ through the radial velocity of the test particle.

3 Observational Constraints from the Rotation Curves of the Galaxies

We shall consider observational constraints on the proposed model from the rotation curves of the galaxies: Milky Way, M33, NGC 2366 and IC 2574.

3.1 Rotation Curve of the Milky Way

Consider the rotation curve of our Galaxy (Milky Way). The circular velocity profile is found to be approximately flat from the solar position ~ 8 kpc to $\sim 15 - 20$ kpc and then decline little with radius, e.g. (Sofue et al. 2009; Nesti & Salucci 2013; Bland-Hawthorn & Gerhard 2016). The rotation curve is modeled as the quadrature sum of the rotation curves of the individual mass components: two BM components, including the central bulge and the stellar disk, as well as the DM halo.

Modern studies based on observational data from the SDSS survey argue in favour of a two-component stellar halo in the Galaxy (Carollo et al. 2007, 2010; Beers et al. 2012). The Galactic halo comprises two broadly overlapping structural components, an inner halo $r < 10 - 15$ kpc, and an outer halo $r > 15 - 20$ kpc. These components exhibit different spatial density profiles, stellar orbits, and stellar metallicities. Recent studies, e.g. (Watkins et al. 2009; Deason et al. 2011; Sesar et al. 2011, 2013; Kafle et al. 2014; Xue et al. 2015), show that the number density of halo stellar population in the Galaxy follows a broken power-law distribution, $v \propto r^{-\alpha}$, with a variety of values for the break radius $16 - 28$ kpc, for the inner slope $1.0 - 2.6$, for the outer slope $2.7 - 4.6$.

Kinematics of the Galactic stellar halo was studied by Kafle et al. (2012), using 4,664 blue horizontal branch stars selected from the SDSS/SEGUE survey. The veloc-

ity dispersion profiles in spherical coordinates and the anisotropy profile were determined up to ~ 60 kpc. They found that the anisotropy is radially biased, $\beta = 0.5$ for $9 < r < 12$ kpc. In the range $r \approx 13 - 18$ kpc, it falls sharply becoming tangentially biased, with a dip $\beta = -1.2$ at $r = 17$ kpc. In the range $r \approx 18 - 25$ kpc, the anisotropy is roughly isotropic, $\beta \sim 0$. The anomaly feature cannot be explained as arising either from halo substructures or from accretion. An additional measurement of $\beta = 0.0^{+0.2}_{-0.4}$ at $r = 24 \pm 6$ kpc is also reported by Deason et al. (2013) in their proper motion studies of the main-sequence halo stars obtained from the Hubble Space Telescope. The feature has been confirmed by King et al. (2015) in a study of the halo with a sample of 19,859 F-type stars from a new Hectospec survey and from published SDSS surveys. Over the entire span of $6 < r < 30$ kpc, the velocity dispersion profiles in spherical coordinates and the anisotropy profile were determined. The velocity dispersion was shown to exhibit unexpected behavior in the range $15 < r < 25$ kpc where the anisotropy estimate declines from 0.5 to around -20.

Consider the HDM model of the Milky Way. Assume that the anomalous feature in the anisotropy profile (Kafle et al. 2012) corresponds to the border between the inner and outer regions in the Galaxy. Take the radius of the border $r_0 = 17$ kpc. The literature data give a variety of values for the circular velocity at 17 kpc, e.g. 195 km s^{-1} (Sofue et al. 2009), 215 km s^{-1} (Bland-Hawthorn & Gerhard 2016), 240 km s^{-1} (Nesti & Salucci 2013). Adopt the circular velocity $v(r_0) = 220 \text{ km s}^{-1}$. At $r_0 = 17$ kpc, the BM mass is equal to the HDM mass (energy), the BM circular velocity is equal to the HDM circular velocity, $v_{bm}(r_0) = v_{hdm}(r_0) = v(r_0)/\sqrt{2} = 156 \text{ km s}^{-1}$. This gives the BM mass $m_b(< r_0) = 9.6 \times 10^{10} m_\odot$. The mass models estimate the total stellar mass of the Milky Way to be $m_\star = (5 \pm 1) \times 10^{10} m_\odot$ (Bland-Hawthorn & Gerhard 2016), $m_\star = (54.3 \pm 5.7) \times 10^9 m_\odot$ (McMillan 2017), $m_\star = 8.3 \times 10^{10} m_\odot$ with an uncertainty of $\sim 5\%$ (Sofue et al. 2009), $m_\star = 0.95^{+0.24}_{-0.30} \times 10^{11} m_\odot$ (Kafle et al. 2014). The gas mass of the Milky Way is estimated to be $1.1 \times 10^{10} m_\odot$ in the HI component (including helium and metals), and $1.2 \times 10^9 m_\odot$ in the H_2 component (McMillan 2017). Thus, the BM mass of the Milky Way obtained in the HDM model is consistent with the literature data.

At $r_0 = 17$ kpc, the radial velocity of the test particle is equal to its circular velocity, $v_c = v_r$. One can estimate the radial velocity through the radial velocity dispersion as $v_r = 2\sigma_r$. The radial velocity dispersion at $r \sim 20$ kpc is $\sigma_r = 100 \text{ km s}^{-1}$ (Kafle et al. 2014). This gives the radial velocity $v_r = 200 \text{ km s}^{-1}$ which is consistent with the circular velocity at $r_0 = 17$ kpc.

In the HDM model, the test particle moves along the parabolic orbit in the outer region $r \geq r_0$. The observational data on the circular velocities in the outer region of the Milky Way, $r \geq 17$ kpc, are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are not suitable for the parabolic orbit and cannot be used to test the HDM model above 17 kpc.

Estimate the enclosed dynamical mass for $r = 60 - 100$ kpc. In the HDM model, the enclosed dynamical mass in the far outer region $r \gg r_0$ is defined by the radial velocity Eq. (6). One can estimate the radial velocity through the radial velocity dispersion as $v_r = 2\sigma_r$. The radial velocity dispersion at $r = 60$ kpc is $\sigma_r = 90 \text{ km s}^{-1}$ (Kafle et al. 2014). This gives the enclosed dynamical mass $m_{dyn}(< 60 \text{ kpc}) \approx 2.2 \times 10^{11} m_\odot$ and the enclosed BM mass $m_b(< 60 \text{ kpc}) \approx 1.1 \times 10^{11} m_\odot$. The radial velocity dispersion at $r = 80$ kpc is $\sigma_r = 80 \text{ km s}^{-1}$ (Kafle et al. 2014). This gives the enclosed dynamical mass $m_{dyn}(< 80 \text{ kpc}) \approx 2.4 \times 10^{11} m_\odot$ and the enclosed BM mass $m_b(< 80 \text{ kpc}) \approx 1.2 \times 10^{11} m_\odot$. The radial velocity dispersion at $r = 100$ kpc is $\sigma_r = 65 \text{ km s}^{-1}$ (Kafle et al. 2014). This gives the enclosed dynamical mass $m_{dyn}(< 100 \text{ kpc}) \approx 2 \times 10^{11} m_\odot$ and the enclosed BM mass $m_b(< 100 \text{ kpc}) \approx 10^{11} m_\odot$. Thus, the prediction of the HDM model in the far outer region $r \gg r_0$ is consistent with the observational data.

Estimate the HDM density from the HDM circular velocity at $r_0 = 17$ kpc. For $v_{hdm}(r_0) = 156 \text{ km s}^{-1}$, $\rho_{hdm} = 3v_{hdm}^2(r_0)/(4\pi Gr_0^2) = 3.1 \times 10^{-25} \text{ g cm}^{-3} = 4.6 \times 10^{-3} m_\odot \text{ pc}^{-3}$. This is consistent with the local DM density at the solar position, $\rho_{dm}(R_\odot) = 0.005 - 0.01 m_\odot \text{ pc}^{-3}$ (Weber & de Boer 2010), $\rho_{dm}(R_\odot) = 0.005 - 0.015 m_\odot \text{ pc}^{-3}$ (Read 2014). Since the prediction of the HDM density is consistent with the local DM density at the solar position, one can expect that the circular velocity at the solar position in the HDM model will be consistent with the observational data. Thus, the HDM model is consistent with the observational data of the Milky Way from the solar position to the far outer region $r \gg r_0$.

3.2 Rotation Curve of M33

Consider the rotation curve of M33 (Corbelli 2003; Corbelli et al. 2014). The circular velocity is steeply rising to 110 km s^{-1} at 4.5 kpc. From 4.5 kpc to 23 kpc, it is approximately flat, with the maximum value 125 km s^{-1} . The similar rotation curve is obtained in (Kam et al. 2017). Ciardullo et al. (2004) derived rotation velocities of the planetary nebulae in M33 out to 10 kpc, with the mean rotation velocity being smaller by $\sim 10 \text{ km s}^{-1}$ than that in (Corbelli 2003). The rotation curve is modeled with the use of the mass com-

ponents: two BM components, including the stellar and gaseous disks, as well as the DM halo.

Consider the HDM model of M33. There is a feature at ~ 7.5 kpc in the curve of the position angle (beginning of the decrease) of M33 corresponding to the warp of the disc (Corbelli et al. 2014). Assume that the feature gives the radius of the border between the inner and outer regions in M33, $r_0 = 7.5$ kpc. The circular velocity at $r_0 = 7.5$ kpc is $v(r_0) = 110 \text{ km s}^{-1}$ (Corbelli et al. 2014). At $r_0 = 7.5$ kpc, the BM mass is equal to the HDM mass (energy), the BM circular velocity is equal to the HDM circular velocity, $v_{bm}(r_0) = v_{hdm}(r_0) = v(r_0)/\sqrt{2} = 78 \text{ km s}^{-1}$. This gives the BM mass $m_b(< r_0) = 1.05 \times 10^{10} m_\odot$. In (Kam et al. 2017) the circular velocity at $r_0 = 7.5$ kpc is $v(r_0) = 108 \text{ km s}^{-1}$. Then, the BM circular velocity and the HDM circular velocity are $v_{bm}(r_0) = v_{hdm}(r_0) = v(r_0)/\sqrt{2} = 76 \text{ km s}^{-1}$. This gives the BM mass $m_b(< r_0) = 1.0 \times 10^{10} m_\odot$. In (Ciardullo et al. 2004) the circular velocity at $r_0 = 7.5$ kpc is $v(r_0) = 100 \text{ km s}^{-1}$. Then, the BM circular velocity and the HDM circular velocity are $v_{bm}(r_0) = v_{hdm}(r_0) = v(r_0)/\sqrt{2} = 71 \text{ km s}^{-1}$. This gives the BM mass $m_b(< r_0) = 8.7 \times 10^9 m_\odot$. The total stellar mass of M33 is estimated to be $(3 - 6) \times 10^9 m_\odot$, and the total gas mass (HI+H₂+He) $\sim 3.2 \times 10^9 m_\odot$ (Corbelli 2003). Thus, the BM mass within 7.5 kpc predicted by the HDM model is consistent with the sum of the stellar and gas masses obtained from observations, neglecting the BM mass beyond 7.5 kpc.

Estimate the HDM density from the HDM circular velocity at $r_0 = 7.5$ kpc. For $v_{hdm}(r_0) = 78 \text{ km s}^{-1}$, $\rho_{hdm}(r_0) = 3v_{hdm}^2(r_0)/(4\pi Gr_0^2) = 4.0 \times 10^{-25} \text{ g cm}^{-3} = 5.9 \times 10^{-3} m_\odot \text{ pc}^{-3}$. For $v_{hdm}(r_0) = 76 \text{ km s}^{-1}$, $\rho_{hdm}(r_0) = 3.8 \times 10^{-25} \text{ g cm}^{-3} = 5.7 \times 10^{-3} m_\odot \text{ pc}^{-3}$. For $v_{hdm}(r_0) = 71 \text{ km s}^{-1}$, $\rho_{hdm}(r_0) = 3.3 \times 10^{-25} \text{ g cm}^{-3} = 4.9 \times 10^{-3} m_\odot \text{ pc}^{-3}$. This is consistent with the value in our Galaxy.

In the HDM model, the test particle moves along the parabolic orbit in the outer region $r \geq r_0$. The observational data on the circular velocities in the outer region of M33, $r \geq 7.5$ kpc, are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are not suitable for the parabolic orbit and cannot be used to test the HDM model above 7.5 kpc.

Estimate the enclosed dynamical mass in the far outer region $r \gg r_0$ with the use of the observational data on the dwarf galaxy AndXXII, the distance to M33 is 59^{+21}_{-14} kpc, the velocity relative to M33 is 50 km s^{-1} (Chapman et al. 2013). The enclosed dynamical mass in the far outer region $r \gg r_0$ is defined by the radial velocity Eq. (6). Calculation gives the enclosed dynamical mass $m_{dyn}(< 59 \text{ kpc}) \approx 1.7 \times 10^{10} m_\odot$ and the enclosed BM mass $m_b(< 59 \text{ kpc}) \approx 8.6 \times 10^9 m_\odot$. Thus, the prediction of the HDM model in the far

outer region $r \gg r_0$ is consistent with the observational data.

3.3 Rotation Curve of NGC 2366

Consider the rotation curve of NGC 2366 (Oh et al. 2011). Several methods have been used to derive the rotation curve of NGC 2366. Oh et al. (2011) derived the rotation curve of NGC 2366 from the bulk velocity field. The circular velocity is rising to 45 km s^{-1} at ~ 2 kpc, to 55 km s^{-1} at ~ 4 kpc, then is approximately flat. For comparison, they presented the rotation curves derived from the Hermite and intensity-weighted mean velocity fields which give smaller circular velocities. Also, the rotation curve of NGC 2366 was considered in (Hunter et al. 2001; van Eymeren et al. 2009), with the circular velocities derived from the Hermite velocity field (van Eymeren et al. 2009) and from the intensity-weighted mean velocity field (Hunter et al. 2001) being smaller than those derived from the bulk velocity field (Oh et al. 2011). The rotation curve is modeled with the use of the mass components: two BM components, including the stellar and gaseous disks, as well as the DM halo.

Consider the HDM model of NGC 2366. There is a feature at ~ 2.5 kpc in the curve of the position angle (beginning of the decrease) of NGC 2366 (Oh et al. 2011). Assume that the feature gives the radius of the border between the inner and outer regions in NGC 2366, $r_0 = 2.5$ kpc. At $r = 2.5$ kpc, the circular velocity for the stellar disk is 15 km s^{-1} , the circular velocity for the gaseous disk is 21 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 26 km s^{-1} . At $r_0 = 2.5$ kpc, the HDM mass (energy) is equal to the BM mass. Adopt the HDM circular velocity equal to the BM circular velocity 26 km s^{-1} . Then, the total circular velocity is 37 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 47 km s^{-1} . The Hermite rotation curve (Oh et al. 2011) gives the circular velocity 45 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 43 km s^{-1} . The Hermite rotation curve (van Eymeren et al. 2009) gives the circular velocity 41 km s^{-1} . The intensity-weighted mean rotation curve (Hunter et al. 2001) gives the circular velocity 41 km s^{-1} .

Estimate the HDM density from the HDM circular velocity at $r_0 = 2.5$ kpc. For $v_{hdm}(r_0) = 26 \text{ km s}^{-1}$, $\rho_{hdm}(r_0) = 3v_{hdm}^2(r_0)/(4\pi Gr_0^2) = 4.1 \times 10^{-25} \text{ g cm}^{-3} = 6.0 \times 10^{-3} m_\odot \text{ pc}^{-3}$. This is consistent with the value in our Galaxy.

One can see in the stellar and gaseous rotation curves of NGC 2366 that most of the BM mass in NGC 2366 lies

out of the radius $r_0 = 2.5$ kpc, within the radius $r_b \sim 6$ kpc. In the HDM model, the test particle moves along the parabolic orbit in the outer region $r \geq r_0$. One can define the radius r_{min} of the parabolic orbit in the range from r_0 to r_b , 2.5–6 kpc. The circular velocity of the test particle at the radius $r_{min} = 2.5–6$ kpc is given by Eq. (3) in which the HDM perturbation is taken equal to the BM gravitational potential. Hence, the HDM circular velocity at $r = 2.5–6$ kpc should be taken equal to the BM circular velocity at the same radius. It is expected that, in the range 2.5–6 kpc, the total rotation curve follows the BM rotation curve, with the enclosed dynamical mass is twice the enclosed baryonic mass. The observational data on the circular velocities in the outer region $r \geq r_0$ of NGC 2366 are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are suitable for the radius $r_{min} = 2.5–6$ kpc of the parabolic orbit thus may be used to test the HDM model in the range 2.5–6 kpc. Above 6 kpc, the data cannot be used to test the HDM model.

At $r = 4$ kpc, the circular velocity for the stellar disk is 16 km s^{-1} , the circular velocity for the gaseous disk is 26 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 30.5 km s^{-1} . Adopt the HDM circular velocity equal to the BM circular velocity 30.5 km s^{-1} . Then, the total circular velocity is 43 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 55 km s^{-1} . The Hermite rotation curve (Oh et al. 2011) gives the circular velocity 54 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 52 km s^{-1} . The Hermite rotation curve (van Eymeren et al. 2009) gives the circular velocity 49 km s^{-1} . The intensity-weighted mean rotation curve (Hunter et al. 2001) gives the circular velocity 48 km s^{-1} .

At $r = 6$ kpc, the circular velocity for the stellar disk is 15 km s^{-1} , the circular velocity for the gaseous disk is 26 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 30 km s^{-1} . Adopt the HDM circular velocity equal to the BM circular velocity 30 km s^{-1} . Then, the total circular velocity is 42 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 57 km s^{-1} . The Hermite rotation curve (Oh et al. 2011) gives the circular velocity 45 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 43 km s^{-1} . The Hermite rotation curve (van Eymeren et al. 2009) gives the circular velocity 46 km s^{-1} . The intensity-weighted mean rotation curve (Hunter et al. 2001) gives the circular velocity 46 km s^{-1} .

One can see that, in the region 2.5–6 kpc, the total rotation curve of NGC 2366 approximately follows the BM rotation curve. However, the circular velocities derived from the observations of NGC 2366 are higher than those pre-

dicted by the HDM model, by $10–14 \text{ km s}^{-1}$ for the bulk rotation curve (Oh et al. 2011), by $3–11 \text{ km s}^{-1}$ for the Hermite rotation curve (Oh et al. 2011), by $1–9 \text{ km s}^{-1}$ for the intensity-weighted mean rotation curve (Oh et al. 2011), by $4–6 \text{ km s}^{-1}$ for the Hermite rotation curve (van Eymeren et al. 2009), by $4–5 \text{ km s}^{-1}$ for the intensity-weighted mean rotation curve (Hunter et al. 2001).

In observations, the circular velocities are derived from the line of sight velocities through the inclination, $1/\sin i$, and position angle, $1/\cos \phi$. The inclination and position angle determined from the HI kinematics in NGC 2366 are not the same as those found from the optical and HI morphologies (Hunter et al. 2001). The HI kinematics gives an inclination of 65° and a position angle of 46° (Hunter et al. 2001); 63° and 43° (van Eymeren et al. 2009); 63.8° and 39.8° (Oh et al. 2011). The outer isophote of the stars gives the values 72° and 32.5° , respectively (Hunter et al. 2001). The HI morphology gives the values 73° and 23.5° , respectively (Hunter et al. 2001). These results indicate that either the HI distribution or the kinematics deviate from axial symmetry. That is, the HI disk must be either elongated or warped or the motions must be non-circular. Using the inclination and position angle of the optical and HI morphologies instead of those of the HI kinematics will give the smaller circular velocities that may eliminate the discrepancy between the predictions of the HDM model and the observational data.

3.4 Rotation Curve of IC 2574

Consider the rotation curve of IC 2574 (Oh et al. 2011). Several methods have been used to derive the rotation curve of IC 2574. Oh et al. (2011) derived the rotation curve of IC 2574 from the bulk velocity field. The circular velocity is rising to 80 km s^{-1} at 11 kpc. For comparison, they presented the rotation curves derived from the Hermite, intensity-weighted mean, single Gaussian and peak velocity fields. The circular velocities of the Hermite, single Gaussian and peak rotation curves are comparable to those of the bulk rotation curve. The circular velocities of the intensity-weighted mean rotation curve are smaller than those of the bulk rotation curve. Also, the rotation curve of NGC 2366, derived from the intensity-weighted mean velocity field, was considered in (Martimbeau et al. 1994), with the circular velocities being comparable to those of the intensity-weighted mean rotation curve (Oh et al. 2011). The rotation curve is modeled with the use of the mass components: two BM components, including the stellar and gaseous disks, as well as the DM halo.

Consider the HDM model of IC 2574. There is a feature at ~ 1.5 kpc in the curve of the inclination (beginning of the decrease) of IC 2574 (Oh et al. 2011). Assume that the feature gives the radius of the border between the inner and outer regions in IC 2574, $r_0 = 1.5$ kpc. At $r = 1.5$ kpc, the circular velocity for the stellar disk is 13 km s^{-1} , the circular velocity for the gaseous disk is 4 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 14 km s^{-1} . At $r_0 = 1.5$ kpc, the HDM mass (energy) is equal to the BM mass. Adopt the HDM circular velocity equal to the BM circular velocity 14 km s^{-1} . Then, the total circular velocity is 20 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 22 km s^{-1} . The Hermite, intensity-weighted mean, single Gaussian and peak rotation curves (Oh et al. 2011) give the circular velocity 16 km s^{-1} . The intensity-weighted mean rotation curve Martimbeau et al. (1994) gives the circular velocity 16 km s^{-1} .

Estimate the HDM density from the HDM circular velocity at $r_0 = 1.5$ kpc. For $v_{hdm}(r_0) = 14 \text{ km s}^{-1}$, $\rho_{hdm}(r_0) = 3v_{hdm}^2(r_0)/(4\pi Gr_0^2) = 3.3 \times 10^{-25} \text{ g cm}^{-3} = 4.9 \times 10^{-3} m_\odot \text{ pc}^{-3}$. This is consistent with the value in our Galaxy.

One can see in the stellar and gaseous rotation curves of IC 2574 that most of the BM mass in IC 2574 lies out of the radius $r_0 = 1.5$ kpc, within the radius $r_b \sim 9$ kpc. In the HDM model, the test particle moves along the parabolic orbit in the outer region $r \geq r_0$. One can define the radius r_{min} of the parabolic orbit in the range from r_0 to r_b , $1.5 - 9$ kpc. The circular velocity of the test particle at the radius $r_{min} = 1.5 - 9$ kpc is given by Eq. (3) in which the HDM perturbation is taken equal to the BM gravitational potential. Hence, the HDM circular velocity at $r = 1.5 - 9$ kpc should be taken equal to the BM circular velocity at the same radius. It is expected that, in the range $1.5 - 9$ kpc, the total rotation curve follows the BM rotation curve, with the enclosed dynamical mass is twice the enclosed baryonic mass. The observational data on the circular velocities in the outer region $r \geq r_0$ of IC 2574 are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are suitable for the radius $r_{min} = 1.5 - 9$ kpc of the parabolic orbit thus may be used to test the HDM model in the range $1.5 - 9$ kpc. Above 9 kpc, the data cannot be used to test the HDM model.

At $r = 3$ kpc, the circular velocity for the stellar disk is 18 km s^{-1} , the circular velocity for the gaseous disk is 10 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 21 km s^{-1} . Adopt the HDM circular velocity equal to the BM circular velocity 21 km s^{-1} . Then, the total circular velocity is 29 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 32 km s^{-1} . The Hermite, single Gaussian and peak rotation curves (Oh et al. 2011) give the

circular velocity 28 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 26 km s^{-1} . The intensity-weighted mean rotation curve (Martimbeau et al. 1994) gives the circular velocity 31 km s^{-1} .

At $r = 6$ kpc, the circular velocity for the stellar disk is 26 km s^{-1} , the circular velocity for the gaseous disk is 26 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 37 km s^{-1} . Adopt the HDM circular velocity equal to the BM circular velocity 37 km s^{-1} . Then, the total circular velocity is 52 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 55 km s^{-1} . The Hermite, single Gaussian and peak rotation curves (Oh et al. 2011) give the circular velocity 53 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 45 km s^{-1} . The intensity-weighted mean rotation curve (Martimbeau et al. 1994) gives the circular velocity 45 km s^{-1} .

At $r = 9$ kpc, the circular velocity for the stellar disk is 24 km s^{-1} , the circular velocity for the gaseous disk is 40 km s^{-1} (Oh et al. 2011). Calculation gives the BM circular velocity 47 km s^{-1} . Adopt the HDM circular velocity equal to the BM circular velocity 47 km s^{-1} . Then, the total circular velocity is 66 km s^{-1} . The bulk rotation curve (Oh et al. 2011) gives the circular velocity 72 km s^{-1} . The Hermite, single Gaussian and peak rotation curves (Oh et al. 2011) give the circular velocity 73 km s^{-1} . The intensity-weighted mean rotation curve (Oh et al. 2011) gives the circular velocity 66 km s^{-1} . The intensity-weighted mean rotation curve (Martimbeau et al. 1994) gives the circular velocity 65 km s^{-1} .

One can see that, in the region $1.5 - 9$ kpc, the total rotation curve of IC 2574 approximately follows the BM rotation curve. The circular velocities predicted by the HDM model are roughly in agreement with the observational data. The bulk rotation curve (Oh et al. 2011) gives the largest circular velocities, by $2 - 6 \text{ km s}^{-1}$ higher than those predicted by the HDM model. The intensity-weighted mean rotation curve (Oh et al. 2011) gives the smallest circular velocities, by $0 - 7 \text{ km s}^{-1}$ lower than those predicted by the HDM model.

4 Conclusion

We have considered the model of the galaxy with HDM to avoid the problems of the Λ CDM model on galactic scales. The HDM particles are assumed to emerge from the decaying matter after the epoch of structure formation. In this case, only BM takes part in the structure formation, and there is no restrictions on the type of DM from the structure

formation. Such a scenario is in agreement with the observational data (Lang et al. 2017; Genzel et al. 2017) showing the baryon dominated high redshift galaxies. Also, the observational data on the density perturbations (Demiański & Doroshkevich 2017) give support to the model with the dominant HDM fraction. We have considered the decay of the proton at the Planck scale into positron and four hypothetical Planck neutrinos. The process may go effectively for the protons falling onto the gravastar. Planck neutrino has been interpreted as a HDM particle.

In the HDM model, the galaxy is divided into two regions. In the inner region, the BM predominates over the HDM while, in the outer region, the HDM predominates over the BM. The border between the regions is at some radius r_0 where the HDM mass is equal to the BM mass. The motion of the test particle is bounded in the inner region (elliptic orbit) and unbounded in the outer region (parabolic orbit). The HDM density is supposed to be constant with radius, $\rho_{hdm} = \text{const}$, and the HDM mass grows with radius as $m_{hdm}(< r) \propto r^3$. At $r \geq r_0$, the maximum HDM perturbation of the gravitational potential is equal to the BM gravitational potential, $\delta\Phi = \Phi$. Hence, the enclosed dynamical mass is twice the enclosed BM mass, $m_{dyn}(< r) = 2m_b(< r)$. In the far outer region $r \gg r_0$, the enclosed dynamical mass is defined by the radial velocity.

Observational constraints on the proposed model have been considered from the rotation curves of the galaxies: Milky Way, M33, NGC 2366 and IC 2574. The radius r_0 in the galaxies has been determined through the features in the anisotropy profile (Milky Way), in the curve of the position angle (M33, NGC 2366), in the curve of the inclination (IC 2574). The HDM density has been estimated from the HDM circular velocity at r_0 . The HDM density in the four galaxies under study is round the same, consistent with the local DM density at the solar position.

The BM mass in the Milky Way and M33 lies mostly within the radius r_0 . The predictions of the HDM model for the BM mass within the radius r_0 in the Milky Way and M33 are in agreement with the literature data. The observational data on the circular velocities in the outer region $r \geq r_0$ of the Milky Way and M33 are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are not suitable for the parabolic orbit and cannot be used to test the HDM model in the outer region $r \geq r_0$. The enclosed BM mass for $r \gg r_0$ has been estimated in the Milky Way and M33, through the radial velocity dispersion at $r = 60 - 100$ kpc in the Milky Way, and through the radial velocity of AndXXII relative to M33. The obtained BM masses in the Milky Way and M33 are in agreement with the observational data.

The BM mass in NGC 2366 and IC 2574 lies mostly out of the radius r_0 , within some radius r_b . One can define the radius r_{min} of the parabolic orbit in the range from r_0 to r_b . It is expected that, in the range from r_0 to r_b , the total rotation curve follows the BM rotation curve, with the enclosed dynamical mass is twice the enclosed baryonic mass. The observational data on the circular velocities in the outer region $r \geq r_0$ of NGC 2366 and IC 2574 are obtained from the line of sight velocities while assuming the elliptic orbit of the test particle. The data are suitable for the radius r_{min} of the parabolic orbit in the range from r_0 to r_b thus may be used to test the HDM model in this range. Above r_b , the data cannot be used to test the HDM model. The total rotation curves of NGC 2366 and IC 2574 in the range from r_0 to r_b approximately follow the BM rotation curves. The predictions of the HDM model for IC 2574 are in agreement with the observational data. The circular velocities derived from the observations of NGC 2366 are higher than those predicted by the HDM model. A possible reason is the use of the inclination and position angle of the HI kinematics which are different to those of the optical and HI morphologies.

References

- Ade, P. A. R., Aghanim, N., Arnaud, M., Ashdown, M., Aumont, J., Baccigalupi C. et al. 2016, A&A, 594, A13.
- Barbieri, J., Chapline, G. 2012, Phys. Lett. B, 709, 114–117.
- Battaner, E., Florido, E. 2000, Fund. Cosmic Phys., 21, 1–154.
- Beers, T. C., Carollo, D., Ivezić, Ž., An, D., Chiba, M., Norris, J. E. et al. 2012, ApJ, 746, 34.
- Bertone, G. 2010, Nature, 468, 389–393.
- Bhatti, M. Z., Sharif, M., Yousaf, Z., Ilyas, M. 2018, Int. J. Mod. Phys. D, 27, 1850044.
- Bland-Hawthorn, J., Gerhard, O. 2016, Annu. Rev. Astron. Astrophys., 54, 529–596.
- Carollo, D., Beers, T. C., Lee, Y. S., Chiba, M., Norris, J. E., Wilhelm, R. et al. 2007, Nature, 450, 1020–1025.
- Carollo, D., Beers, T. C., Chiba, M., Norris, J. E., Freeman, K. C., Lee, Y. S. 2010, ApJ, 712, 692–727.
- Chapline, G. 2003, Int. J. Mod. Phys. A, 18, 3587–3590.
- Chapman, S. C., Widrow, L., Collins, M. L. M., Dubinski, J., Ibata, R. A., Rich, M. et al. 2013, MNRAS, 430, 37–49.
- Ciardullo, R., Durrell, P. R., Laychak, M. B., Herrmann, K. A., Moody, K., Jacoby, G. H., Feldmeier, J. J. 2004, ApJ, 614, 167–185.
- Corbelli, E. 2003, MNRAS, 342, 199–207.
- Corbelli, E., Thilker, D., Zibetti, S., Giovanardi, C., Salucci, P. 2014, A&A, 572, A23.
- Deason, A. J., Belokurov, V., Evans, N. W. 2011, MNRAS, 416, 2903–2915.
- Deason, A. J., Van der Marel, R. P., Guhathakurta, P., Sohn, S. T., Brown, T. M. 2013, ApJ, 766, 24.
- Demiański, M., Doroshkevich, A. 2017, arXiv:1701.03474.

- Famaey, B., McGuagh, S. 2012, *Living Rev. Relativ.*, 15, 10.
- Genzel, R., Schreiber, N. M., Förster, Übler, H., Lang, P., Naab, T., Bender, R. et al. 2017, *Nature*, 543, 397–401.
- Georgi, H., Glashow, S. 1974, *Phys. Rev. Lett.*, 32, 438–441.
- Hunter, D. A., Elmegreen, B. G., van Woerden, H. 2001, *ApJ*, 556, 773–800.
- Kafle, P. R., Sharma, S., Lewis, G. F., Bland-Hawthorn, J. 2012, *ApJ*, 761, 98.
- Kafle, P. R., Sharma, S., Lewis, G. F., Bland-Hawthorn, J. 2014, *ApJ*, 794, 59.
- Kam, S. Z., Carignan, C., Chemin, L., Foster, T., Elson, E., Jarrett, T. H. 2017, *AJ*, 154, 41.
- Khokhlov, D. L. 2011a, *Ap&SS*, 333, 209–212.
- Khokhlov, D. L. 2011b, *Ap&SS*, 335, 577–580.
- Khokhlov, D. L. 2011c, *Open Astron. J.*, 4 SI 1, 151–153.
- Khokhlov, D. L. 2013, *Ap&SS*, 343, 787–790.
- Khokhlov, D. L. 2014, *Phys. Lett. B*, 729, 1–2.
- Khokhlov, D. L. 2015, *Ap&SS*, 360, 27.
- Khokhlov, D. L. 2017, *Int. J. Mod. Phys. Appl.*, 4, 8–11.
- King III, C., Brown, W. R., Geller, M. J., Kenyon, S. J. 2015, *ApJ*, 813, 89.
- Kroupa, P. 2012, *PASA*, 29, 395–433.
- Kroupa, P. 2015, *Can. J. Phys.*, 93, 169–202.
- Landau, L., and Lifshitz, Ye. 1960, *Mechanics*, Pergamon Press, Oxford.
- Lang, P., Förster Schreiber, N. M., Genzel, R., Wuyts, S., Wisnioski, E., Beifiori, A. et al. 2017, *ApJ*, 840, 92.
- López-Corredoira, M. 2017, *Found. Phys.*, 47, 711–768.
- Martimbeau, N., Carignan, C., Roy, J.-R. 1994, *AJ*, 107, 543–554.
- Mazur, P., Mottola, E. 2004, *Proc. Nat. Acad. Sci.*, 101, 9545–9550.
- McMillan, P. J. 2017, *MNRAS*, 465, 76–94.
- Nesti, F., Salucci, P. J. 2013, *J. Cosm. Astropart. Phys.*, 07, 016.
- Oh, S.-H., de Blok, W. J. G., Brinks, E., Walter, F., Kennicutt, R. C., Jr. 2011, *AJ*, 141, 193.
- Ostriker, J. P., Steinhardt, P. J. 1995, *Nature*, 377, 600–602.
- Peebles, P. J. E. 1980, *The large-scale structure of the universe*, Princeton University Press, Princeton.
- Prantzos, N., Boehm, C., Bykov, A. M., Diehl, R., Ferrière, K., Gues-soum, N. et al. 2011, *Rev. Mod. Phys.*, 83, 1001–1056.
- Read, J. I. 2014, *J. Phys. G: Nucl. Part. Phys.*, 41, 063101.
- Reyes, R., Mandelbaum, R., Seljak, U., Baldauf, T., Gunn, J. E., Lom-briser, L., Smith, R. E. 2010, *Nature*, 464, 256–258.
- Stabile, A., Capozziello, S. 2014, *Galaxies*, 2, 520–576.
- Sesar, B., Jurić, M., Ivezić, Ž. 2011, *ApJ*, 731, 4.
- Sesar, B., Ivezić, Ž., Stuart, J. S., Morgan, D. M., Becker, A. C., Sharma, S. et al. 2013, *AJ*, 146, 21.
- Sofue, Y., Honma, M., Omodaka, T. 2009, *PASJ*, 61, 227–236.
- Trimble, V. 1987, *Ann. Rev. Astron. Astrophys.*, 25, 425–472.
- van Eymeren, J., Trachternach, C., Koribalski, B. S., Dettmar, R.-J. 2009, *A&A*, 505, 1–20.
- Watkins, L. L., Evans, N. W., Belokurov, V., Smith, M. C., Hewett, P. C., Bramich, D. M. et al. 2009, *MNRAS*, 398, 1757–1770.
- Weber, M., de Boer, W. 2010, *A&A*, 509, 25.
- Weinberg, D. H., Bullock, J. S., Governato, F., de Naray, R. K., Peter, A. H. G. 2015, *Proc. Nat. Acad. Sci.*, 112, 12249–12255.
- Xue, X.-X., Rix, H.-W., Ma, Z., Morrison, H., Bovy, J., Sesar, B., Janesh, W. 2015, *ApJ*, 809, 144.