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Tracking, Tracker Layout and RICH Performance

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Abstract

We first examine the role played by station T11 and by the y layers in stations T1, T2 and T10 on the PID performance of both RICH 1 and RICH 2. We study the conditions under which station T11 and all y layers can be removed from the tracking setup, keeping unchanged the particle identification quality, with special emphasis on performance at high momentum. The performance has been assessed both through the angular resolution on track direction, and the particle misidentification rates after RICH reconstruction. The two methods lead to similar conclusions. The role of station T1 (as a whole) on particle identification performance is also considered within a sample of events including pile–up effects and realistic material for the VELO. Issues related to track curvature generated by the fringe field in the RICH 2 gas volume are also considered.

1 Introduction

Reconstruction of the Cherenkov angle from photon hits recorded in the RICH detectors requires a measurement of the charged track direction. The baseline (Technical Proposal, TP) set of tracking stations included horizontal strips [1] in T1 and T2, for RICH 1 and in T10 and T11 for RICH 2. These were foreseen to provide a precise measurement of the y–coordinates at the entry and exit of both RICH 1 and RICH 2. Equipped with horizontal strips, the tracking system provides an accuracy for both coordinates and slopes which is, indeed, of comparable quality in the vertical and horizontal transverse directions, as will be illustrated below.

The track–finding algorithms [2, 3] do not use these horizontal strips. Furthermore, no coordinate measurement in T11 is used, either for seeding [4] or for track following [2].

The tracking layers with long horizontal strips (T10, T11) are difficult to construct and suffer from a high occupancy. Due to lack of space in RICH 1 region, there is also a strong interest in reducing the length (in z) of T1 and T2, and elimination of the horizontal-strip layers would help here.

Therefore, a study has been made in order to assess the precise influence on RICH performance of station T11 as a whole, and of the y–layers in tracking stations T1, T2 and T10. The information provided by station T1 in order to improve the track direction at entry of RICH 1 will also be considered ; because of a likely redundancy between T1 and VELO information, one could expect that the improvement provided by T1 on top of VELO information is small enough that it could also be removed from the tracking setup, as far as the RICH performance is concerned[1].

The present note is divided into four Parts. In the first, we examine the role of station T11 in giving the appropriate accuracy on charged track direction in RICH 2 ; its removal is considered together with an increase of the radiator depth which is possible because a 33 cm length along the beam axis becomes free under this assumption. The PID performance of RICH 2 [5] is then re–examined in some detail to assess the influence of this.

Some other questions relating to RICH 2 are also considered. They are related to the fact that charged particles have bent trajectories inside the gas volume due to the non–negligible magnetic fringe field at this location. It is shown that for low and medium momenta (below 80 GeV) the bending effects are significant. These can be accounted for at the photon generation level, and also in the ring reconstruction at the expense of an additional error contribution (curvature error), which will anyway have to be implemented in order to reconstruct the (future) RICH 2 data.

In Part II, we focus on the influence of the y layers in station T10 on the PID performance of RICH 2. The effect of replacing these by inclined layers of small stereo angle is considered ; we also examine whether removing the stack of y layers in T10 degrades significantly the RICH 2 PID. For this purpose, the dependence on the stereo angle of the standard stereo layers in T7 to T10 is also studied.

In Part III, we examine the contribution of the y layers in stations T1 and T2 to the PID performance of RICH 1.

Finally, Part IV is devoted to examining the RICH system identification performance

[1] A final decision should take into account other effects like, for instance, a possible loss in K_S reconstruction efficiency.

on a data set obtained with a realistic material budget and with pile up conditions. In this case, we address within realistic conditions (especially for the VELO), the actual contribution of station T1 to the RICH particle identification performance.

Part I : Effects of T11 on RICH 2 Performance

For this study, we have used a data sample of 2500 $B \to \pi^+\pi^-$ events generated assuming an Al–Be beam pipe. The VELO design is still that given in the TP [1] ($\simeq$ 10% radiation length thickness, almost half the thickness now anticipated in [6]) and all tracking stations have been assumed each of 1.6 % X_0. The material budget of RICH 1 is unchanged with respect to TDR [5], while those of RICH 2 has been updated in order to account for a thicker exit window (4.48% X_0) which practically accounts also for the material of the mirror support.

Therefore, background conditions are more favourable compared to previous studies where a stainless steel beam pipe was assumed, which generated many more secondary particles throughout the detector. As the influence of T11 on RICH 2 performance is only a problem of accuracy (T11 is not used by in the tracking), the changed background conditions should not in principle influence the result [5, 8].

2 True Track Trajectory inside RICH 2

The RICH simulation as used in the former TDR studies [5] contained an approximation in the photon generation which was necessary to refine for the present study. The (true) entry and exit points of the particle trajectory in the RICH volume, as calculated by GEANT, are stored in a SICB data bank (the "RIRW" bank). In the simulation, Cerenkov photons are generated assuming the track trajectory to be a straight line joining these two points, whereas, in reality, this trajectory curves in the fringe field of the magnet.

As the fringe field inside RICH 1 is very low (about 10 G), this is enough to define properly the track direction inside the radiator volume ; indeed, for tracks of physics interest, this direction is approximated with good precision by the the straight line joining the entry and exit points recorded in the RIRW bank.

However, inside the RICH 2 volume, as the fringe field is relatively high (about 350 G), the straight line approximation is somewhat crude for low and medium momentum tracks. Therefore, using the straight line approximation for the charged track direction introduces a bias in the Cerenkov photon generation because of the curvature of the actual trajectory ; additionally, this curvature provides a new type of error in Cerenkov ring reconstruction which has not yet been accounted for in the RICH PID [5]. Therefore, there is some interest in finding a good approximation to the exact track trajectory inside the RICH 2 gas volume.

Beside Monte Carlo truth information already quoted (entry and exit point coordinates from RICH 2, the momentum and charge), the other useful information which can contribute in determining at best the true track trajectory inside the RICH 2 volume are the exact hit coordinates at station T10, especially the exact exit point from this station.

From Monte Carlo truth information, the best estimate of the track direction at the entry to RICH 2 is the vector joining the exit point from station T10 and the entry point in RICH 2 ; in this case, one only neglects the track curvature produced by a $\simeq$ 10 cm swimming of the particle through a magnetic field of about 350 G. Then, we know the 3–momentum vector and the charge of the particle at entry of RICH 2 and one can follow its trajectory inside the field by propagating the particle using the Runge–Kutta method with the known field map.

However, following this procedure, one still does not account for the multiple scattering undergone by the particle at the RICH entrance window and inside the radiator volume ; note that multiple scattering effects on the RICH mirrors and behind (mirror support, exit window) influence neither the photon generation nor the ring reconstruction[2].

There is a simple iterative procedure which happens to provide a remarkable approximation to the charged track trajectory inside the whole RICH 2 volume when multiple scattering is acting. It can be described by the following algorithm :

- Step 1 : Define the track 3–momentum $\vec{p}$ using the (exact) exit point at station T10, the entry point to RICH 2, and the true absolute momentum. The charge is also known.

- Step 3 : Use the Runge–Kutta procedure, currently used by the LHCb tracking software [2, 3], in order to transport the track from the entry point ($\vec{x}_{in}$) to the z location of the exact exit point from RICH 2 ($\vec{X}_{out}$) ; one thus reaches a point ($\vec{x}_{out}$) slightly displaced in the tranverse (x, y) directions.

- Step 3 : Correct the direction of $\vec{p}$ by an angle equal to the angle between $[\vec{X}_{out} - \vec{x}_{in}]$ and $[\vec{x}_{out} - \vec{x}_{in}]$ and restart at Step 2 up to convergence.

Convergence is defined by requiring $|\vec{X}_{out} - \vec{x}_{out}|$ smaller than some bound (typically $\simeq$ 10 microns), or some maximum number of iteration steps (typically 10), or both. Practically, the distribution of iteration steps is of exponential type with a mean value of 2.4 steps. When examining the final distribution of the transverse components of $\vec{X}_{out} - \vec{x}_{out}$ we observe no bias (residual mean values are below the 1 micron level), the y residual is a narrow peak (0.7 microns r.m.s.) and extends up to about ± 2 microns. The convergence cut (10 microns) affects more the x residual which is nevetheless narrow (3.8 microns r.m.s.) ; as the bending plane is $x - z$, this behaviour is as expected.

In order to verify the numerical accuracy of this procedure, an additional, non–standard SICB bank was created[3] to store information on particle trajectories at a plane

[2] It should be noted that the contribution of the fringe field to the track curvature in this region should also not have any influence at the reconstruction level. This means that the tracking information used in order to reconstruct the track direction in RICH 2 (the so–called AXTP 3 and 4 bank contents) should be provided at appropriate z locations : at the entrance window (about $z \simeq 954$ cm) and at the mirror mean position (about $z \simeq 1088$ cm), instead of the RICH exit window ((about $z \simeq 1150$ cm). Otherwise, one is introducing a bias in the track direction for low and medium momentum particles ; this bias plays in opposite directions for positively and negatively charged particles.

[3]We gratefully acknowledge F. Ranjard for this work.

located in the middle of RICH 2. 500 events were produced with these additional banks and a comparison was made between the Monte Carlo truth information and the corresponding information calculated using the iterative procedure described above.

At the middle of RICH 2 volume, the x and y coordinate and slope residuals are well fitted by fixed width Breit-Wigner expressions. The corresponding resolutions are 24 microns r.m.s. for both x and y coordinates ; for the x–slope the resolution is 34 μrad (r.m.s.) and for the y–slope 31 μrad (r.m.s). These numbers should be compared with the emission point and chromatic errors, which are [5] 310μrad and 420μrad (r.m.s) respectively. Therefore, even if mathematically not perfect, the algorithm just sketched provides a good approximation of the true track inside RICH 2 volume, quite appropriate in order to make more realistic Cerenkov photon generation inside RICH 2.

For practical purposes, and taking into account that the fringe field is relatively low, one can further approximate the estimated trajectory by a sequence of straight line segments of known lengths in order to emit the appropriate number of photons from each of them. In the working routine, we choose to define them by asking $\Delta z \simeq 10$ cm between the two end–points ; this turns out to approximate true trajectories by a sequence of $\simeq 20$ straight line segments. In the course of this study, we also discovered that 1–2 % of tracks have $p \simeq 10$ MeV and curl repeatedly inside the RICH 2 gas volume ; a small fraction is slightly above the Cerenkov threshold (16 MeV for electrons) and presumably will give an increase in background photons. Proper treatment of these curling tracks has not been considered in the present studies.

3 Scaling and Non–Scaling Errors

PID with Ring Imaging Cerenkov counters—RICH or DIRC, for instance— basically relies on reconstructing the Cerenkov cone opening angle knowing each photon direction and generally [7, 8, 5] (but not necessarily) the charged track direction. The resolution on the Cerenkov angle associated with a ring depends on the angular accuracy on the photon directions, but also on the angular accuracy of the charged track direction. However, these two kinds of error contribute in quite different ways.

In the likelihood expression maximized in the RICH PID software [7, 8], the basic "measured" quantities are the angles between the direction of the charged track and those of each photon. So, even if photon errors are uncorrelated, the errors on the angles just defined are certainly all correlated, as it is the same charged track error function which affects a given track, whichever is the photon to which the track is associated. It is easy to use the full error covariance matrix in a local approach [9, 5, 10] (e.g. on a track by track basis) as it can be inverted analytically [10]. In a global approach to PID, where all tracks and all photons are considered altogether, the problem is more complicated and it is not easy to deal with the full error covariance matrix when correlation terms are non–zero [7].

Using simple examples involving Cerenkov angle resolution and Cerenkov light emission, it is easy to show that[4] the Cerenkov angle resolution (σ), the charged track direction

[4]It is easy to check this numerically using simple Monte Carlo techniques in order to study the so–called local approach to PID. In this case, one can vary separately the error on the charged track direction

angular accuracy (σ_{track}), and the angular accuracy (σ_γ) of each of the n photon directions are related by :

$$\sigma^2 = \sigma^2_{track} + \frac{\sigma^2_\gamma}{n} \tag{1}$$

This expression illustrates that errors come into the problem with different properties : the charged track direction does not scale with the number n of photons, in contrast to the angular accuracy of photons. This expression gives a reasonable description of the observed ring resolution for RICH 2 (see below) as a function of the generated Cerenkov photon number.

The charged track angular uncertainty itself can be written :

$$\sigma^2_{track} = \sigma^2_{geom} + \frac{C^2}{p^2} \quad , \quad C \simeq 13.6 \; 10^{-3} \; [\mathrm{GeV}]\sqrt{\frac{X}{X_0}} \tag{2}$$

for $\beta = 1$ particles. The first term reflects the geometrical accuracy generated by digitization errors on the track direction, while the second term describes multiple scattering effects affecting the charged track direction. Both contributions are certainly statistically independent and thus no crossed term is expected. This relation shows that the dominant contribution to the error for high momentum particles is the geometical one.

These two formulae have been used as parametrizations for the study presented throughout this paper.

4 Curvature Error Contribution to Cerenkov Angle Resolution

As stated above, the track curvature inside the RICH 2 volume cannot be neglected in Cerenkov photon generation. For ring reconstruction purposes, however, it is necessary to use an appropriate mean track direction which can be computed quickly.

The best mean direction estimator of a curved track is obtained by sampling the tangent with a large frequency along the trajectory and by calculating the sample mean value. We have compared this mean value with various quantities expected to be good estimators of the sample mean value :

- the mean value of slope parameters at entry and exit of the RICH volume (the one currently used in the RICH reconstruction [5, 8]),
- the slope parameters at the middle of RICH 2,
- the chord joining the points at entry and exit of the RICH *sensitive* volume (see footnote 2).

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and the number of photons (possibly the angular resolution of each photon too). In this way Eq. (1) can be verified empirically.

	x-slope	y-slope
mean entry/exit slope	369.5	13.5
slope at mid RICH 2	258.1	13.3
entry/exit chord	25.6	9.7

Table 1: Resolution r.m.s. for the slope inside RICH 2 volume, comparing the true mean slope with different estimators. Numbers are in μrad.

The results are given in Table 1. They clearly show that the best estimator of the mean slope, useful for RICH reconstruction, is the chord.

While sampling the tangent along the track trajectory, in addition to the mean trajectory, one can calculate the standard deviation of the slope. This is given in Fig. 1 in bins of particle momenta with a fit function superimposed, parametrized by Eq. (2). For particles of momenta between 1 and 5 GeV, the curvature error amounts to 2.26 ± 1.06 mrad, between 5 and 10 GeV, this error is still large 1.18 ± 0.42 $m$rad ; at large momenta it clearly tends to zero as could be expected. The parametrized curve is $\sigma^2 = p_1^2/p^2 + p_2^2$ using the notation of the inset in Fig. 1, automatically provided by PAW.

This source of error is not negligible compared to the other contributions to the Cerenkov angle resolution [1], even at physically accessible large momenta. This error affects mostly the projected direction in the $(x$–$z)$ bending plane, while it is negligible in the $(y$–$z)$ non–bending plane ; from 60 to 80 GeV this error is 82 ± 9 μrad and for $p \geq 80$ GeV, it is still 46 ± 16 μrad. Therefore, this uncertainty contributes by slightly degrading the PID performance comapred with the simulations in the TDR where curvature effects were neglected.

Fortunately, even if due to the existence of a fringe field which affects the charged track direction inside RICH 2, this error is of scaling type (*i.e.*, it should be added in quadrature to the other contributions to the single photon angular resolution). This is because it affects specifically the photon direction at its specific emission time and could be folded in with the chromatic uncertainty associated with each photon. Its effect is largest below $p = 30$ GeV, e.g. in a momentum range where the RICH 1 contribution to the PID dominates and tends to compensate the degradation produced by the curvature uncertainty in RICH 2.

The fit information given in Fig. 1 allows to parametrize the curvature error contribution to the single photon resolution on a track by track basis, as a function of the particle momentum.

5 The Role of T11 in RICH 2 PID

We now address the question of the exact role of station T11 for the RICH 2 performance.

In the absence of T11, the track direction information is provided solely by the track fit in the (track) seeding region ; as station T10 is very close to the RICH 2 entrance window, the track direction inside the RICH 2 gas volume is obtained by extrapolating

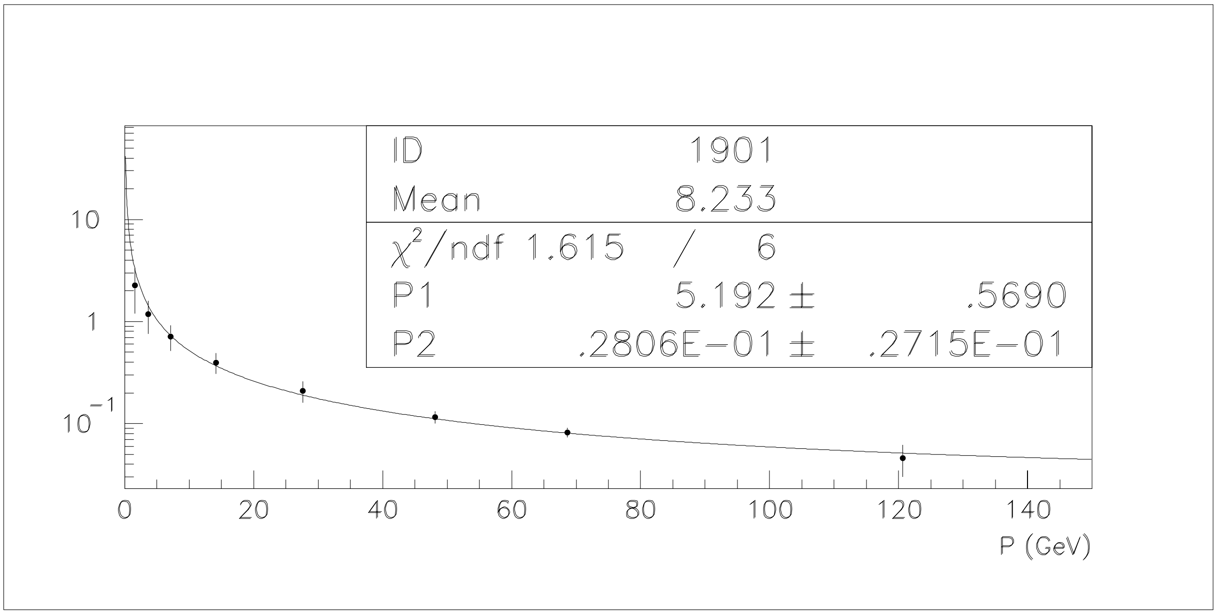

Figure 1: Curvature error as a function of the particle momentum. Ordinate is in millirads, abscissa in GeV.

the track parameters at station T10 to the z location of the entrance and exit points from RICH 2 (the so–called AXTP3 and AXTP4 bank contents). The geometric part of the uncertainty on the charged track direction at T10 is already small (it integrates out the measurement errors in stations T7 to T10, at least) ; the only information not accounted for by the x and y slope values[5] returned by the tracking software is the multiple Coulomb scattering at station T10, at the RICH 2 entrance window and inside the gas volume.

If T11 is used, the geometric part of the track uncertainty is further reduced by the large lever arm represented by the distance between T10 and T11 ($\simeq$ 2 meters) ; however, we will see shortly that this improvement has only a marginal influence for high momentum particle identification.

Thus, the main contribution of T11 is rather to provide a measurement of the multiple Coulomb scattering undergone by the track between T10 and T11. This includes multiple scattering effects which are indeed relevant for the RICH performance issue (T10, RICH 2 entrance window, gas volume), but also additional effects – affecting the track after the radiator – of no relevance for this purpose : multiple scattering effects in the mirror material, in the mirror support and in the RICH exit window.

Within the sample we used for our studies, the T10 radiation thickness is 1.6% X_0 (according to present estimates it should be increased to $\simeq 3\ \%\ X_0$). Summing up the "relevant" radiation thickness of RICH 2 gives $\simeq 4\ \%\ X_0$, while the "irrelevant" radiation thickness of RICH 2 amounts to $\simeq 9.3\ \%\ X_0$. Stated otherwise, only half[6] of the multiple

[5]Instead, tracking errors on slopes take into account these multiple scattering effects and can be used in computing the Global Likelihood function used by the RICH PID [7].

[6]Indeed, the irrelevant material thickness "benefits" of having a smaller lever arm with respect to T11 compared to the relevant material thickness, which is located farther from T11 and closer to the RICH 2

Digitization [mrad]	x-slope	y-slope
T11 on	0.042 ± 0.005	0.048 ± 0.005
T11 off	0.062 ± 0.006	0.068 ± 0.007
Multiple Scatt. [mrad GeV]		
T11 on	1.525 ± 0.116	1.867 ± 0.140
T11 off	1.456 ± 0.119	2.219 ± 0.172

Table 2: Fit values of the digitization and the multiple Coulomb scattering contributions to the angular track error at T10, with or without using T11 information. The parametrization follows Eq. (2).

scattering effects measured by T11 is indeed relevant to the PID, while the second half contribute as a kind of background.

For the purpose of RICH 2 reconstruction, in connection with using or not using station T11, what is relevant is certainly the x–slope and y–slope information at T10. We show in Fig. 2 the curve for x–slope and y–slope at T10 as functions of p, using or not T11 information. As before, the fitted function is Eq. (2) and the fit parameter values and errors can be read off from Fig. 2 ; they are also gathered in Table 2. We have not tried determining the numerical correlation level between these parameter values.

One clearly sees that the geometrical error, which dominates at high momentum, is improved by using T11 information. The improvement is slightly larger for the x-slope (about 50%) than for the y-slope (about 40%). Comparing these numbers with other contributions to the ring resolution as given in the RICH TDR [5], one can estimate that, with a mean number of photons of 18–20, the influence of improving the digitization error by the T10–T11 lever arm is little.

Concerning the multiple scattering error coming from fit, we observe no improvement for the x-slope and a very small improvement for the y-slope (a 2 σ effect). Even if these two kinds of errors are correlated and somewhat mixed up numerically, the improvement provided by T11 information looks small.

6 RICH 2 Performance with/without Station T11

We have investigated the change in the PID performance of RICH 2 with and without using station T11. Therefore, we have run the RICH particle identification code on our event sample (see the header of Part I). However, we have introduced a modification ; if station T11 is removed from the setup, this leaves free $\simeq$ 33 cm which can be used in order to increase the depth of the radiator volume available in RICH 2. It is easy to verify that such an increase corresponds to increasing the Cerenkov photon yield by $\simeq$ 18%. Therefore, we have run the RICH PID code using station T11 in normal conditions, while we have artificially increased the photon yield per centimeter by 18% when removing station T11. The Cerenkov photons were generated along the curved trajectories as

entrance window.

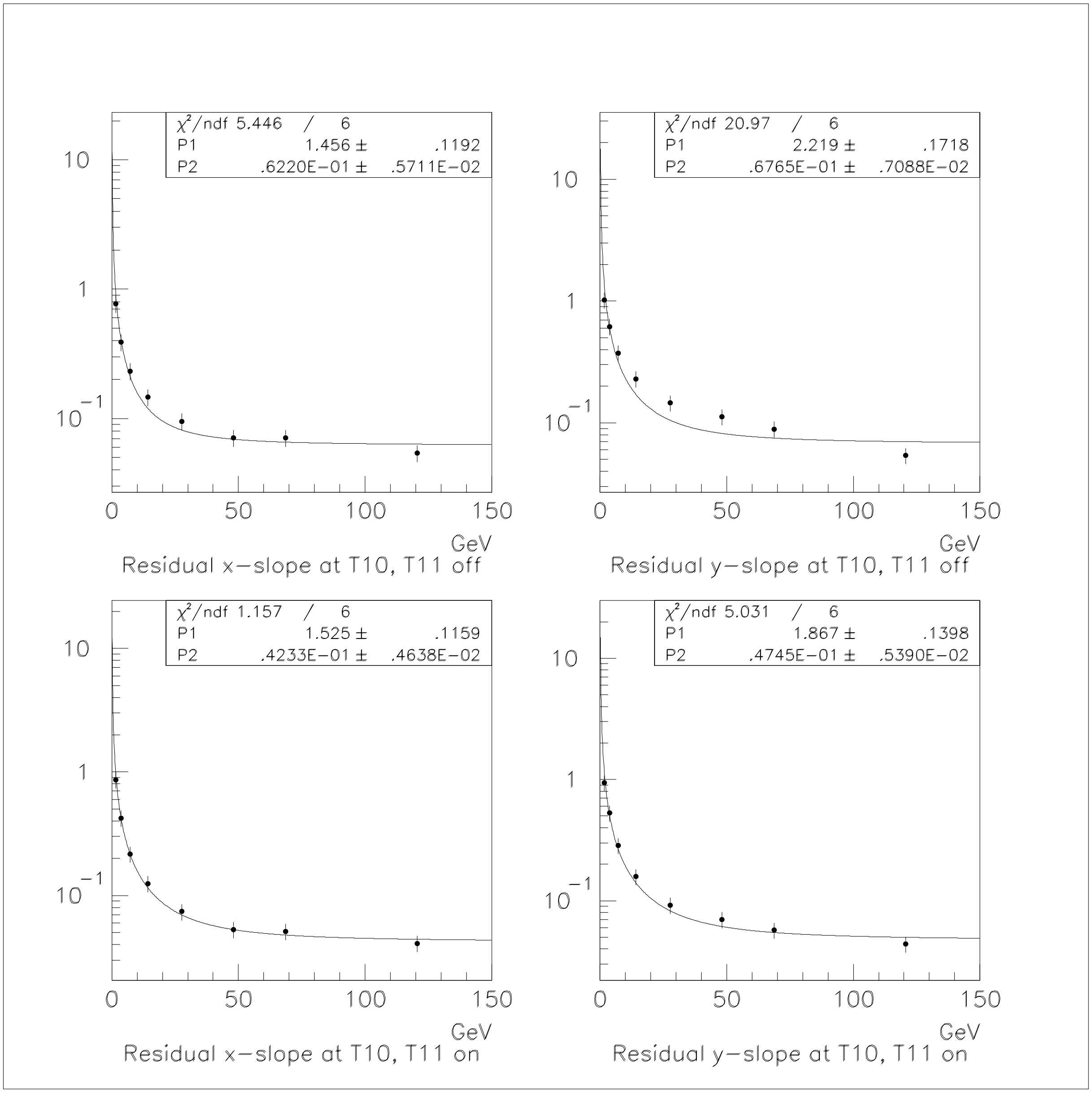

Figure 2: Reconstructed slope residuals at T10 with and without using T11 as functions of the particle momentum. Ordinates are in mrad and abscissa in GeV.

explained above. The main results are summarized in Table 3 for three ranges of charged particle momentum.

The charged particle tracks were reconstructed using the C++ fitting procedure [2], but track pattern recognition was not performed for this study[7].

Even with 2500 events, there are still significant statistical fluctuations, especially for the limited kaon track sample, however a trend is visible which is more clearly seen in the

[7]We gratefully acknowledge M. Needham for his help and patience, while we tried becoming familiar with the new tracking software.

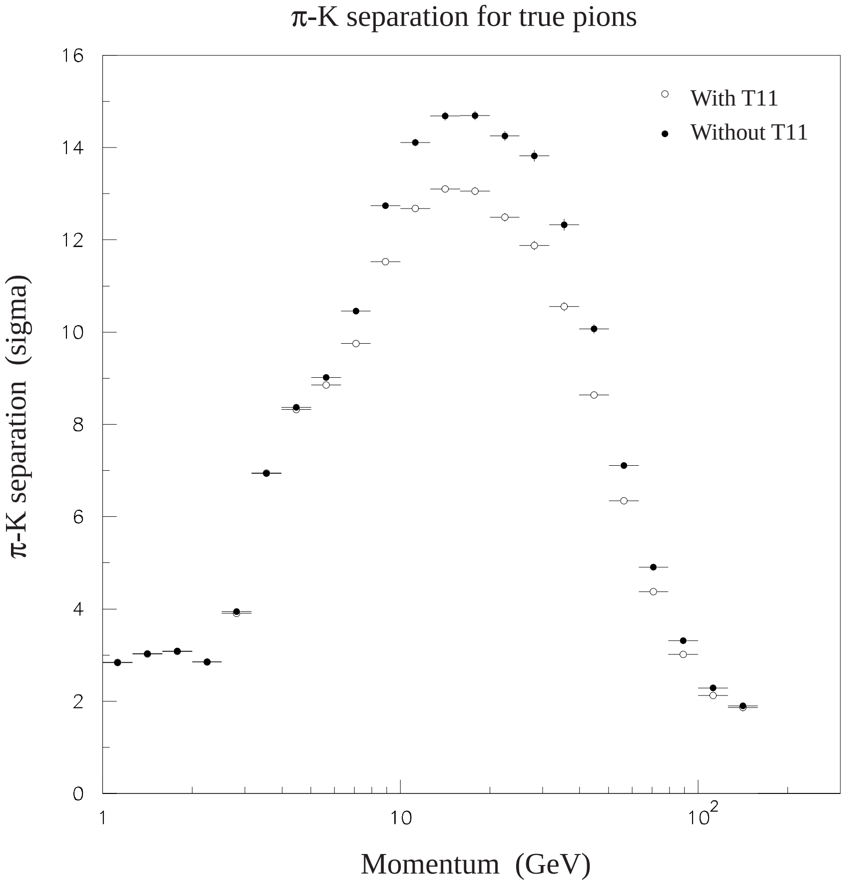

Figure 3: Number of sigma separation between pion and kaon hypotheses for true pions, as a function of momentum. Data set of 2500 events generated with an Al–Be beam pipe ; tracks are reconstructed following the full reconstruction chain. The two histograms are identical below $p = 5$ GeV.

pion identification performance. Table 3 does not exhibit any noticeable degradation of PID performances which could be attributed to T11 removal, when the additional 18% photon yield is included. One should even note that pion misidentification at medium momenta is slightly improved (a 3 to 4 σ effect).

The significance of π–K separation, measured by N_σ, is plotted as a function of the track momentum in Figure 3. This figure shows that the high-momentum performance is not degraded by the removal of T11. Again, one even observes some improvement in the medium momentum range (between 10 and 30 GeV), where the π–K separation is better ; this presumably has some connection with the better pion misidentification in the medium p region commented on just above (see Table 3).

One possible explanation for this effect is that T11, when used, contributes an increased number of poor quality track stubs reconstructed by the Extended Tracking which perturb the working of the PID algorithm. Another possible contribution could be that the observed improvement in the intermediate momentum region is an effect of the 18% increase of the photon yield.

	PIONS			KAONS		
p range [GeV]	%	T11 on	T11 off	%	T11 on	T11 off
$p < 20$	eff	85.8	85.7	eff	78.9	79.1
	purity	98.3	98.0	purity	65.8	67.6
	$\pi \rightarrow$Kp	2.3 ± 0.1	2.0 ± 0.6	K$\rightarrow$e$\mu\pi$	6.8 ± 0.4	7.0 ± 0.4
$20 < p < 70$	eff	55.8	56.8	eff	95.9	96.2
	purity	97.7	96.2	purity	93.2	91.6
	$\pi \rightarrow$Kp	1.4 ± 0.1	1.0 ± 0.1	K$\rightarrow$e$\mu\pi$	0.8 ± 0.2	0.6 ± 0.2
$p > 70$	eff	29.4	29.2	eff	78.4	81.9
	purity	95.1	94.7	purity	50.4	48.2
	$\pi \rightarrow$Kp	11.6 ± 0.9	12.1 ± 1.0	K$\rightarrow$e$\mu\pi$	19.0 ± 3.2	18.1 ± 3.2

Table 3: Particle identification performance (efficiency, purity, misidentification probability) for true pions and kaons in the low, mid and high-momentum bands. In the "T11 off" configuration the Cherenkov photon yield was increased by 18%.

This has been tested by investigating the $\pi - K$ separation performance with T11 on, with T11 off and no increase of the photon yield and, finally, with T11 off and a 18% increase of the Cerenkov photon yield. For this purpose 10 000 events have been generated with a stainless steel beam pipe and the RICH reconstruction has been performed with smearing the true track directions as explained in Part III (see below). The results are shown in Fig. 4. They confirm, under harder background conditions, the conclusions already reached. They additionally prove that the 18% increase of the photon yield indeed improves the $\pi - K$ separation up to about 100 GeV, with the bulk of the effect at medium momenta. Finally, the almost perfect superimposition of the dots with the full histogram also proves that the effect of solely using T11 information is practically invisible.

Therefore, we conclude that removing station T11 from the Tracker setup while at the same time increasing the RICH 2 radiator depth by the 33 cm then made available, gives an identical PID performance. Indeed, the potential usefulness of T11, in providing a measurement of the multiple scattering in T10, RICH 2 entrance window and radiator gas, is largely diluted by scattering in the mirror assembly and exit window.

7 Photon yield in RICH 2

The 18% increase of the Cerenkov photon yield made possible by extending RICH 2 by 33 cm into the region liberated by T11, has allowed particle identification performance to remain identical to the TDR configuration [5]. However, the increased photon yield alone certainly does not give a dramatic improvement in particle identification. This being left aside, there are even better reasons to increase as much as possible the photon yield in RICH 2. It is indeed important to emphasise that the photon hit count from RICH 2 is not very large (about $N_{\mathrm{pe}} = 19$ [5]) and leaves little margin to accommodate any loss compared with the number of hits anticipated, e.g. due to a possible ageing effect of the

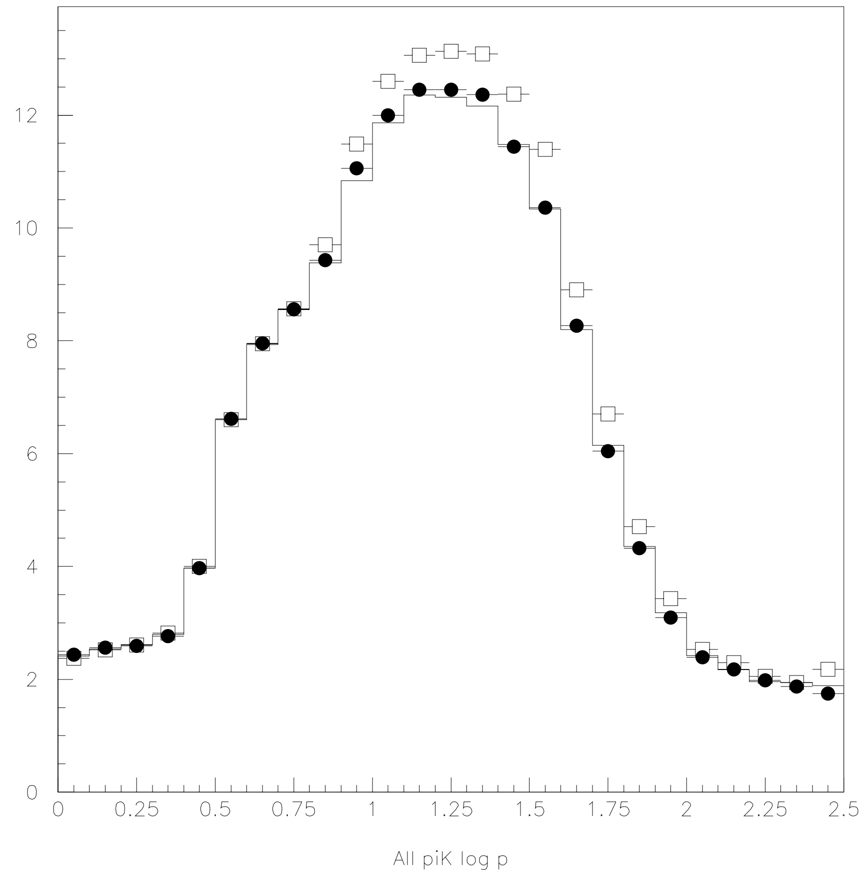

Figure 4: Number of sigma separation between pion and kaon hypotheses for true pions, as a function of momentum. The data set consists of 10 000 events generated with a stainless steel beam pipe. The histogram corresponds to the analysis without taking into account T11 information and with the nominal photon yield ; the black dots correspond to using the T11 information. Finally the empty squares represent the results when T11 information is removed together with a 18% increase of the Cerenkov photon yield.

photocathodes.

In order to illustrate this point, Fig. 5 shows the global misidentification rates for high momentum ($70 < p < 150$ GeV) pions and kaons for different assumptions of the photon yield. The fitted curves are parametrized as given in Eq. (1) and misidentification rates should have similar behaviours with respect to angular errors as the resolution function. The fit quality supports this functional shape and the sharing of resolution effects between scaling and non–scaling contributions.

The value $R_{\rm pe} = 1$ corresponds to $\simeq 19$ photoelectron hits, obtained with the RICH 2 radiator length as defined in the RICH TDR [5], and is the reference value here. The data sample used in order to get this figure is 10 000 events generated with a stainless steel beam pipe[8]. This Figure clearly showns that the RICH TDR (nominal) photon yield is

[8]Our smaller reference sample with Aluminium–Beryllium beam pipe confirms the functional shape with a turning point also around $R_{\rm pe} = 1$; the main difference is that the misidentication rate for kaons

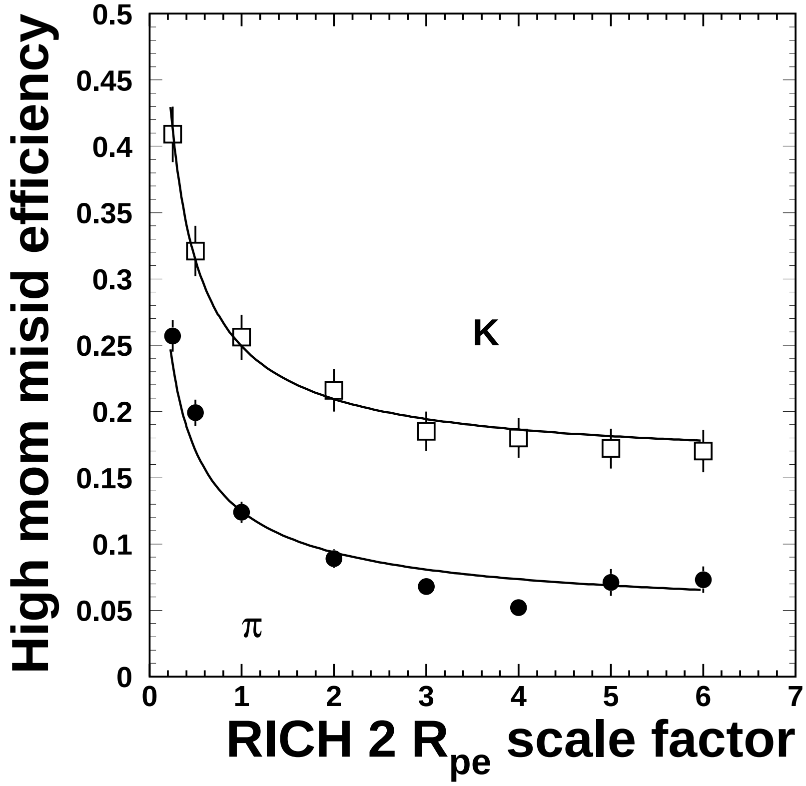

Figure 5: High momentum misidentification for pions and kaons, as a function of the relative photon yield in RICH 2

delicately placed : although a relatively large increase in the photon yield (a factor 2 to 3) would not improve dramatically the misidentification rates, any photon loss could result in a significant degradation of performance. This observation emphasizes the pertinence of exploiting the space liberated by the removal of T11 for increased photon yield.

Part II : Influence of the T10 y–Layers on RICH 2 Performance

As stated in the Introduction, none of the y layers is used in the tracking ; these serve solely to improve the knowledge of charged track directions inside RICH 1 (T1,T2) and RICH 2 (T10,T11) and, thus, the PID performance of the LHCb RICH system.

We have already seen in Part I that station T11, as a whole, does not improve RICH 2 PID and can be removed. In this part, we study the effect of the T10 y–layers, having

scales down to about 20% in the neighborhood of this point.

removed station T11 from track reconstruction and particle identification. Correspondingly, we have increased the Cerenkov photon yield by 18%, as explained in the previous Part.

We next study whether these y–layers can be replaced by inclined ones with a relatively small stereo angle. It is also interesting, for several purposes, to see whether these layers cannot be simply removed, because the y information provided by the stereo planes of the tracking stations could be already sufficient as they are, or whether the stereo angle value in the standard stereo layers of the track seeding region (stations T7 to T10) can be changed to a more beneficial value for the RICH.

8 Changing the T10 y–Layers to Stereo Layers

By changing y layers to stereo layers, one certainly degrades the position and slope information in the vertical plane ; at the same time, this change provides some improvement for the position and slope information in the horizontal plane. Whatever the balance between these complementary effects, the relevant information, from the point of view of RICH 2 particle identification quality, is the track angular resolution and not the x– or y– slope resolution considered in isolation. We have, using obvious notations :

$$\sigma^2_{track} = \sigma^2_x + \sigma^2_y \quad , \tag{3}$$

and thus any degradation of the track y–slope resolution is somewhat washed out. To give an example, let us consider that $\sigma_x = \sigma_y = \sigma$. Let us assume that the y–slope resolution is then degraded to $\sigma_y = \sigma(1 + \lambda)$. Then, at leading order, we have :

$$\sigma_{track} \simeq \sqrt{2}\sigma \left[1 + \frac{\lambda}{2}\right] \tag{4}$$

instead of $\sqrt{2}\sigma$; therefore, a large degradation of the y–slope accuracy ($\lambda = 30\%$, for instance) results in a limited degradation of the track direction accuracy (15% in the present example).

Using our reference event sample, we have first studied the effects produced by having stereo layers instead of the y layers in station T10. We have considered these effects at stereo angles of 90° (y–layers), 30°, 20°, 15° on our 2 500 events ; we have also some information on 5°, but this is subject to large fluctuations as it comes from a 500 event subsample. It should be noted that, in this exercise, half of the y layers have been rotated by the quoted stereo angle, and the other half by the opposite value in order to keep the usual symmetry [1].

In Fig. 6, we show the dependence of the y–slope resolution and of the track angular resolution (in space) as a function of the stereo angle. The data point at 0° is the value obtained by simply removing the T10 y–stack from track reconstruction ; the data point at 100° is the same and has been drawn in order to guide the eye.

The data point at 90° is the usual case of having y plane measurements (stereo angle is $\pm 90°$). In this Figure, we have plotted the angular information for tracks with $p \geq 80$ GeV, where the resolution problem is more crucial for the RICH 2 performance[9].

One clearly observes in both upper and lower plots, that the slope information degrades linearly at small angles. However, starting at $\simeq 20°$ a flat plateau extends up to 90° (standard y layers). Therefore, stereo layers with an angle relative to vertical of $\simeq 20°$ performs as well as y layers from the point of view of tracking information ; this means, of course, that such a change does not affect at all the performance of RICH 2.

By comparing the upper and lower figures in Fig. 6, one also observes that the effect of degrading the y–slope information is significantly smaller for the track angular resolution : Between 10° and 20° , the y–slope resolution is degraded by about 20%, while the track angular resolution is degraded by only 10%, as expected. The apparently anomalous value at 5° is presumably a statistical fluctuation associated with the small sample size.

One should additionally remark that just removing the T10 y stack degrades the track angular resolution by only 20%. It is therefore wise to examine the actual effect of this degradation on RICH 2 performance.

9 RICH 2 performance versus Stereo Angle

As stated above, we do not expect changes of RICH 2 performance when changing its y layers to stereo layers with angle 20° ; any difference between these two cases has almost certainly to be attributed to statistical fluctuations.

Another attractive solution, still to be examined, is whether the T10 y stack cannot be simply removed, possibly by changing the 5° stereo angle in the (track) seeding region (e.g. from T7 to T10) to a slightly larger value.

We have examined the PID performance for tracks with momenta above 70 GeV. There is, of course, a strong correlation between angular resolution on the track direction and the misidentification rates ; indeed, both carry the same information, except that the former is less sensitive to background effects (for instance, coming from poor tracks in the Extended Tracking) than the latter. We have examined the data on our 2 500 event sample, and made additional checks on 10 000 events generated under the same conditions.

Our results are summarized in Table 4. Most information reported there deals with the 2 500 event sample ; some additional information on 10 000 events is given within parentheses. Angular resolutions are given in μrad, and misidentification rates in percent.

Comparing numerically the x and y slope resolutions and the track angular accuracy when having a y stack in station T10, or 20° stereo planes instead, clearly illustrates that any possible degradation remains comfortably within statistical errors.

In the central three data columns of Table 4, we give the results obtained with 3 values of the stereo angle for stations T7 to T10 (the track seeding region) ; the stereo angle in stations T1 to T6 is always kept at its nominal value (5°). Examining the resolutions

[9] The discussion which follows would be the same for data above $p \geq 40$ GeV. A real degradation begins to take off at lower momenta, where the main PID device in the LHCb detector becomes RICH 1, practically in isolation.

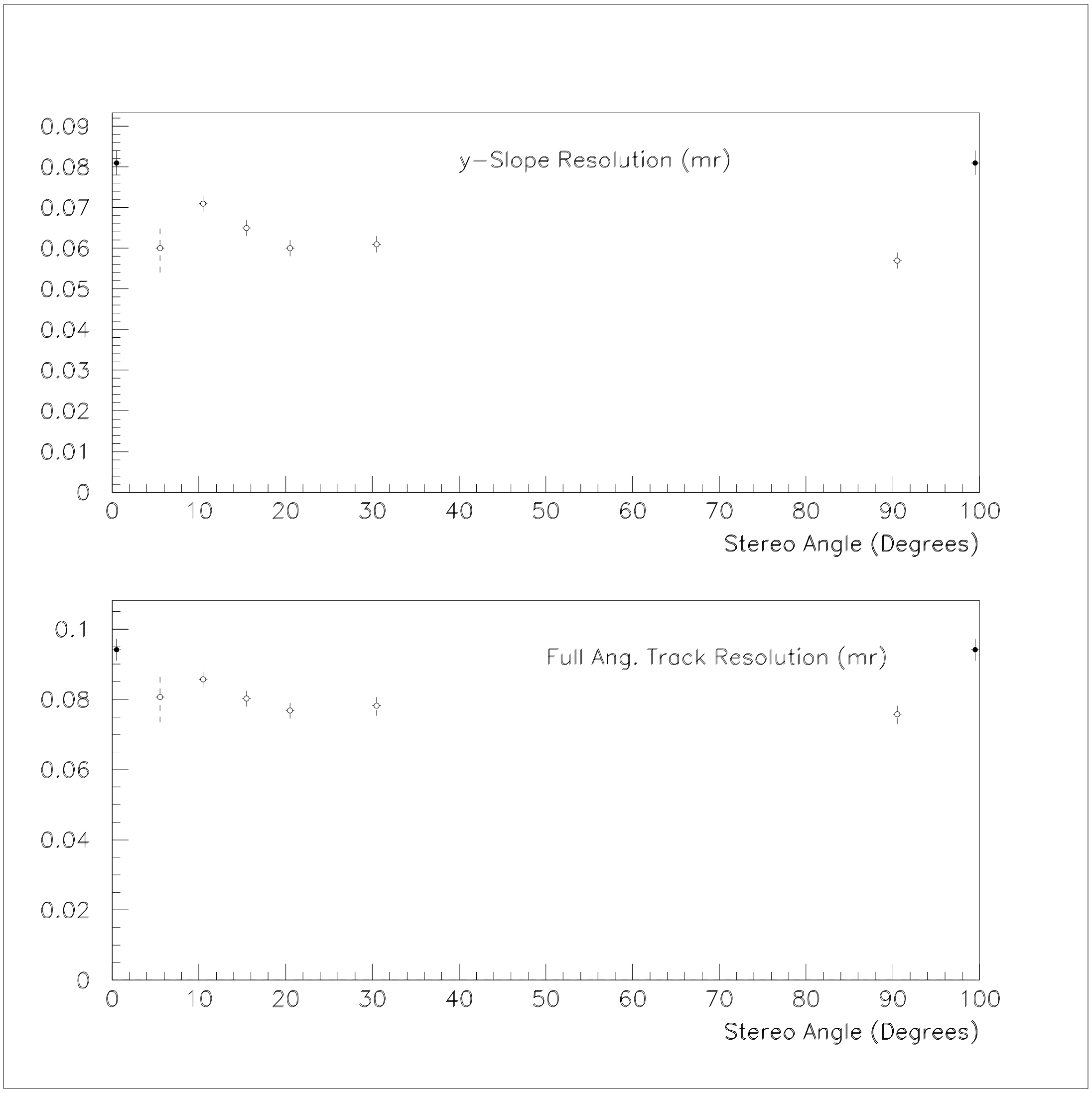

Figure 6: y-slope uncertainty and global track angle uncertainty (in mr) as a function of the stereo angle in the T10 stack devoted to y–measurements (named T10 y stack, even when consisting of stereo layers). The stereo angle in the usual stereo planes of all tracking station is kept fixed at its nominal value (5°).

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(three first lines) indeed shows that there is some degradation when merely removing the T10 y–stack (track resolution evolves from 76 to 95 μrad) ; however, increasing the stereo angle to 10° significantly limits the degradation (from 76 to 82 μrad).

The question of how significant this is, cannot be disconnected from the other sources of errors [5] which affect the Cerenkov ring reconstruction within the RICH PID.

Presently, all errors are treated as scaling in the RICH PID ; in this case, the degradation of about 20μrad from geometrical accuracy is clearly negligible : the single photon resolution is increased from 557μrad to 560μrad. This means that the Cerenkov angle resolution for 20 photons is "degraded" from 124.5μrad to 125.2μrad in the standard RICH

PID [5]. This should mean that differences between percentages in Table 4 are mostly statistical fluctuations.

In a track by track approach, the errors listed above scale with the photon number (uncorrelated errors), in contrast to the error on the charged track direction which contributes in a correlated way with each photon. Assuming about 20 photons, the Cerenkov angle resolution in this approach is 145μrad at reference and degrades from 145μrad to 156μrad in the worst option (stereo angle of $5°$), that is a 11μrad degradation.

Moreover, the numbers just given do not account for the contribution of the curvature error in RICH 2, originating from the fringe field and discussed above. This error (of scaling type and momentum dependent) washes out a little more effect of a global 11μrad degradation.

It is now interesting to see how this transfers to RICH 2 PID performance at high momenta. Comparing the misidentification rates, we see that statistical fluctuations are significant for kaons, even in the 10 000 event sample, although pion numbers are more robust. However, as pion and kaon misidentification rates are closely related, one can draw clearer conclusions from the pion misidentification rate evolution. Then, we can assess that we do not observe degradation effects larger than 1σ and this should be partly (at least) attributed to statistical fluctuations. Indeed, the 10 000 event pion misidentification rates reported in the last line of Table 4 cannot be understood otherwise.

Therefore, if one relies on the pion misidentification rate, one clearly concludes from this Table that removing the y stack of T10, assuming that T11 has been already removed and compensated for by an increased Cerenkov photon yield, does not spoil RICH 2 performance. In this case, all stereo layers in the Tracker system keep their value at $5°$ angle.

Part III : y–Layer Influence on RICH 1 Performance

To investigate the importance to the RICH system of y-measuring layers in tracking stations 1 and 2, the dependence of the particle identification on angular track resolution in this projection was mapped out.

A large sample of simulated events was subjected to the full tracking and RICH reconstruction chain. For physics quality tracks [8] passing through RICH 1, however, the direction information returned by the track fit was replaced by a direction obtained by smearing the true track trajectories independently in the x and y planes. These smearings were single, momentum independent Gaussians. In x the smearing was set to reproduce the results of the full tracking. In y a range of resolutions were explored, with the entire sample being reprocessed for each setting, and the particle identification performance analysed.

Figure 7 gives the results of this study, showing the variation of pion and kaon identification efficiency against the angular resolution in y. The efficiencies were calculated for physics quality tracks of $1 < p < 150\,\mathrm{GeV/c}$ and are defined as in [8]. Also indicated by arrows are the observed resolutions for the detector simulated for the RICH TDR [5] (marked 'TDR'), which included y measuring planes in stations 1 and 2, and a simulation without these planes (marked 'No y'). It can be seen that removing these planes degrades

	with T10 y stack (Reference)	no T10 y stack stereo 5°	no T10 y stack stereo 7.5°	no T10 y stack stereo 10°	with T10 y stack stereo 20°
$\sigma(t_x)$	49.9 ± 1.3	48.2 ± 1.2	48.1 ± 1.1	49.9 ± 1.2	47.5 ± 1.2
$\sigma(t_y)$	57.1 ± 1.6	81.4 ± 2.4	70.8 ± 2.1	65.4 ± 2.1	59.5 ± 1.8
$\sigma(ang.res.)$	75.8 ± 2.1	94.6 ± 2.7	85.6 ± 2.4	82.3 ± 2.4	76.2 ± 2.2
$K \to e/\mu/\pi$ (%)	15.79 ± 2.96 (16.63 ± 1.67)	20.13 ± 3.29 (19.63 ± 1.64)	14.87 ± 2.92 (17.43 ± 1.56)	18.00 ± 3.14	21.19 ± 3.33 (19.00 ± 1.60)
$\pi \to K/p$ (%)	11.87 ± 0.95 (11.43 ± 0.52)	14.13 ± 1.02 (11.71 ± 0.49)	12.28 ± 0.96 (12.16 ± 0.50)	10.57 ± 0.90	12.21 ± 0.96 (11.81 ± 0.49)

Table 4: First data column is the standard setup with a y stack in T10; In second to fourth data columns, the y stack is physically removed and the stereo angle for the remaining stereo planes in T7 to T10 is modified as indicated. In the last data column, the T10 y stack is replaced by two stereo layers at 20°, while all other stereo planes keep their stereo angle at nominal TDR [1] value (5°). Numbers given within parentheses deal with 10 000 event statistics. Angular resolutions are given in μrad, and misidentication rates correspond to tracks with $70 < p < 150$ GeV.

the vertical resolution by about a factor of 2. The particle identication performance, however, is identical in both cases, and does not degrade until the resolution becomes coarser than 1 mrad. This result is as expected, since the intrinsic RICH 1 angular precision [5] on single photon Cherenkov angle is 1.45 mrad for C_4F_{10} and 2.00 mrad for the aerogel, meaning that the particle identification performance is less dependent on the tracking precision than is the case in RICH 2, which has a single photon resolution [5] of 0.58 mrad.

From these studies it can be concluded that the y measuring planes in stations 1 and 2 may be removed, without degrading the particle identification. Therefore, the improvement in precision of track angle in the $y - z$ projection, provided by the y-layers of T1 and T2, has a negligible effect.

Part IV : Effects of T10y and T1 with Pile–Up Events

The present design of the LHCb Experiment is somewhat thicker in terms of radiation length than foreseen in the TP [1] ; additionally, in running conditions, we will

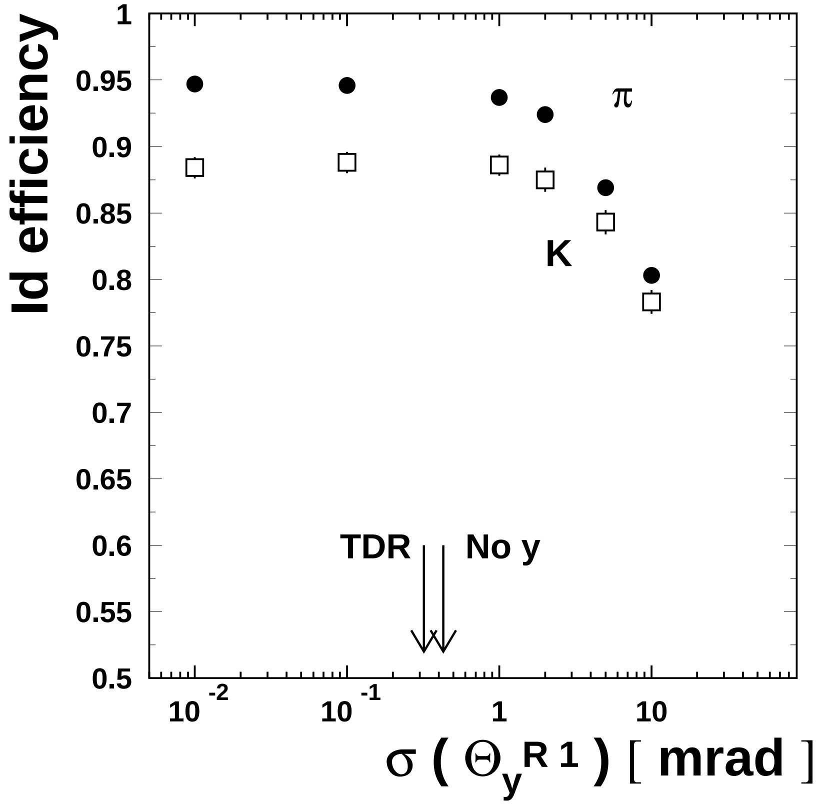

Figure 7: Variation in the pion and kaon identification efficiencies with angular tracking resolution in y at RICH 1. The arrows indicate the measured resolutions with ('TDR') and without ('No y') y-measuring planes in stations 1 and 2.

have to deal with pile–up events and spill over which correspond to higher background conditions.

Therefore, we have made complementary studies in order to assess precisely the role of the y stack in T10 and the role of T1 in more realistic conditions, using Monte Carlo data implementing pile–up effects. The material thickness of the detector has also been updated by using the latest VELO material thickness [6] and a tracking station thickness of 2% radiation length.

We have performed our RICH performance studies on 10 000 events using efficiencies, purities and misidentication rates evaluated in this sample. The sharing of events between single, double, . . . interaction(s) is shown in Fig 8, where one sees that single interaction events represent half of the data sample.

For the purpose of this study, we consider as reference the RICH performance with a standard tracking setup, where station T11 has been removed and replaced by a 18% increase of the Cerenkov photon yield in RICH 2. Performance is then examined by removing only the T10 y stack from this setup[10] on the one hand, and, on the other hand,

[10]We can perform this exercise by only removing the y measurement from the Outer Tracker part

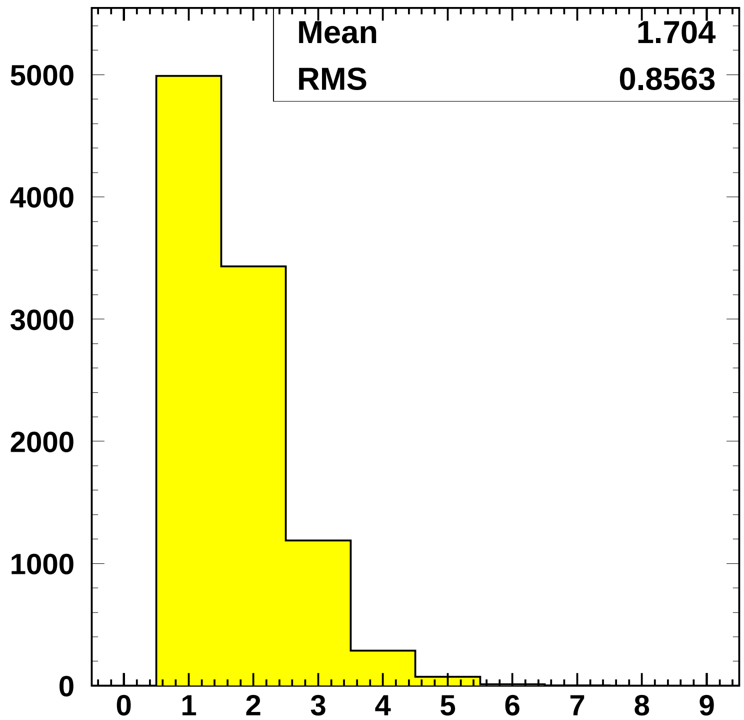

Figure 8: Data Sample with Pile–up. Distribution of interaction number per event ; the total sample size is 10 000 events obtained with a luminosity of 2 10^{32}.

by removing the T10 y stack together with the whole station T1. All important results are gathered in Table 5 for the whole data set and in Table 6 for the subset of single interaction events.

As T1 is located just downstream from the VELO (with no thick device in between), the role of station T1 certainly needs to be addressed. Indeed, except for large multiple scattering effects at exit of the VELO, VELO information and T1 information should be quite redundant. This is why the role of station T1 should only be investigated with a realistic radiation thickness[11] for the VELO [6].

It is interesting to compare first our former results with low material (see Table 3) with the new results obtained with a more realistic estimation of material thicknesses. For

of station T10. In this simulation, the corresponding Inner Tracker Station information is actually x measurements.

[11]Actually, the data sample we use now is always generated with the T1 material, whatever is done with T1 information (used or unused). Therefore, when T1 is "removed", its radiation thickness continues to contribute to the multiple scattering just upstream of RICH 1. This is equivalent to having a vertex detector exit window thicker by 2% radiation length than given in [6]. As a consequence, our results with T1 "removed" are certainly somewhat pessimistic since it is equivalent to having a VELO exit window of 4% radiation length instead of 2%.

PILE–UP SINGLE INTERACTIONS		Standard Setup	No T10 y	No T10 y No T1	"T11 off" (less material)
PIONS	%				
$p < 20$	eff	83.9	82.9	81.5	85.7
	purity	97.2	96.9	96.5	98.0
	$\pi \to$Kp	2.28 ± 0.05	2.44 ± 0.05	2.71 ± 0.05	2.3 ± 0.1
$20 < p < 70$	eff	54.5	54.3	54.2	56.8
	purity	96.0	96.2	96.0	96.2
	$\pi \to$Kp	1.55 ± 0.10	1.85 ± 0.10	1.70 ± 0.10	1.0 ± 0.1
$p > 70$	eff	30.0	28.9	30.5	29.2
	purity	92.2	93.0	93.5	94.7
	$\pi \to$Kp	13.29 ± 0.73	14.51 ± 0.76	13.43 ± 0.73	12.1 ± 1.0
KAONS	%				
$p < 20$	eff	76.8	76.8	75.1	79.1
	purity	62.3	62.4	59.8	67.6
	K$\to$e$\mu\pi$	9.25 ± 0.32	9.79 ± 0.33	10.99 ± 0.35	7.0 ± 0.4
$20 < p < 70$	eff	93.7	94.9	94.3	96.2
	purity	89.5	88.0	88.0	91.6
	K$\to$e$\mu\pi$	1.57 ± 0.24	1.70 ± 0.25	2.17 ± 0.29	0.6 ± 0.2
$p > 70$	eff	75.8	73.4	72.2	81.9
	purity	43.8	40.0	41.3	48.2
	K$\to$e$\mu\pi$	21.30 ± 2.46	20.44 ± 2.44	25.64 ± 2.64	18.1 ± 3.2

Table 5: RICH particle identification performance for pile– up events with realistic material budget. Here, we consider single interaction events in isolation. T11 has been removed and consequently the Cherenkov photon yield was increased by 18%. The momentum ranges are in GeV. In the last data column we recall the results of Table 3 corresponding to much thinner material for the VELO and the stations.

this purpose, we have introduced the relevant former results in the last data column of Table 5. When restricting to single interaction events (5 000 events), we are in principle under the most favorable conditions compared with the full set of pile–up events. We thus observe, as a consequence of larger multiple scattering effects that efficiencies and purities go down slightly, whichever the exact tracker layout we consider.

Several effects are visible, not easy to attribute : statistical fluctuations or fewer fake tracks (through the Extended Tracking) may contribute. For instance, the degradation of pion misidentification at medium momenta when only removing the y stack of T10, which is then improved by also removing T1, is hard to interpret. We also see that performance with pions at medium and high energies benefits from removing simultaneously T10y and T1. Whatever the reason, performance with pions does not feel a significant degradation by the removal of these.

The kaon performance exhibits the same ambiguous situation. low momentum kaons are certainly not much affected by removing T1 and T10y. At medium to high momenta, if there is a degradation it is very small, and the numbers obtained while removing T1, might indicate that fluctuations are still large[12].

As noted above, these results indicate that the effects of removing the y stack of T10 are small. Statistical (or background) effects mask a little bit the effects of removing T1 ; however, one expects that RICH 1 performance does not suffer very much from this, because of the high quality track information of the nearby VELO.

Looking at real pile–up events (see Table 6) reveal the same trend as single interaction events in isolation. In this case (possibly because of the 10 000 event statistics), we more clearly see that the removal of T10y has negligible effects, while there is some indication of lower performance at low momentum when T1 information is not used. Indeed, the kaon misidentification rate changes from 12% to 13% (a 4σ effect) and the pion misidentification rate from 2.5% to 2.8% (a 10σ effect). A part of this effect should be attributed to the T1 material which is still present, even when its tracking information is unused.

On the other hand, comparing the corresponding yields in Tables 5 and 6 teaches us that the degradation produced by piling up interactions are much more noticeable than the effects of removing the T10 y stack and/or station T1.

Part V : Conclusions

The study above was originally performed in order to study the influence of the information provided by several of the tracking stations. As a by–product, it led us to identify a problem connected with the curvature of tracks within the RICH 2 volume produced by the fringe field, not negligible along about 2 meters at this location. This curvature error influences the resolution on the Cerenkov angle, mostly for low and medium momentum tracks traversing the whole LHCb detector. We have shown that this uncertainty cannot be neglected but can easily be parametrized as a function of the momentum.

On the other hand, analyzed in terms of RICH 2 Cerenkov photon yields, particle misidentification rates show that the present (nominal) yield is located at a transition region where any loss could have dramatic consequence on PID performance. Therefore, increasing this yield is certainly beneficial ; removing station T11, by liberating some longitudinal space, allows to increase the RICH 2 radiator depth and corresponds to increasing the photon yield by 18%. Even if small this improvement is welcome.

For the main subject of this study, *i.e.* the role of some tracking stations, we reached several conclusions :

- Station T11 mainly serves to measure multiple scattering effects affecting tracks which traverse RICH 2. This might give some improvement for low and medium momentum tracks, but this improvement has no impact on the PID at these momenta ; additionally, about half of the measured multiple scattering affects the track direction after it leaves RICH 2 sensitive volume, which reduces the value of

[12] Sharply degrading the kaon misidentification rate at high momentum by simply removing T1 is probably a statistical fluctuation.

PILE–UP ALL EVENTS		Standard Setup	No T10 y	No T10 y No T1
PIONS	%			
$p < 20$	eff	82.4	81.6	80.0
	purity	96.8	96.6	96.2
	$\pi \to$Kp	2.49 ± 0.03	2.52 ± 0.03	2.83 ± 0.03
$20 < p < 70$	eff	54.0	53.6	53.7
	purity	96.2	96.3	96.2
	$\pi \to$Kp	2.21 ± 0.07	2.28 ± 0.07	2.33 ± 0.08
$p > 70$	eff	29.7	29.2	30.3
	purity	92.3	93.1	92.6
	$\pi \to$Kp	15.25 ± 0.51	15.36 ± 0.52	14.65 ± 0.51
KAONS	%			
$p < 20$	eff	74.0	73.9	72.9
	purity	60.1	59.9	57.5
	K$\to$e$\mu\pi$	12.01 ± 0.23	12.08 ± 0.23	13.13 ± 0.24
$20 < p < 70$	eff	92.4	92.6	92.2
	purity	88.3	87.3	86.5
	K$\to$e$\mu\pi$	2.03 ± 0.18	2.02 ± 0.18	2.31 ± 0.19
$p > 70$	eff	74.3	73.7	73.2
	purity	42.3	42.0	42.6
	K$\to$e$\mu\pi$	19.79 ± 1.54	19.91 ± 1.54	21.14 ± 1.58

Table 6: RICH particle identification performance with the full set of pile– up events, using a realistic material budget. T11 has been removed and consequently the Cherenkov photon yield was increased by 18%. The momentum ranges are in GeV.

this measurement. Therefore, removing station T11 does not degrade RICH 2 PID for particles whatever their momentum.

- The y layers in station T10 can be changed to layers with inclined strips with a stereo angle of 20°, keeping the PID performance unchanged.

- The y layers in stations T1 and T2 give indeed an improved determination of track directions inside RICH 1. However, in the momentum range where RICH 1 is the dominant PID device, this improvement does not provide any significant change in the RICH particle identification performance.

- A precise analysis under various background conditions has also shown that simply removing the y stack of station T10 affects in a very marginal way the RICH PID. The conclusion was similar if one removes all T1 information from the analysis.

Therefore, our main conclusion is that all y layers in the tracking setup can be removed without spoiling the PID performance of the LHCb RICH system. Moreover, station T11 and, to a large extent station T1, play no (significant) role in particle ID performance.

Since most of these elements (e.g. all elements considered above, except for a part T1) are not used by the tracking software, there is no strong reason to keep them. Altogether, removing T1, T11 and the y layers in T2 and T10, turns out to reduce the LHCb detector radiation thickness by 8–9% X_0.

The marginal role played by station T1 in the numbers given above is visible partly because its material thickness still contributes in the simulation as an effective increase of the VELO exit window thickness. This means that really removing T1 (including its material) *and* changing the composition of some elements of the VELO exit window from aluminium to beryllium, should allow the recovery of an almost complete equivalence between T1 and VELO information from the point of view of RICH 1 information.

However, the role of station T1 should also be considered under in the context of other issues, like K_S reconstruction efficiency, before making any recommendation.

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