Frascati Physics Series Vol. XXXV (2004), pp. 457-465
HEAVY QUARKS AND LEPTONS - San Juan, Puerto Rico, June 1-5, 2004

MEASUREMENT OF THE TOTAL HADRONIC CROSS-SECTION AT e^+e^- MACHINES

Ivan Logashenko
Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
Boston University, Physics Department, Boston, MA 02215, USA

ABSTRACT

The current status of measurements of the cross-section $e^+e^- \to hadrons$ at low energies ($\sqrt{s} < 2$ GeV) is reviewed. Recent results of direct and ISR measurements are discussed.

1 Introduction

Measurement of the total cross-section $e^+e^- \to hadrons$, often expressed as the dimensionless ratio

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{1}$$

has been an important topic of the high energy physics since early 70s. $R(s)$ plays a special role in many high precision theoretical calculations of fundamental quantities such as the Higgs mass, the running QED and QCD coupling constants, c and b quark masses. The most demanding current application for $R(s)$ is the calculation of a_μ, the anomalous magnetic moment of muon. The E821 experiment in BNL has measured the value of a_μ to the 0.5 ppm precision [1]. The SM prediction for the same value is known to 0.7 ppm, where the uncertainty in dominated by the knowledge of $R(s)$ [2]. The measured difference of 2-3 standard deviations between the experimental result and the theoretical value is on the edge of discovering forces beyond the Standard Model. This makes the improvement of the accuracy of $R(s)$ determination very important.

The hadronic contribution to a_μ is calculated via numerical integration of $R(s)$ with the proper kernel function:

$$a_\mu^{had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2} \frac{R(s)K(s)}{s^2} ds. \tag{2}$$

At high energies ($\sqrt{s} > (5-10)$ GeV) the value of the integral can be calculated within the framework of pQCD with high precision. At lower energies the only source of $R(s)$ is the experimental measurement of the cross-section $e^+e^- \to$ *hadrons*. The energy region below 2 GeV gives by far the dominant contribution both to the value of the integral (2) and to its uncertainty. Moreover, it turns out that the systematic error of the measurements, not the statistical one, limits the final uncertainty.

In the following section we'll describe the on-going and near-future experiments focused on the high precision measurement of $R(s)$ at $\sqrt{s} < 2$ GeV. Measurements of $R(s)$ at higher energies are described elsewhere [3, 4].

2 Measurement at Novosibirsk

The energy range $0.36 < \sqrt{s} < 1.4$ GeV has been studied at the electron-positron collider VEPP-2M (Novosibirsk, Russia). Two experiments, the CMD-2 and the SND, started in 1992 and 1995, respectively, and continued up to 2000, when the collider was shut down. Two energy scans, covering the whole available energy range $0.36 < \sqrt{s} < 1.4$ GeV with small steps were performed over these years. Measurement of $R(s)$ was one of the major physics goals for both experiments.

In the VEPP-2M energy range the number of modes and the multiplicity of $e^+e^- \to$ *hadrons* events are small, while the kinematic distributions for different modes are quite different. That makes an exclusive approach, where the cross-section for each channel $e^+e^- \to hadrons$ is measured independently with unique acceptance, radiative and other corrections, the only viable option. The inclusive approach, where the total cross-section $e^+e^- \to hadrons$ is measured directly, becomes usable at $\sqrt{s} > 2$ GeV [3]. In the energy range under consideration the dominant contribution to $R(s)$ comes from the $e^+e^- \to \pi^+\pi^-$ mode. Above $\sqrt{s} > 1$ GeV the contribution from $e^+e^- \to 4\pi$ becomes important and, eventually, dominant. Other modes, $e^+e^- \to 3\pi$, KK, etc., give an important contribution at the narrow ω and φ resonances.

The high precision measurement of the $e^+e^- \to \pi^+\pi^-$ cross-section was performed at the CMD-2 detector. Design features of the detector allowed for a factor 2-3 reduction in the systematic error compared to the previous experiments in the same energy range. The results of the measurement in the energy range $0.6 < \sqrt{s} < 1$ GeV were already published [5, 6]. The analysis of much larger data set, covering the whole VEPP-2M energy range $0.36 < \sqrt{s} < 1.4$ GeV, is in progress.

The analysis scheme is the following. Events with two tracks in the final state, $e^+e^- \to e^+e^-, \mu^+\mu^-, \pi^+\pi^-$, are selected simultaneously in the same fiducial volume. The background events, mainly from the cosmic background, are separated using the spatial distribution of the vertex. Then the three final states are separated with the help of the unbinned likelihood fit using either the momenta information (at energies $\sqrt{s} < 0.6$ GeV) or the energy deposition information (at energies $\sqrt{s} > 0.6$ GeV). The pion formfactor is calculated as follows:

$$|F_\pi|^2 = \frac{N_{\pi\pi}}{N_{ee}} \cdot \frac{\sigma(e^+e^- \to e^+e^-)}{\sigma(e^+e^- \to \pi^+\pi^-)|_{F_\pi=1}}, \tag{3}$$

where $N_{ee,\pi\pi}$ are the numbers of the corresponding final states detected and $\sigma(e^+e^- \to e^+e^-, \pi^+\pi^-)$ are the calculated detection cross-sections, which include all the acceptance and radiative corrections. The cross-section for the 2π final state is calculated for point-like pions ($F_\pi = 1$). Since muons and pions cannot be separated by their energy deposition, at $\sqrt{s} > 0.6$ GeV the ratio $N_{\mu\mu}/N_{ee}$ was fixed in the fit to the value calculated within the QED with acceptance and radiative corrections taken into account. At energies $\sqrt{s} < 0.6$ GeV all three final states are separated, therefore, in addition to the $e^+e^- \to \pi^+\pi^-$ cross-section, the $e^+e^- \to \mu^+\mu^-$ cross-section is also measured, providing an additional consistency test. The ratio $(N_{\mu\mu}/N_{ee}) \cdot (\sigma_{ee}/\sigma_{\mu\mu})$ is shown in Fig. 1. The experimental value (0.989 ± 0.015) is consistent with the expected value

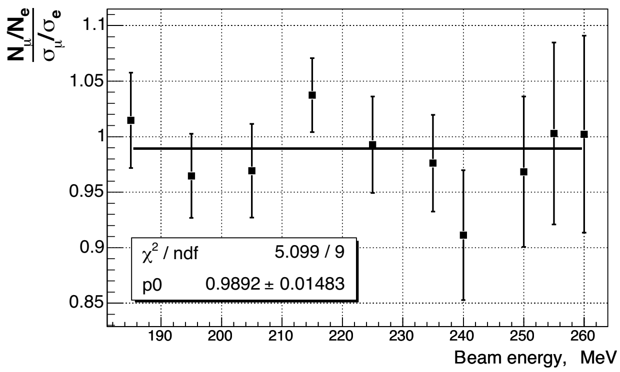

Figure 1: *Ratio* $(N_{\mu\mu}/N_{ee})\cdot(\sigma_{ee}/\sigma_{\mu\mu})$*, measured at CMD-2. The average value,* 0.989 ± 0.015*, is consistent with the expected value of 1.*

of 1 within 0.8 statistical deviation.

The hadronic cross-section used for the calculation of $R(s)$ and the hadronic cross-section used to extract parameters of the resonances are two different entities. The first is the "bare" one, although the final state radiation (FSR) is considered as apart of the cross-section; the latter is the "dressed" one, but the FSR effects should be excluded. Therefore, for the purpose of the pion formfactor measurement, the radiative corrections to the $e^+e^- \to \pi^+\pi^-$ cross-section take into account the initial and the final state radiation, but not the vacuum polarization (VP). When $R(s)$ is calculated, the "bare" cross-section is calculated from the "dressed" one by applying corrections for VP and FSR.

The cross-sections of other hadronic modes are calculated in the following way:

$$\sigma_f = \frac{N_f - N_b}{L \cdot (1 + \delta_f) \cdot \varepsilon_f}, \tag{4}$$

where N_f is the number of detected events of a particular final state, N_b is the number of background events, L is the luminosity, δ_f is the radiative correction and ε_f is the detection efficiency. The luminosity is measured using the large angle Bhabha scattering with the systematic precision of 1-3%. The radiative correction takes into account the effects of the initial state radiation. The typical systematic uncertainty of the calculation is 1%. The detection efficiency is calculated with the help of Monte-Carlo simulation, and its systematic uncertainty varies between 2% and 10% for different final states.

The main cross-sections measured at VEPP-2M are listed in Table 1 and are shown in Fig. 2. The data analysis is still under way, therefore some of the results are preliminary.

Table 1: The cross-sections $e^+e^- \to hadrons$, measured at VEPP-2M with CMD-2 and SND detectors. Some results are preliminary. References to the published results are shown in the third column.

Mode	Systematic error	References
$e^+e^- \to \pi^+\pi^-$	0.6%–5% (0.6% at ρ-meson)	[5], [6]
$e^+e^- \to 3\pi$	1.3%–5% (1.3% at ω-meson)	[7], [8]
$e^+e^- \to 4\pi$	5%–10%	[9], [10]
$e^+e^- \to K^+K^-$	3%–6%	[11]
$e^+e^- \to K_SK_L$	1.7%–8% (1.7% at φ-meson)	[6], [11], [12]

The VEPP-2M collider was decommissioned in 2000 to prepare the experimental hall for the new collider, VEPP-2000 [13]. Direct high-precision measurement of the $e^+e^- \to hadrons$ cross-section up to $\sqrt{s} < 2$ GeV is one of its main goals. The CMD-3 and the upgraded SND detectors are expected to start data taking in 2006-2007.

3 Radiative return experiments

In addition to the direct measurements there were indirect measurements of $e^+e^- \to hadrons$ cross-sections at e^+e^- colliders. It is possible to connect the vector spectral functions of hadronic τ decays with the isovector part of $e^+e^- \to 2\pi, 4\pi$ cross-sections. The large τ data set collected by ALEPH, CLEO and OPAL allowed for the high precision measurements of these cross-sections from the threshold up to 1.8 GeV. There is significant discrepancy between these results and the direct measurements, unexplained at the moment. The details of comparison are described elsewhere [14].

The radiative return is another approach to a measurement of the $e^+e^- \to$ *hadrons* cross-section at e^+e^- factories, complementary to the traditional en-

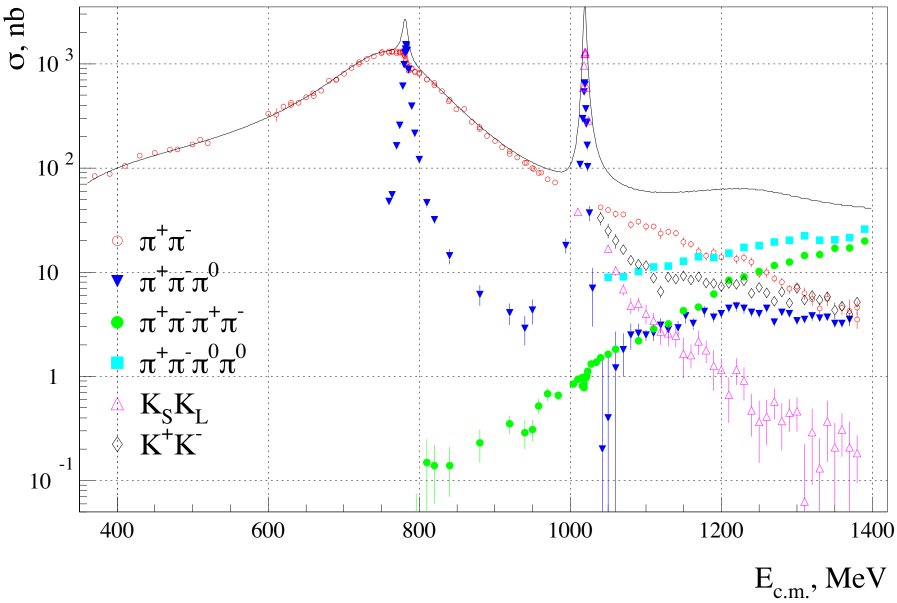

Figure 2: *The cross-sections of different modes $e^+e^- \to hadrons$, measured at CMD-2. The line represents the total cross-section $e^+e^- \to hadrons$.*

ergy scan [15]. In this new method one measures the distribution of the invariant mass of the hadronic system in $e^+e^- \to \gamma + hadrons$, where a photon is radiated by the initial state electrons. The cross-section $\sigma(e^+e^- \to hadrons)$ is then calculated as:

$$\sigma(e^+e^- \to hadrons) \cdot H(M^2_{hadrons}) = M^2_{hadrons} \cdot \frac{d\sigma(e^+e^- \to \gamma + hadrons)}{dM^2_{hadrons}}. \quad (5)$$

The radiator function $H(M^2_{hadrons})$ is obtained from theory.

Two reasons make this approach promising at the particle factories. First, it allows to measure the cross-section in the wide energy range while the collider stays at single energy. This is particularly important at the factories designed to operate in the narrow energy range. Second, the high luminosity of the factories allows to overcome $O(\alpha)$ suppression due to a requirement for the ISR photon in the final state.

The KLOE collaboration has recently announced the first results of the

$e^+e^- \to \pi^+\pi^-$ cross-section measurement using the ISR approach [16]. The c.m. energy for the DAFNE φ-factory is 1.02 GeV, which makes the ISR photon relatively soft. In this case the FSR presents significant background to the main, ISR, process. In order to reduce its contribution, the analysis is restricted to events with low-angle, undetected ISR photons ($\Theta_\gamma < 15^0$ or $\Theta_\gamma > 165^0$). This and other kinematic restrictions limit the measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ at KLOE to $0.35 < s < 0.95$ GeV^2.

The statistical uncertainty of the KLOE measurement is negligible. The total systematic uncertainty is 1.3%, with main contributions from the experimental technique (0.9%), the luminosity measurement (0.6%) and the calculation of the radiator function (0.5%). The comparison of the KLOE and CMD-2 results is shown in Fig. 3. While the contributions to the dispersion integral (2) from two data sets are consistent, the differential cross-sections differ beyond the claimed systematic errors. This discrepancy is not yet understood.

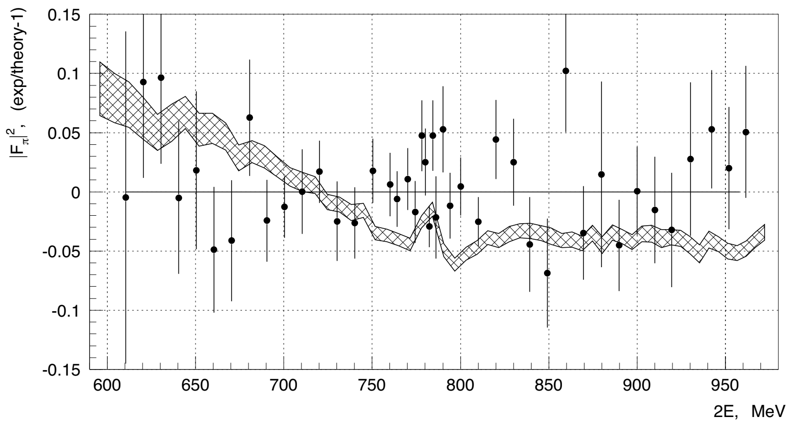

Figure 3: *Comparison of the pion formfactor measured at CMD-2 and KLOE. The quantity shown is $|F_\pi|^2(exp.)/|F_\pi|^2(theory)-1$, where $|F_\pi|^2(theory)$ is the fit to the CMD-2 data. Data points show the CMD-2 data, the hatched corridor represents the KLOE results. Only statistical errors are shown. The CMD-2 data are corrected for the vacuum polarization.*

The BaBar collaboration actively pursues the measurement of $\sigma(e^+e^- \to \mathit{hadrons})$ at low s using ISR approach [17]. There are two significant differences between BaBar and KLOE techniques. First, the c.m. energy for the PEP-II collider is 10.58 GeV, while the interesting energy range for the cross-section measurement starts at the 2π threshold. That makes the ISR photon very hard and FSR negligible for most hadronic states. Unlike the KLOE approach, the ISR photon is detected and is well separated from the hadronic state. Second, in BaBar analysis the $e^+e^- \to \mu^+\mu^-\gamma$ is used as the monitoring process. The cross-section is calculated as:

$$\sigma(e^+e^- \to hadr) = \frac{N(hadr+\gamma)\cdot\varepsilon_{\mu\mu}(1+\delta_{FSR}^{\mu\mu})}{N(\mu\mu\gamma)\cdot\varepsilon_{hadr}(1+\delta_{FSR}^{hadr})}\sigma(e^+e^- \to \mu^+\mu^-), \qquad (6)$$

where N is the number of detected events, ε is the detection efficiency and δ_{FSR} is the correction for the final state radiation. In this approach many effects, such as the vacuum polarization, the radiative correction to the initial state, some detector inefficiencies, are cancel out. This should work particularly well for the $e^+e^- \to \pi^+\pi^-$ channel, where the hadronic state and the monitoring process have a very similar signature.

The BaBar collaboration plans to measure exclusively all main hadronic cross-sections from the threshold up to 3-4 GeV. The first results have already been published [18].

4 Conclusions

Recent years showed a significant progress in the measurement of $\sigma(e^+e^- \to hadrons)$ at low s. The data analysis of the direct measurements performed at VEPP-2M is getting finalized and most of the results are expected to be published in the next 1-2 years. The first results from the new approach, the radiative return measurements, were presented by the KLOE and BaBar collaborations. The new technique shows its great potential and these results will most likely dominate the field for the next 5-7 years. The new high precision results are expected from the VEPP-2000 collider once it starts the operation. These efforts, if successful, will allow to reach a precision below 0.4 ppm for the hadronic contribution to $(g-2)_\mu$.

5 Acknowledgments

The author would like to thank S.I. Eidelman, F.V. Ignatov, B.I. Khazin and E.P. Solodov for their help in preparation of this paper. This work is supported in part by grants RFBR-03-02-16280, RFBR-04-02-16217 and RFBR-04-02-16223.

References

1. G. W. Bennett *et al.*, Phys. Rev. Lett. **92** (2004) 161802.
2. M. Davier *et al.*, Eur. Phys. J. C **31** (2003) 503.
3. J.Z. Bai *et al.*, Phys. Rev. Lett. **88** (2002) 101802.
4. S.A. Dytman, Nucl. Phys. B (Proc. Suppl.) 131 (2004) 32.
5. R.R. Akhmetshin *et al.*, Phys. Lett. **B527** (2002) 161.
6. R.R. Akhmetshin *et al.*, Phys. Lett. **B578** (2004) 285.
7. R.R. Akhmetshin *et al.*, Phys. Lett. **B476** (2000) 33.
8. M.N. Achasov *et al.*, Phys. Rev. **D66** (2002) 032001; M.N. Achasov *et al.*, Phys. Rev. **D68** (2003) 052006.
9. R.R. Akhmetshin *et al.*, Phys. Lett. **B466** (1999) 392; R.R. Akhmetshin *et al.*, Phys. Lett. **B595** (2004) 101.
10. M.N. Achasov *et al.*, J. Exp. Theor. Phys. 96 (2003) 789.
11. M.N. Achasov *et al.*, Phys. Rev. **D63** (2001) 072002.
12. R.R. Akhmetshin *et al.*, Phys. Lett. **B551** (2004) 27.
13. S.I. Eidelman and S.I. Serednyakov, Nucl. Phys. B (Proc. Suppl.) 131 (2004) 19.
14. M. Davier *et al.*, Eur. Phys. J. C **27** (2003) 497.
15. S. Binner, J.H. Kühn and K. Melnikov, Phys. Lett. **B459** (1999) 279; M. Benayoun *et al.*, Mod. Phys. Lett. **A14** (1999) 2605.
16. A. Aloisio *et al.*, hep-ex/0407048
17. M. Davier, Nucl. Phys. B (Proc. Suppl.) 131 (2004) 82.
18. B. Aubert *et al.* Phys. Rev. **D69** (2004) 011103.