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SUPERSYMMETRY AND THE STRONG CP PROBLEM*

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ABSTRACT

We show how supersymmetric grand unified theories (SSGUT) provide a natural solution to the strong CP problem. Such theories contain an additional global $U(1)_{PQ}$ chiral (Peccei-Quinn) symmetry which is conserved up to color anomalies. Hence the QCD θ angle is irrelevant. Moreover the PQ symmetry is spontaneously broken at the grand unified scale. As a result the standard Weinberg-Wilczek axion has a mass

$$m_a \sim \frac{\Lambda^2_{QCD}}{\Lambda_{GUM}} \sim 10^{-10} \text{ eV}$$

which effectively decouples from ordinary matter.

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1. Introduction

The standard model of strong and electroweak interactions is by now well established [1]. An excellent description of nature at low energies is provided with 20 adjustable parameters [2]. There are of course problems of naturalness associated with some of these parameters. In particular in this paper we shall focus on the so-called gauge hierarchy [3] and strong CP problems [4]. It has recently been suggested that a possible solution to the gauge hierarchy problem may be found in grand unified super symmetric models [5-7]. The most attractive scenario (including the grand unified gauge group G_{GUT}) requires the symmetries $G_{GUT} \otimes$ super symmetry to be good above the grand scale Λ_{GUM}. Below Λ_{GUM}, G_{GUT} breaks spontaneously to the standard model $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, preserving super symmetry. Super symmetry is then assumed to break spontaneously as a result of nonperturbative effects [5-7] at about 1 TeV. These nonperturbative effects may be due to instantons as suggested by Witten [5] or by fermion condensates of a new strong gauge group supercolor [5-7] which must be appended to G_{GUT}. In either case scalars are then naturally protected from obtaining quadratically divergent mass corrections down to a TeV scale. In this paper we shall show that supersymmetric GUTS (SSGUT) also provide a natural framework for solving the strong CP problem [4]. There exists in these models a $U(1)_{PQ}$ (Peccei-Quinn) [8] symmetry which results in a vanishing strong CP violating angle

$$\bar{\theta} = \theta + \arg \det M \quad .$$

In addition, we show that $U(1)_{PQ}$ is spontaneously broken at the grand unified scale Λ_{GUM}. As a result the standard Weinberg-Wilczek axion [9] has a decay constant F_a of order Λ_{GUM} (F1). Hence, the axion mass m_a is of order $\Lambda^2_{QCD}/F_a \sim 10^{-10}$ eV and more important, it effectively decouples from ordinary matter since its Yukawa coupling g_Y is of order $m_f/F_a \sim 10^{-19}$-10^{-17} where m_f are ordinary quark or lepton masses.

Dine, Fischler and Srednicki [7] have also discussed the strong CP problem and the PQ symmetry in a supersymmetric scenario. They obtain F_a of order $\Lambda_{SC} \sim 1-10$ TeV, where Λ_{SC} is the supercolor condensate scale. Although the coupling of their axion is suppressed relative to the standard Weinberg-Wilczek axion, it is nevertheless ruled out by astrophysical constraints for a light axion. Namely, Dicus, Kolb, Teplitz and Wagoner [11] have shown that if red giant stars are to lose energy at an acceptable rate via axion emission, then F_a must be of order 10^9 GeV or greater. In this paper we have extracted the DFS mechanism and placed it in its natural setting, i.e., a SSGUT (F2). The astrophysical constraints are clearly satisfied and no new scales must be introduced.

In sect. 2 we introduce a supersymmetric GUT with $G_{GUT} = SU(5)_G$ [14]. We then identify $U(1)_{PQ}$. We show that it is conserved up to a color anomaly and spontaneously broken at Λ_{GUM}.

In sect. 3 we discuss some open problems for the simple SU(5) SSGUT discussed in sect. 2 and possible solutions.

Finally, in the appendix, we derive the results discussed in sect. 2. We also show how a similar solution to the strong CP problem can work in the standard SU(5) GUT (F3). It requires a Higgs 5 and $\bar{5}$ and also a complex 24 and 1 of elementary scalars.

2. Supersymmetric GUT

We consider in this paper the following supersymmetric version of the standard $SU(5)_G$ GUT. It includes a real super gauge field V_G for $SU(5)_G$ and the following complex left-handed chiral superfields:

$$\begin{array}{rl} \Phi_1(x,\theta) & (1) \\ \Phi_{24}(x,\theta) & (24) \\ H(x,\theta) & (5) \\ \bar{H}(x,\theta) & (\bar{5}) \\ Y_i(x,\theta) & (10) \\ \bar{X}_i(x,\theta) & (\bar{5}) \end{array} \tag{2.1}$$

where we have explicitly given their SU(5) transformation properties, and the index i = 1,...,3 labels the three generations of ordinary quarks and leptons. The superfields Φ_1 and Φ_{24} contain complex scalar fields which will acquire vacuum expectation values at a scale Λ_{GUM} breaking $SU(5)_G$ down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. H and $\bar{H}$ include the standard Higgs 5 and $\bar{5}$ which are necessary to break $SU(2)_L \otimes U(1)_Y$ down to Q_{EM} and to give mass to the quarks and leptons which are included in the superfields Y_i and $\bar{X}_i$ (F4).

The Lagrangian for this model contains the terms:

$$\mathscr{L} = \mathscr{L}_{gauge} - V \tag{2.2}$$

where by $\mathscr{L}_{gauge}$ we mean the pure gauge Lagrangian plus all the kinetic terms for matter fields and also matter-gauge couplings. The term V contains all the necessary Yukawa couplings and scalar interactions. Explicitly we have

$$V = \int d^2\theta \left\{ \lambda_1 \Phi_1^3 + \lambda_2 \Phi_1 \Phi_{24}^2 + \lambda_3 \Phi_{24}^3 + h_1 \bar{H} \Phi_1 H \right.$$

$$\left. + h_2 \bar{H} \Phi_{24} H + g_{ij} Y_i Y_j H + g'_{ij} Y_i \bar{X}_j \bar{H} \right\} \tag{2.3}$$

$$+ \text{Hermitian conjugate}$$

where $SU(5)_G$ traces are understood. We note that $\mathscr{L}$ is classically scale invariant. If a mass term were added it would destroy our $U(1)_{PQ}$ symmetry necessary to solve the strong CP problem. The first three terms with couplings λ_i (i = 1,2,3) are necessary to provide $SU(5)_G$ symmetry breaking directions. Since $\mathscr{L}$ is scale invariant, however, the symmetry breaking scale is arbitrary (F5). The next two terms with couplings h_i (i = 1,2) are necessary to give mass to the color triplet scalars contained in the Higgs superfields H and $\bar{H}$. Finally, the last two terms contain the standard Yukawa couplings which enable the 3 families of quarks and leptons to obtain mass.

$\mathscr{L}$ is also invariant under two global Abelian symmetries $U(1)_X \otimes U(1)_Z$. The transformation properties of the superfields of eq. (2.1) under X and Z are given in table 1. Note that eq. (2.3) contains the most general potential invariant under both $SU(5)_G \otimes U(1)_X \otimes U(1)_Z$ and supersymmetry. It is easy to verify that the charge Q_Z is anomally free. Moreover, a linear combination of the standard hypercharge Q_Y and Q_Z shall remain unbroken and can be identified as B-L, i.e.,

$$B - L = \frac{1}{5}\,(2Q_Y + Q_Z)$$

Q_X is conserved up to an $SU(5)_G$ anomaly. Indeed, when $SU(5)_G$ spontaneously breaks down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, Q_X has also an $SU(3)_C$ anomaly and can be used to rotate away the QCD θ angle associated with $F\tilde{F}$. The symmetry $U(1)_X$ is thus the standard $U(1)_{PQ}$ (Peccei-Quinn) [8] symmetry. The new property in this model is that $U(1)_{PQ}$ is spontaneously broken at the scale Λ_{GUM} by the expectation values of φ_1 and φ_{24}, the scalar components of the superfields Φ_1 and Φ_{24}. As a result of this spontaneous symmetry breaking we obtain the standard Weinberg-Wilczek axion [9]. However, in this case the axion decay constant F_a is of order Λ_{GUM} and not $G_F^{-1/2}$ as was the case previously [8,9]. This has two direct consequences. First, it implies that the axion mass m_a is of order $\Lambda^2_{QCD}/F_a \sim 10^{-10}$ eV. Secondly, the axion couples to ordinary fermions via Yukawa couplings g_Y of order m_f/F_a where m_f is the fermion mass. Thus for a typical fermion mass $m_f \sim 1$ GeV we obtain $g_Y \sim 10^{-17}$.

The axion also couples to bosons, e.g., the Higgs doublets via couplings $gm_h h^*ha$ (cubic couplings) or $g^2 h^*ha^2$ (quartic coupling) where g is of order $m_h/F_a \sim 10^{-15}$. Thus we find that the axion essentially decouples from all ordinary matter. The necessary ingredients for our supersymmetric solution to the strong CP problem were a complex scalar singlet (Φ_1) and (24) dimensional representation (Φ_{24}) of $SU(5)_G$ and two scalar Higgs multiplets H and $\bar{H}$. The fields Φ_{24}, H and $\bar{H}$ were absolutely necessary ingredients for the supersymmetric $SU(5)_G$ GUT. Φ_1 was added to obtain the $U(1)_{PQ}$ symmetry.

In the appendix we derive the above results. We also present a simple generalization of our model to a non-supersymmetric SU(5) GUT (F3).

3. Open Problem for SSGUTS

In this section we would like to discuss some open problems which must be faced in any supersymmetric GUT.

A. Gauge hierarchy problem - Supersymmetry per se does not solve the gauge hierarchy problem. It enables scalars to be protected from obtaining quadratic mass corrections down to the scale of spontaneous supersymmetry breaking. In order for the standard Higgs doublets to obtain mass no greater than $\sim G_F^{-1/2}$, supersymmetry in such a scenario must be preserved down to $\sim$ 1 TeV. Hence, the gauge hierarchy problem has been rephrased. Why is the scale of supersymmetry breaking so much less than the grand unification scale. Two suggestions have been made in the literature regarding this question:

1. The first requires a new strong interaction (supercolor) and new fermions carrying this strong charge. It has been shown that fermion condensates spontaneously break supersymmetry [5-7]. Thus if the new fermions condense at low energies when supercolor becomes strong, supersymmetry is spontaneously broken and a huge gauge hierarchy is natural. Such a scenario has, however, two intrinsic problems:

a) Supercolor condensates typically introduce unwanted light pseudo-Nambu-Goldstone bosons. Such a program would thus be counterproductive in view of the solution to the strong CP problem (discussed in sect. 2) which is obtained neglecting supercolor condensates.

b) Recently, T. Banks and V. Kaplunovsky [16] have shown in certain cases that fermions in real representations tend not to condense if their non-Abelian charge can be screened by gluons. This result may have grave consequences for any supercolor scenario. In particular, the supercolor scenario is clearly dead if their results generalize to the statement that fermions in arbitrary representations will not condense when their non-Abelian charge can be screened by massless scalars (F6).

2. The second suggestion relies on "coset" instantons [17] of $SU(5)_{GUT}$ to spontaneously break supersymmetry (thus eliminating supercolor [5]). This would provide a natural solution to the gauge hierarchy problem since the breaking scale would be of order $\sim e^{-8\pi^2/g^2(\Lambda_{GUM})}\Lambda_{GUM}$. Such a scenario is extremely aesthetic. It is, however, still unclear whether such a mechanism actually works in a supersymmetric field theory.

B. Fine tuning - In standard SU(5) GUT there are two separate issues associated with the gauge hierarchy problem. The first has to do with understanding the extremely small ratio $G_F^{-1/2}/\Lambda_{GUM}$ which we discussed previously. The second is associated with the fact that the standard 5 of Higgs contains a color triplet state in addition to the standard Higgs doublet. Since the color triplets mediate proton decay they must have mass of order Λ_{GUM} whereas the doublets are essentially massless at this scale. Thus the parameters (h_1,h_2) [eq. (2.3)] must be fine tuned to obtain this result. In an SSGUT the same fine tuning must be made with, however, two huge distinctions.

a) It is assumed that the Higgs doublets are exactly massless at

Λ_{GUM}. They obtain mass only after supersymmetry breaking at ~ 1 TeV.

b) In a supersymmetric model the fine tuning which is necessary at Λ_{GUM} is not subject to radiative corrections as long as supersymmetry is preserved [5]. It is thus extremely plausible that the ratio h_1/h_2 which is necessary to obtain massless Higgs doublets may be determined by Clebsch-Gordon coefficients of a larger group which includes both the singlet Φ_1 and the 24 (Φ_{24}) in a single representation. An extremely interesting example would be the group E_6 where Φ_{24} and Φ_1 may be contained in the 351' and the Higgs 5 and $\bar{5}$ may be contained in a 27 along with a complete family of quarks and leptons. This possibility is now under study. It would of course also have to satisfy the relation for $(\lambda_1, \lambda_2, \lambda_3)$ [eq. (A.1)] in the potential for Φ_1 and Φ_{24} [eq. (2.3)] in order to obtain the breaking $E_6 \to SU(5) \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

C. Scale invariance and dilatons - The model of sect. 2 [eq. (2.3)] is classically scale invariant.

Hence at Λ_{GUM}, when SU(5) breaks down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, dilatation invariance is also spontaneously broken. This results in a massless Nambu-Goldstone boson - the dilaton. Unlike non-supersymmetric theories, as long as supersymmetry is preserved the minimum of the effective potential of eq. (2.3) remains flat in any finite order of perturbation theory [5]. Thus the dilaton cannot obtain mass until the supersymmetry is spontaneously broken. We thus expect the dilaton δ to obtain mass m_δ of order ~ 1 TeV. It should couple to ordinary matter via a Yukawa coupling $g_{\delta ff}$ of order $\sim m_f/F_\delta$, where m_f is the fermion mass and F_δ is the dilaton decay constant, i.e., $F_\delta \sim \Lambda_{GUM}$. The dilaton would thus be as unobservable as the axion in such a theory.

D. Low energy spectrum - In addition to the standard fermion mass spectrum, the scalar mass spectrum is of particular concern.

Any realistic model must be able to give a large negative mass^2 to the Higgs doublets in order to break the electroweak interactions in the standard way. Moreover, all supersymmetric partners to desired states should obtain large positive mass^2. This includes the scalar partners to ordinary quarks and leptons as well as the fermionic partners to the gauge bosons and Higgs doublets. Although it has been shown to be possible to achieve these goals in a supercolor scenario [7], it is not clear whether they may be obtained in a SSGUT without supercolor [18].

Appendix

Consider the supersymmetric Lagrangian (2.2) and (2.3). We denote the scalar components of Φ_1 and Φ_{24}^A as φ_1, φ_{24}^A , respectively. (A is an $SU(5)_G$ index.) A solution to the potential that preserves supersymmetry and breaks $SU(5)_G$ to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is subject to the following constraints

$$\frac{3\lambda_3}{\sqrt{15}} v_2 + 2\lambda_2 v_1 = 0$$
$$3\lambda_1 v_1^2 + \lambda_2 v_2^2 = 0 \tag{A.1}$$

where $v_1 = \langle\varphi_1\rangle$ and $v_2 = \langle\varphi_{24}^Y\rangle$ and

$$Y = \frac{1}{\sqrt{60}} \begin{pmatrix} -2 & & & & 0 \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ 0 & & & & 3 \end{pmatrix} \tag{A.2}$$

The vacuum expectation values break $U(1)_{PQ}$ as well. If we parameterize

$$\varphi_1 = v_1 + \eta_1 + i\eta_2 \quad ; \quad \varphi_{24}^Y = v_2 + \xi_1 + i\xi_2$$

the corresponding axion can be read off to be

$$a = \frac{v_1\eta_2 + v_2\xi_2}{\sqrt{v_1^2 + v_2^2}} \quad . \tag{A.3}$$

The tree level couplings to bosons are generally given by the relations $\left(m_B^2/F_a\right)aB^*B$ and $\left(m_B^2/F_a^2\right)aaB^*B$ (where $F_a = \sqrt{v_1^2 + v_2^2}$) and its tree level couplings to fermions are given by $(m_F/F_a)a\bar{F}\gamma_5F$. Two exceptions to this general form have to be discussed:

(i) The trilinear coupling of the axion to the scalar fields in H and $\bar{H}$ (ah^*h) vanishes because of parity.

(ii) The axion of (A.3) has no couplings to the ordinary quarks and leptons which are the fermionic components of Y_i and $\bar{X}_i$. But in a realistic model one has to take into account the $SU(2)_L \otimes U(1)_Y$ breakdown caused by a vacuum expectation value v_3 $(v_3/v_1 \sim 10^{-15})$ of the scalar component h of H. Since this breaks $U(1)_{PQ}$ the axion has to be slightly redefined

$$a = \frac{v_1\eta_2 + v_2\xi_2 + v_3h_2}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \quad . \tag{A.4}$$

This redefined axion couples to quarks and leptons $g_Y a\bar{f}\gamma_5 f$ with $g_Y = m_f/F_A \sim 10^{-19}\text{-}10^{-17}$.

The existence of a $U(1)_{PQ}$ symmetry does not necessarily imply supersymmetry. In fact, one can "generalize" the model described above to a usual (non-supersymmetric) $SU(5)_G$ model including <u>complex</u> scalars $\phi_1(1)$, $\phi_{24}(24)$, H(5) and $\bar{H}(\bar{5})$ in addition to three families of fermions $\psi_i(10)$ and $\bar{\chi}_i(\bar{5})$.

$$\begin{aligned} \mathscr{L} = {} & \mathscr{L}_{gauge} - V\left(\phi_1^*\phi_1 \, , \phi_{24}^*\phi_{24} \, , HH^* \, , \bar{H}\bar{H}^* \, , \phi_1^*\phi_{24}\right) \\ & + g_{ij}\,\psi_i\,\psi_j\,H + g'_{ij}\,\psi_i\,\bar{\chi}_j\,\bar{H} + h.c. \\ & + \lambda_1\left(\bar{H}\,\phi_{24}^2\,H + h.c.\right) + \lambda_2\left(\bar{H}\,\phi_1^2\,H + h.c.\right) \quad . \end{aligned} \tag{A.5}$$

This Lagrangian has two global U(1) symmetries $U(1)_X \otimes U(1)_Z$ as given in table 2. $U(1)_X$ again has color anomalies and is broken at Λ_{GUM}. The axion and its couplings to ordinary matter are similar to those in the SSGUT model.

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Footnotes

(F1) We assume that Λ_{GUM} is of order $\sim 10^{17}$ GeV in a SSGUT. For a further discussion of this point see ref. [10].

(F2) The idea of having a $U(1)_{PQ}$ symmetry which is spontaneously broken at a very high scale is originally due to Kim [12]. We thank K. Lane for emphasizing to us the importance of such a possibility. Recently, Dine, Fischler and Srednicki [13] have presented a more economical version of Kim's scenario.

(F3) Similar ideas have been presented in a recent paper by Wise, Georgi and Glashow [15]. We note that the complex scalar φ_1 is in fact superfluous for their solution.

(F4) In supersymmetric theories we are required to have two inequivalent Higgs representations H and $\bar{H}$ in order to give mass to all the quarks and leptons. This is in contrast to standard $SU(5)_G$ where a single Higgs 5 dimensional representation is sufficient. Moreover, the scalar 24 dimensional representation is necessarily complex in a supersymmetric model since all chiral matter fields are complex.

(F5) The dilaton which results from this symmetry breaking is discussed in sect. 3.

(F6) We thank L. Susskind for emphasizing the above consequences.

References

[1] S. Weinberg, Phys. Rev. Lett. 19 (1976) 2364; A. Salam, Proc. 8th Nobel Symposium (Wiley, New York, 1968); S. L. Glashow, J. Illiopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.

[2] See, for example, a recent review by P. Langacker, to appear in Phys. Rep.

[3] E. Gildener, Phys. Rev. D14 (1976) 1667; S. Weinberg, Harvard preprint (1976); A. Buras et al., Nucl. Phys. B135 (1978) 66; Y. Kazama and Y. P. Yao, Fermilab-Pub-81/18-THY.

[4] G. 't Hooft, Phys. Rev. Lett. 37 (1976) i; Phys. Rev. D14 (1976) 3432; R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172; C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Lett. 63B (1976) 334.

[5] E. Witten, Princeton University preprint (1981).

[6] S. Dimopoulos and S. Raby, SLAC preprint SLAC-PUB-2719 (1981).

[7] M. Dine, W. Fischler and M. Srednicki, Princeton IAS preprint (1981).

[8] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791.

[9] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 279; see also W. A. Bardeen and S.-H. H. Tye, Phys. Lett. 74B (1978) 229.

[10] S. Dimopoulos, S. Raby and F. Wilczek, Santa Barbara preprint, NSF-ITP-81-31 (1981).

[11] D. A. Dicus, E. W. Kolb, V. L. Teplitz and R. V. Wagoner, Phys. Rev. D18 (1978) 1829.

[12] J. E. Kim, Phys. Rev. Lett. 43 (1979) 103.

[13] M. Dine, W. Fischler and M. Srednicki, IAS preprint (1981).

[14] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[15] M. B. Wise, H. Georgi and S. L. Glashow, Harvard preprint, HYTP-81/A019 (1981).

[16] T. Banks and V. Kaplunovsky, Tel Aviv University preprint TAUP 930-81 (1981).

[17] See G. 't Hooft, Ref. [4]; V. Baluni, IAS preprint (1980); I. Affleck, Harvard preprint (1980).

[18] See S. Dimopoulos and H. Georgi, in the Proc. of the Second Workshop on Grand Unification, University of Michigan (1981); see also the discussions by P. Fayet, CERN preprint TH.2864-CERN, (1980) in the context of a supersymmetric version of the standard model of strong and electroweak interactions.

Table 1. The global $U(1)_X \otimes U(1)_Z$ symmetry in the supersymmetric SU(5) model. $U(1)_X$ is the Peccei-Quinn symmetry.

$U(1)_X$

$$\Phi_1(x,\theta) \to e^{i\alpha}\,\Phi_1\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$\Phi_{24}(x,\theta) \to e^{i\alpha}\,\Phi_{24}\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$H(x,\theta) \to e^{i\alpha}\,H\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$\bar{H}(x,\theta) \to e^{i\alpha}\,\bar{H}\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$Y_i(x,\theta) \to e^{i\alpha}\,Y_i\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$\bar{X}_i(x,\theta) \to e^{i\alpha}\,\bar{X}_i\left(x, e^{(3/2)i\alpha}\theta\right)$$

$$V_G(x,\theta,\theta^*) \to V_G\left(x, e^{(3/2)i\alpha}\theta\ ,\ e^{-(3/2)i\alpha}\theta^*\right)$$

$U(1)_Z$

$$H(x,\theta) \to e^{-2i\alpha}\,H(x,\theta)$$

$$\bar{H}(x,\theta) \to e^{2i\alpha}\,\bar{H}(x,\theta)$$

$$Y_i(x,\theta) \to e^{i\alpha}\,Y_i(x,\theta)$$

$$\bar{X}_i(x,\theta) \to e^{-3i\alpha}\,\bar{X}_i(x,\theta)$$

all other fields remaining unchanged.

Table 2. The charges of the global $U(1)_X \otimes U(1)_Z$ symmetry in the non-supersymmetric model.

	ϕ_1	ϕ_{24}	H	$\bar{H}$	ψ	χ
X	2	2	− 2	− 2	1	1
Z	0	0	2	− 2	− 1	3