SLAC-PUB-1341
(T)
November 1973

DUAL LIGHT-CONE MODEL FOR THE STRUCTURE FUNCTIONS OF DEEP INELASTIC SCATTERING AND ANNIHILATION*

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ABSTRACT

Abstracted from a dual model, we present a concrete ansatz for the light-cone spectral function $F(\alpha, \beta, t)$ recently discussed by Gatto and Preparata and give the correct continuation prescription from deep inelastic scattering to the annihilation region. We discuss the scaling properties of this ansatz and, in particular, show that the deep inelastic structure functions fulfill a new type of Gribov-Lipatov reciprocity relation.

(Submitted to Phys. Letters B)

* Work supported by the U. S. Atomic Energy Commission and the Max Kade Foundation.

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In a recent paper [1] we have constructed a dual model for the Compton amplitude [2] $A(s,t;q_1^2,q_2^2)$ with the following interesting properties:

(i) The model exhibits generalized Mandelstam analyticity. In particular, it incorporates broad resonances without introducing ancestors. Mandelstam analyticity allowed us to continue the amplitude from the deep inelastic scattering to the annihilation region.

(ii) The good factorization properties of the original Veneziano amplitudes are preserved. The model reduces to a Veneziano type amplitude in the limit of linear trajectories.

(iii) The scaling behavior is intimately connected with the current algebra fixed pole at J = 1. This is achieved by a six-point function ansatz similar to the one of Ademollo and Del Giudice. [3]

In the scaling limit the full amplitude $A(s,t;q_1^2,q_2^2)$, originally written in dual six-point function variables, is reduced to (here we only write the st-term, the ut-term is given by crossing)

$$A(s,t;q_1^2,q_2^2) = -\int_0^1 d\alpha \int_{-(1-\alpha)}^{1-\alpha} d\beta \frac{F(\alpha,\beta,t)}{\left[\frac{q_1^2+q_2^2}{2}(1-\alpha)+\frac{q_1^2-q_2^2}{2}\beta+s\alpha\right]} \tag{1}$$

where

$$F(\alpha,\beta,t) = N\alpha^{-\alpha_t\left(t,\frac{4\alpha}{(1+\alpha)^2-\beta^2}\right)}\left(\frac{(1-\alpha)^2-\beta^2}{4}\right)^c\left(\frac{(1+\alpha)^2-\beta^2}{4}\right)^{-c+c'+\alpha_t\left(t,\frac{4\alpha}{(1+\alpha)^2-\beta^2}\right)-2} \tag{2}$$

and

$$\alpha_t(t,\gamma) = t\alpha_t'(t(1-\gamma)^2)+\alpha(0) \tag{3}$$

The trajectory (3) (and analogous trajectories have been introduced in the s, q_1^2 and q_2^2 channels) was a choice [4] which allowed for Mandelstam analyticity even for positive arguments (provided that $|\alpha'(t)| \leq 0(|t|^{-1/2})$) but maintained the good properties (ii). Here c and c' correspond to constant "trajectories" in the mixed channels having lepton number ≥ 1. The significance of these constants will become clear in the following discussion. For linear trajectories, i.e., $\alpha_t' = \text{const.}$, and $c' = c + 2$ our model coincides with the dual model of Ademollo and Del Giudice [3] which, however, lacks Mandelstam analyticity.

Surprisingly, our form of the Compton amplitude (1) is exactly the same as derived by Gatto and Preparata [5] from a light-cone dominated current commutator. It also can be understood as a DGS-Nakanishi representation [6] with all mass terms neglected. Thus, even though the derivation of Eqs. (1) and (2) from a dual model was very instructive, we can look at our model amplitude as a self-supporting ansatz bearing in mind, however, the origin of the constants c and c'. In the following we shall discuss this particular ansatz for the light-cone spectral function in respect to deep inelastic electroproduction and annihilation.

The corresponding deep inelastic scattering structure function ($s > 0$, $q_1^2 = q_2^2 = q^2 < 0$) $F(x) = \nu W_2 = \frac{2\nu}{\pi} \operatorname{Im} A(s, 0; q^2, q^2)$ is given by

$$F(x) = \int_{-(1-x)}^{1-x} d\beta \; F(x, \beta, 0)$$

$$= N x^{-\alpha(0)+1} (1-x)^{2c+1} \int_{-1}^{+1} d\beta' \left(\frac{1-\beta'^2}{4}\right)^c \left(\frac{(1+x)^2 - (1-x)^2 \beta'^2}{4}\right)^{-c+c'+\alpha(0)-2} \tag{4}$$

where $x = -\frac{q^2}{2\nu}$. Equation (4) clearly exhibits Regge behavior for $x \to 0$ while at threshold, i.e., $x \to 1$, we find $F(x) \sim (1-x)^{2c+1}$.

In the Regge region $A(s, t; q_1^2, q_2^2)$ is dominated by the current algegra fixed pole [7] (which was one of the pillars of our model)

$$A_{FP}(s, t; q_1^2, q_1^2) = -\frac{1}{s} \int_0^1 \frac{d\alpha}{\alpha} \int_{-(1-\alpha)}^{1-\alpha} d\beta \; F(\alpha, \beta, t) \tag{5}$$

Current algebra now requires that the residue be of the form

$$-\int_0^1 \frac{d\alpha}{\alpha} \int_{-(1-\alpha)}^{1-\alpha} d\beta \; F(\alpha, \beta, t) = F_{el}(t) \tag{6}$$

where $F_{el}(t)$ is the electromagnetic (pion) form factor. For $t = 0$ this essentially gives the Adler sum rule [8] as can easily be verified by comparison with Eq. (4).

In the dual model [1] the electromagnetic form factor $F_{el}(t)$ is obtained through factorization at the lowest s-channel pole (i.e., the pion pole) which led to

$$F_{el}(t) = \tilde{N} \int_0^1 dy \; y^{-\alpha_t(t, y)} \, (1-y)^c \tag{7}$$

We can easily check that $F_{el}(t) \sim |t|^{-c-1}$ as $t \to \infty$. This proves the Drell-Yan relation [9] between the threshold behavior of the structure function and the large momentum decrease of the form factor. For large t, expression (7) is also consistent with the left-hand side of Eq. (6). However, in order to achieve consistency for general t we have to include satellite terms [10] $\alpha_t \to \alpha_t - 1$, $\alpha_t - 2$, etc. in $F(\alpha, \beta, t)$.

Now we shall consider the continuation of $A(s, 0; q_1^2, q_2^2)$ to the deep inelastic annihilation region $s, q^2 > 0$, $q_{1,2}^2 = q^2 \pm i\epsilon$. This has previously been discussed by Gatto and Preparata [5] whose ideas we shall closely follow. In this region we expect $A(s, 0; q_1^2, q_2^2)$ to scale with the structure function

$\overline{F}(x) = \nu \overline{W}_2 = \frac{2\nu}{\pi} \text{Im } A(s,0; q^2 + i\epsilon, q^2 - i\epsilon)$. The correct relation between $\overline{F}(x)$ and the analytic continuation of $F(x)$ in x to $x > 1$ now reads

$$\overline{F}(x) = -\text{Re } F(x) + G(x) \tag{8}$$

where

$$G(x) = \frac{1}{2} \text{disc}_x \; \Gamma(x)$$

and

$$\Gamma(x) = \lim_{\lambda \to \infty} \int_{C_\lambda} d\beta \; F(x, \beta, 0) \tag{9}$$

with C_λ shown in Fig. 1.

We differ from Gatto and Preparata [5] in the integration path C_λ. From the explicit form of $F(\alpha, \beta, 0)$ we can easily deduce how their continuation procedure fails. [11] The inequality [5]

$$(\text{Im } F(x))^2 \leq G(x) \, (G(x) + 2 \, \text{Re} F(x)) \tag{10}$$

implies $\text{Im} F(x) = 0$ if $G(x) = 0$. If we now take C_λ as in Ref. [5] we obtain $G(x) = 0$ as long as $c' + \alpha(0) - 2 < -\frac{1}{2}$ since C_λ can then be closed at infinity around the upper half-plane. However, we find that $\text{Im } F(x) \neq 0$ for $x > 1$ and noninteger c which contradicts inequality (10).

For $c' + \alpha(0) - 2 < -\frac{1}{2}$ the contour C_λ can be deformed, e.g., around the left-hand cut (dotted line in Fig. 1) which explicitly gives [12] $(x \geq 1)$

$$G(x) = -N \sin^2 \pi c \; x^{-\alpha(0)+1} \int_{-(1-x)}^{1-x} d\beta \left(\frac{(1-x)^2 - \beta^2}{4} \right)^c \left(\frac{(1+x)^2 - \beta^2}{4} \right)^{-c+c'+\alpha(0)-2}$$

$$= N \sin^2 \pi c \; x^{-\alpha(0)+1} (x-1)^{2c+1} \int_{-1}^{+1} d\beta' \left(\frac{1-\beta'^2}{4} \right) \left(\frac{(1+x)^2 - (1-x)^2 \beta'^2}{4} \right)^{-c+c'+\alpha(0)-2} \tag{11}$$

For integer c (i.e., for multipole behavior of the electromagnetic form factor) this leads to

$$\bar{\bar{F}}(x) = -F(x) \tag{12}$$

where F(x) turns out to be the analytic continuation of F(x) (and thus $\bar{\bar{F}}(x)$ is, in principle, determined by F(x) though, in practice, this is a difficult task). Equation (11) is now consistent with inequality (10) as can be deduced from Eq. (4). Furthermore, we see that the Drell-Yan relation remains valid even in the annihilation region since G(x) has the same threshold behavior as F(x).

For $c' + \alpha(0) - 2 \geq -\frac{1}{2}$ (the physical meaning of c' will be discussed later) Γ(x) does no longer exist. However, we still can derive the general relation (8) with G(x) given in Eq. (11) by first taking the discontinuity of the integral (9) and then letting λ go to infinity. This is justified because, in our model, the discontinuity of the integral (9) vanishes for $|\beta| \geq x - 1$. In other words, the infinite part of Γ(x) has no discontinuity and, hence, does not contribute to G(x). This choice of c' only affects the (singular) behavior of F(x) and G(x) at $x \to \infty$, but does not give rise to violation of scaling as argued by Gatto and Preparata [5].

The continuation procedure so far discussed is fairly academic as it bears very little experimental significance. It seems much more appealing to us to look for a Gribov-Lipatov type of reciprocity relation [13] which connects F(x) and $\bar{\bar{F}}(x)$ in their physical regions. In fact, we find

$$\bar{\bar{F}}(x) = -x^{2c'-1} F\left(\frac{1}{x}\right) \tag{13}$$

For integer c, i.e., in the absence of branch cuts, this can be directly read off from Eq. (4) whereas for noninteger c one has to be a little more careful about the branch cuts (taken care of by G(x)) in deriving Eq. (13). The constant c' (i.e., the mixed channel "trajectory") now controls the large momentum transfer behavior of the electromagnetic $(2^+) \to (1^-)$ transition form factor (e.g., $A_2 \to \rho\gamma$) which was shown to be [10]

$$F_{trans}(t) \sim |t|^{-c'-1} \tag{14}$$

This leads to a Drell-Yan type of relation between the asymptotic behavior of the transition form factor (14) and the large x behavior of $\overline{F}(x)$ provided that F(x) is known. For c' = 2 Eq. (13) gives back the original Gribov-Lipatov reciprocity relation which, in our model, corresponds to $F_{trans}(t) \sim t^{-3}$.

From both the Drell-Yan and the generalized Gribov-Lipatov relation we explicitly see how the light-cone carries some notion of compositeness which provides a new point of view. The relation between dual and light-cone models in general deserves further investigations.

We would like to thank R. Blankenbecler for helpful comments and S. D. Drell for the warm hospitality extended to us at SLAC.

REFERENCES

[1] G. Schierholz and M. G. Schmidt, Stanford Linear Accelerator Center Report, to be published.

[2] Our notation is $T_{\mu\nu} = P_{\mu} P_{\nu} A + \cdots$ where $P = p_1 + p_2$.

[3] M. Ademollo and E. Del Giudice, Nuovo Cimento 63A, 639 (1969).

[4] L. Gonzales Mestres and R. Hong Tuan, Orsay preprint (1973).

[5] R. Gatto and G. Preparata, Nucl. Phys. B 47, 313 (1972).

[6] S. Deser, W. Gilbert and E.C.G. Sudarshan, Phys. Rev. 115, 731 (1959); N. Nakanishi, Progr. Theoret. Phys. 26, 337 (1961).

[7] Note that the variable α is exactly the infinite momentum variable appearing in the parton model. See for example: S.J. Brodsky, F. E. Close and J. F. Gunion, Stanford Linear Accelerator Center Report No., SLAC-PUB-1243 (1973).

[8] S. L. Adler, Phys. Rev. 143, 1144 (1966); J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[9] S.D. Drell and T. M. Yan, Phys. Rev. Letters 24, 181 (1970).

[10] For details see Ref. [1].

[11] The source, why the continuation procedure developed by Gatto and Preparata fails, lies in the wrong symmetry argument leading to their Eqs. (2.16) and (2.17). From $F(\alpha,\beta) = F(\alpha, -\beta)$ we obtain

$$\int_{-\lambda}^{0} d\xi\, F(x(1+\frac{\xi}{\lambda}), \xi + i\epsilon) = \int_{-\lambda}^{0} d\xi\, F(x(1+\frac{\xi}{\lambda}), -\xi - i\epsilon)$$

$$= \int_{0}^{\lambda} d\xi\, F(x(1-\frac{\xi}{\lambda}), \xi - i\epsilon)$$

which results in our different integration path C_λ. This path guarantees the symmetry in q_1^2 and q_2^2.

[12] Note that one has to be very careful applying symmetry arguments since the symmetry depends on the choice of the branch cuts.

[13] V. N. Gribov and L. N. Lipatov, Phys. Letters 37B, 78 (1971); Yad. Fiz 15, 781 (1972), (Sov. J. Nucl. Phys. 15, 438 (1972)); for scaling models and further literature see: G. Schierholz, Stanford Linear Accelerator Center Report No., SLAC-PUB-1302, to be published in Phys. Letters B.

FIGURE CAPTION

1. Integration path C_λ.

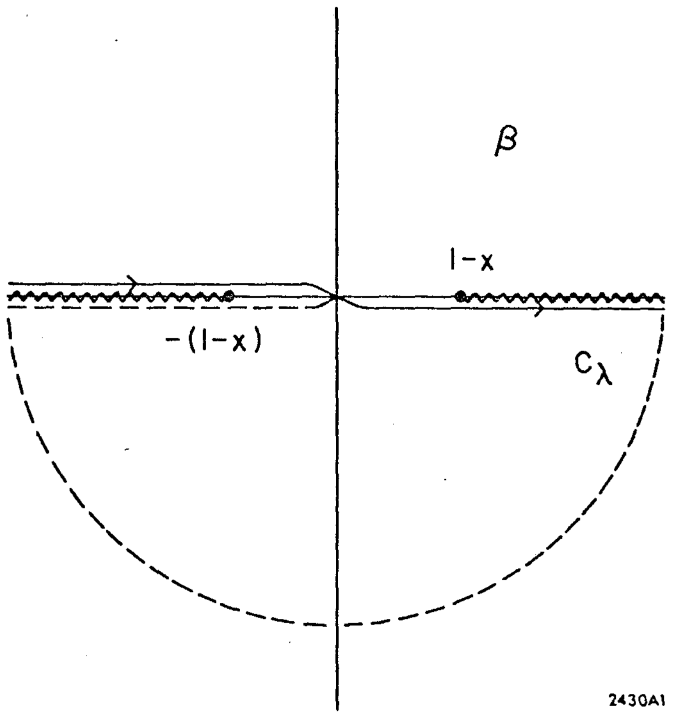

Fig. 1