

FAKE-GRIND STUDIES OF 50-GeV 4-PRONG PION-INDUCED INTERACTIONS  
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## ABSTRACT

A set of experiments in the 25-foot bubble chamber was simulated and analyzed using FAKE and GRIND. The reactions generated were  $\pi^- + p \rightarrow \pi^- + p + \pi^+ + \pi^- + X$  at 50 GeV/c, where X represents no missing neutral, a fast  $\pi^0$ , a slow  $\pi^0$ , or two fast  $\pi^0$ 's. Setting errors of 150 $\mu$  and 500 $\mu$  were employed. Fits were attempted in each case to pion hypotheses with no neutral, a missing  $\pi^0$ , or a missing neutron. The 4c events were uniquely identifiable and were not faked by other events. The majority of the 1c events were identifiable on the basis of  $\chi^2$ . Events with slow  $\pi^0$ 's were, of course, the most likely to confuse the fitting program. A sample of 4c events in  $K^+ p \pi^+ \pi^-$  was briefly studied.

## I. INTRODUCTION

During the 1968 Summer Study, FAKE and GRIND were used to simulate and analyze strong-interaction experiments in the 12- and 25-foot bubble chambers as a function of chamber parameters.<sup>1</sup> Only events without missing neutrals were FAKE. It is appropriate now to extend such an investigation to reactions in the 25-foot chamber where additional neutrals are produced. The parameters of the 25-foot chamber for this study have been fixed at:

Size - a cylinder 4 m in diameter and 3 m high  
Field - 40 kG  
Setting error - 150 $\mu$  or 500 $\mu$ .

During the 1969 Summer Study optimism was expressed that this chamber's optical properties might be much better than anticipated. The value of  $\epsilon = 150\mu$ , while a factor of two worse than any existing chamber, represents a probable lower limit for the 25-foot, while  $\epsilon = 500\mu$  has been adopted as a more pessimistic value.

The current study is very limited in scope. Only one kind of reaction,  $\pi^- p \rightarrow p \pi^+ \pi^- \pi^- + n \pi^0$  ( $n = 0, 1, 2$ ), has been generated. The beam momentum has been chosen to be 50 GeV/c. Thus we are considering an energy region where one

would hope that the 25-foot chamber (and the fitting program GRIND as well) would not be strained to yield well-analyzable events.

There have been a number of "hand calculations" in the 1968 and 1969 Summer Studies addressed to the same questions considered here. We anticipate no great surprises in this Monte-Carlo study but rather a corroboration of these estimates under experimentally more realistic conditions.

## II. FAKE AND EVENT GENERATION

For the principal study, four reactions were generated:

1.  $\pi^- p \rightarrow p_p \pi^+ \pi^- \pi^-$  ("4c")
2.  $\pi^- p \rightarrow p_p \pi^+ \pi^- \pi^- \pi^0$  ("FAST  $\pi^0$ ")
3.  $\pi^- p \rightarrow N_p^{*+} \pi^+ \pi^- \pi^-$  ("SLOW  $\pi^0$ ")  
 $\quad \quad \quad \downarrow$   
 $\quad \quad \quad p \pi^0$
4.  $\pi^- p \rightarrow p_p \pi^+ \pi^- \pi^- \pi^0 \pi^0$  (" $\pi^0 \pi^0$ ")

In the above reactions, a subscript "p" means "peripheral"; this particle was given an angular distribution of  $e^{6t}$  in the center-of-mass. Therefore, in reactions 1, 2, and 4 the proton is slow in the laboratory, and in reaction 3 both the proton and  $\pi^0$  are slow. The remaining particles are phase-space distributed. Thus in reactions 2 and 4 the  $\pi^0$ 's tend to have high momentum.

Figures 1-3 display the momentum distributions of particles in these events. Only those events which passed the tests described in Sec. 3 were included. In Fig. 1 we observe a depletion of events with very small recoil proton momenta. This is a result of an uncorrected "bug" in GRIND, which excluded some events with short stopping protons from the sample of analyzable events. This bias, however, does not materially alter our conclusions.

The errors incorporated by FAKE are those described in Ref. 2 and correspond to an 8-point measurement of each track with optimal distribution of the points. Two sets of events were generated, one with  $\epsilon$ , the measuring error, equal to  $150\mu$ , and another with  $\epsilon = 500\mu$ . In the following, the results of studying events under these two conditions will be described separately where appropriate.

## III. GRIND AND EVENT ANALYSIS

GRIND was run with conventional parameters to restrict the fitting procedure (number of steps, standard deviations, etc.) The constraint equations were required to be satisfied to within 1 MeV for a fit to be considered acceptable. Typically, only 1 or 2 steps were required for fits.

The measured beam momentum was averaged with an input value of  $50.00 \pm 0.05$  GeV/c ( $\pm 0.1\%$ ), but the measured azimuth and dip were used in the fit. Because of this, our analysis sample has been limited to only those events which had a beam track of minimum length 50 cm. In addition, a maximum length of 400 cm was required to permit 4 meters of potential outgoing track length. Not all tracks are this long since secondary interactions occur on outgoing tracks and the tracks are considered measurable only up to those scatterings. For this reason, a minimum length of 50 cm was required of all (non-stopping) outgoing tracks.

After the above cuts were made, sample sizes ranged from 23 to 55 so-called "good events."

The events were fitted to the hypotheses

$$\begin{aligned}\pi^- p &\rightarrow p \pi^+ \pi^- \pi^- \\ \pi^- p &\rightarrow p \pi^+ \pi^- \pi^- \pi^0,\end{aligned}$$

with both permutations of  $\pi^+$  and  $p$  being tried. A 4c fit was considered acceptable if it had a  $\chi^2$  probability of  $\geq 0.01\%$ , while a 1c fit had to have a probability  $\geq 2\%$ . Fits passing this test were excluded if they could have been rejected by a bubble-density measurement on the scanning table. In reality, since protons are likely to be more peripheral than those generated here,  $\pi^+ - p$  ambiguities will rarely if ever occur. A sub-sample of each type of event ( $\sim 50\%$ ) was also fitted to hypotheses where there were missing neutrons and  $K\bar{K}$  pairs replaced  $\pi^+ \pi^-$  or  $\pi^\pm \pi^0$  pairs. This was done only for  $\epsilon = 150\mu$ . The results of this analysis are described separately in Sec. 7.

In summarizing the results of the fitting program, great emphasis has been placed on the specific  $\chi^2$  value for each hypothesis. This is appropriate to do in a study such as this, where the errors are exactly understood. In practice, however, many events which we here "distinguish" on the basis of small differences in  $\chi^2$  probability would be considered hopelessly ambiguous and not useful for physics analysis.

#### IV. 4C EVENTS

In a sample of 55 good events with  $\epsilon = 150\mu$ , 53 had a unique fit to the correct 4c hypothesis (n, K fits excluded, see Sec. 7). Two events had no good fit. The sample for  $\epsilon = 500\mu$  consisted of 53 good events, all of which had unique 4c fits (n, K fits excluded). These results and those for other reactions, are summarized in Table I.

The distributions of measured longitudinal ( $\parallel$ ) and transverse ( $\perp$ ) momentum imbalance for these events are shown in Figs. 4 and 5.

We see that 80% of the events have

	$\epsilon = 150\mu$	$\epsilon = 500\mu$
$ \Delta p_{\parallel}  <$	300 MeV/c	1150 MeV/c
$\Delta p_{\perp} <$	60 MeV/c	240 MeV/c

while 50% of the events have

	$\epsilon = 150\mu$	$\epsilon = 500\mu$
$ \Delta p_{\parallel}  <$	100 MeV/c	150 MeV/c
$\Delta p_{\perp} <$	35 MeV/c	100 MeV/c

#### V. $\pi^0$ EVENTS

1. FAST  $\pi^0$ 's. In a sample of 47 good events with  $\epsilon = 150\mu$ , 46 fit only the correct 1c hypothesis (for n, K fits see Sec. 7). One event had a 1c - 1c ambiguity in which the correct hypothesis had the smaller  $\chi^2$ .

The event sample for  $\epsilon = 500\mu$  consisted of 44 events, of which 41 fit only the correct hypothesis (n, K fits excluded). Of the remaining 3 events with 1c - 1c ambiguities, two were resolved correctly on the basis of  $\chi^2$ .

The measured missing-mass-squared distribution, for the correct assignment of track identities, is shown in Fig. 7.

2. SLOW  $\pi^0$ 's. For 52 events with  $\epsilon = 150\mu$ , 47 had a unique 1c fit. Three events with 1c - 1c ambiguities were correctly resolved by  $\chi^2$ , and two events had no fit. (For n, K fits see Sec. 7.)

Of 51 events with  $\epsilon = 500\mu$ , 45 had a unique 1c fit, and two 1c - 1c ambiguities were correctly resolved by  $\chi^2$ . Of the remaining 4 events, one had a wrong unique 1c fit, one ambiguous event was wrong by  $\chi^2$ , one event had no fit, and finally one event had a 4c fit with  $P(\chi^2) = 0.09\%$ . (This was the only case in the entire study where a 1c event faked a 4c.) These results are summarized in Table I.

The missing-mass-squared distribution for fast and slow  $\pi^0$  events is shown in Fig. 6.

#### VI. $\pi^0 \pi^0$ EVENTS

The sample with  $\epsilon = 150\mu$  consists of 24 good events, only one of which has a 1c fit. (For n, K fits see Sec. 7.) For  $\epsilon = 500\mu$ , only 2 of 23 events have a 1c fit (see Table I).

This apparent excellent distinction from single  $\pi^0$  events is somewhat illusory. From Fig. 6, we see that for the  $\pi^0\pi^0$  events the missing mass is very large since both  $\pi^0$ 's tend to have high momentum. Only the very lowest missing mass events can therefore be confused with single  $\pi^0$  reactions.

## VII. NEUTRON AND KAON FITS

About half of the FAKE events (all in the case of  $\pi^0\pi^0$ ) of each reactions were fitted in GRIND to a set of additional hypotheses:

$$\begin{aligned}\pi^- p &\rightarrow \pi^+ \pi^+ \pi^- \pi^- n \\ &\rightarrow p K^+ K^- \pi^- (\pi^0) \\ &\rightarrow p K^+ \pi^- \pi^- \bar{K}^0 \\ &\rightarrow p \pi^+ K^- \pi^- K^0\end{aligned}$$

including all mass permutations. Only the sample with  $\epsilon = 150\mu$  was considered. The results are summarized in Table II.

Over 50% of 4c events fit also as 4c's with a  $K^+K^-$  pair replacing the  $\pi^+$  and one  $\pi^-$ . The energy constraint is likely to allow this when both  $\pi^+$  and  $\pi^-$  are fast; for the 16 ambiguous events the particles in question have momenta  $\geq 10$  GeV. On the basis of  $\chi^2$ , we would erroneously choose a 4c fit with kaons 10% of the time.

Fifty percent of the fast  $\pi^0\pi^0$  events are also ambiguous with 1c kaon fits (about half with  $K^+K^-$  replacing  $\pi^+\pi^-$ , and half with  $K^\pm K^0$  replacing  $\pi^\pm\pi^0$ ). Here we again misidentify  $\sim 10\%$  of the events by  $\chi^2$ .

The SLOW  $\pi^0\pi^0$  events almost always have a competing kaon fit, but only one fits best with a missing neutron.

Finally, one-fourth of the  $\pi^0\pi^0$  events fit with charged or neutral kaons. Clearly there is a problem in event identification if the possibility of kaons in the final state is allowed. Missing neutron fits, however, are not a source of contamination in the events studied here. For a setting error of  $500\mu$ , we might expect the problem to be significantly worse.

## VIII. REACTION $K^+p \rightarrow pK^+\pi^+\pi^-$

A small number of events was generated of the reaction  $K^+p \rightarrow pK^+\pi^+\pi^-$  at 50 GeV/c, with both  $\epsilon = 150$  and  $500\mu$ . These events were fitted to the appropriate  $K^+p$  hypothesis with no missing neutrals but with the 6 possible permutations of positive particle mass assignments. The results of the fits illustrate very vividly the problems to be faced in attempting to do such physics in a bubble chamber.

Of 14 events with  $\epsilon = 150\mu$ , eleven had an unresolvable (by ionization) 4c - 4c ambiguity involving a  $\pi^+$  replacing a fast  $K^+$ . Nine of the eleven events were correctly identified by  $\chi^2$ .

Of 14 events with  $\epsilon = 500\mu$ , twelve had an unresolvable 4c - 4c ambiguity, of which eight could be correctly identified by  $\chi^2$ .

The problem of  $K^+ - \pi^+$  ambiguity is of course important even at today's higher bubble-chamber experiment energies.

#### IX. CONCLUSIONS

The utility of the 25-foot chamber for 4c pion physics is clearly established so long as K,  $\pi$  ambiguities do not require resolution. In the same sense, the chamber will reliably identify events with a missing  $\pi^0$  (i.e., few will be lost from the sample) although multi- $\pi^0$  events with small missing masses will undoubtedly create a significant background.

#### X. ACKNOWLEDGMENTS

I have profited greatly from conversations with Drs. L. Eisenstein and N. Gelfand. Dr. Richard Abbott was of great assistance in running the programs. The completion of this work was in no small way due to the enthusiastic cooperation of the staff of the Computing Center at the University of Colorado in Boulder.

#### REFERENCES

- <sup>1</sup>M. Derrick et al., FAKE Studies on Some Strong and Weak Interactions in the 12-Ft and 25-Ft Bubble Chambers, National Accelerator Laboratory 1968 Summer Study Report A. 1-68-89-II, Vol. I, p. 47.
- <sup>2</sup>M. Derrick et al., FAKE Studies on Some Strong and Weak Interactions in the 12-Ft and 25-Ft Bubble Chambers, National Accelerator Laboratory 1968 Summer Study Report A. 1-68-89-I, Vol. I, p. 29.

Table I. Results of Fitting to Pion Hypotheses.

Reaction	$\epsilon(\mu)$	No. of Events	No. Fitting Correct Hypothesis Uniquely	No. Ambiguous but Correctly Selected by $\chi^2$	No. of Wrong Fits by $\chi^2$	No. not Fitting Any Hypothesis
4c	150	55	53	0	0	2
"	500	53	53	0	0	0
FAST $\pi^0$	150	47	46	1	0	0
"	500	44	41	2	1	0
SLOW $\pi^0$	150	52	47	3	0	2
"	500	51	45	2	3 <sup>a</sup>	1
$\pi^0 \pi^0$	150	24	-	0	1 <sup>b</sup>	23
"	500	23	-	0	2 <sup>b</sup>	21

<sup>a</sup>One event fakes 4c with prob. = 0.09%

<sup>b</sup>1c fits

Table II. Results of Fitting to Larger Set of Hypotheses.

Reaction	No. of Events	No. Fitting Correct Hypothesis Uniquely	No. Ambiguous but Correctly Selected by $\chi^2$	No. of Wrong Fits by $\chi^2$	No. not Fitting Any Hypothesis
4c	29	10	16	3	0
FAST $\pi^0$	18	9	7	2	0
SLOW $\pi^0$	34	3	13	17 <sup>a</sup>	1
$\pi^0 \pi^0$	24	-	0	6	18

<sup>a</sup>One event fits missing neutron.

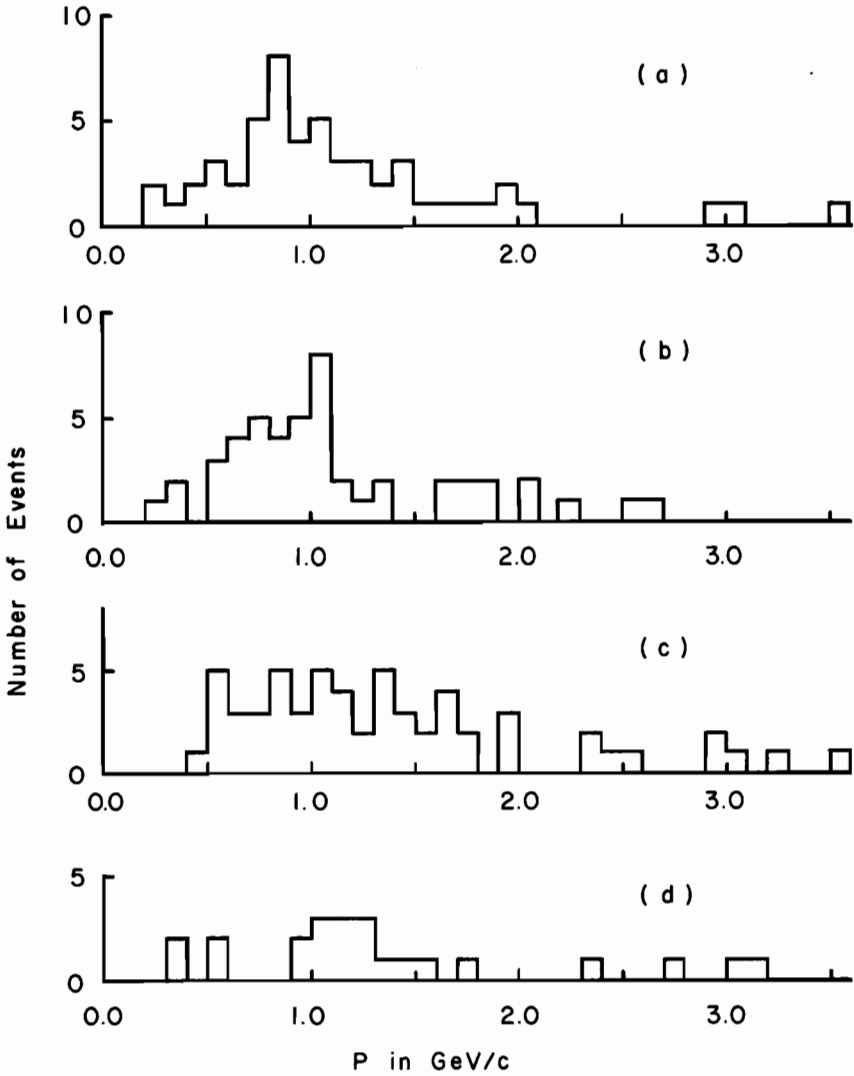


Fig. 1. Outgoing proton momenta for some of the generated events: a) 4c, b) FAST  $\pi^0$ , c) SLOW  $\pi^0$ , d)  $\pi^0 \pi^0$ .



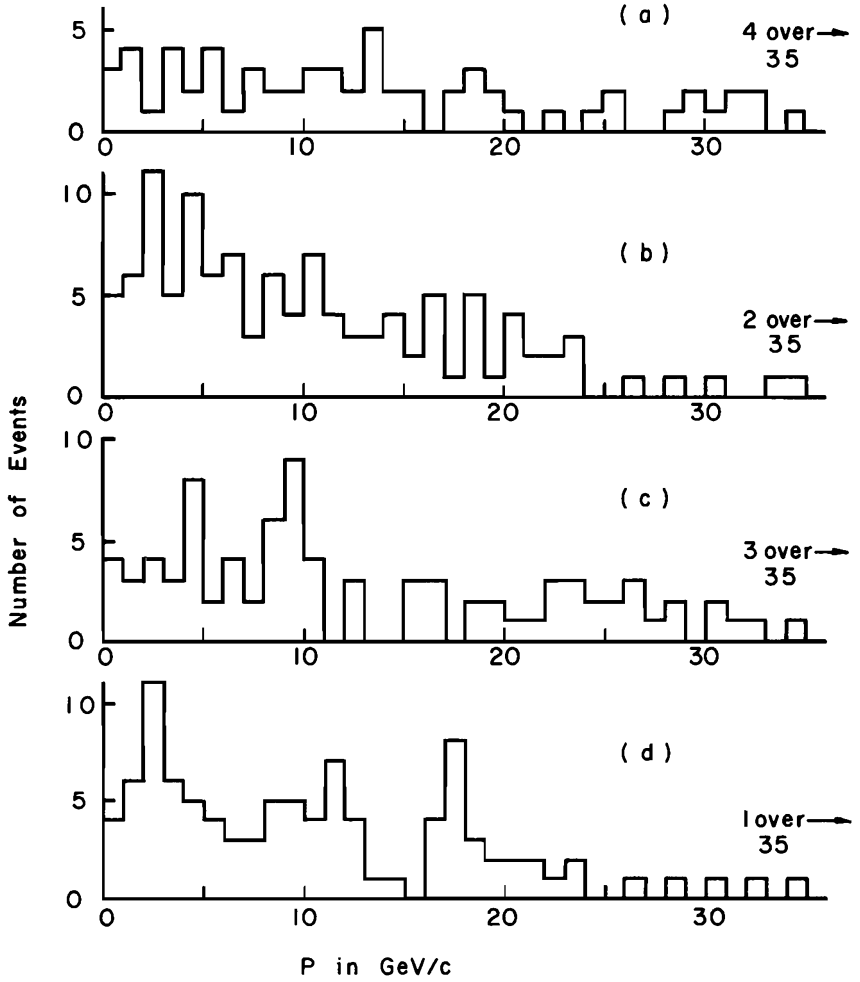


Fig. 2. Momenta of outgoing charged tracks for some of the generated events: a)  $4c$ , b) FAST  $\pi^0$ , c) SLOW  $\pi^0$ , d)  $\pi^0 \pi^0$ .

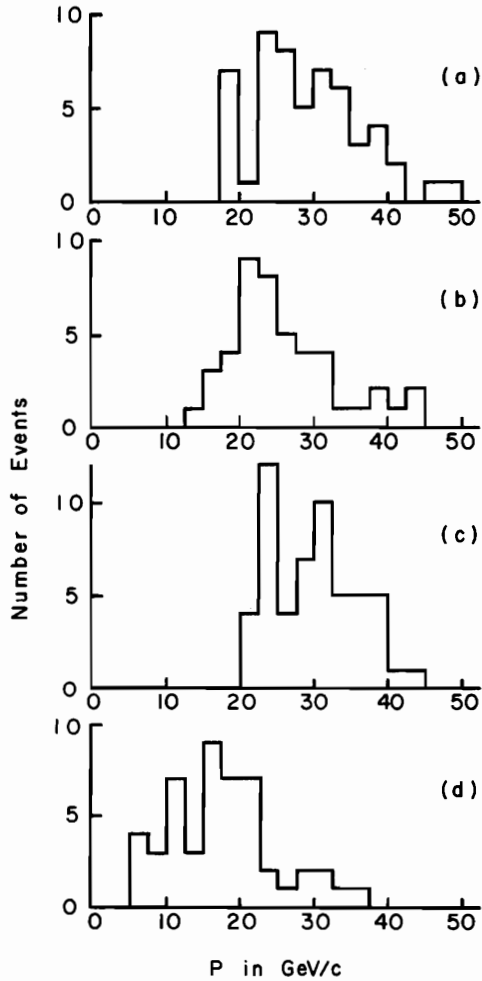


Fig. 3. Momentum of fastest outgoing particle for some of the generated events. The  $\pi^0$  is included in b) and c), a) 4c, b) FAST  $\pi^0$ , c) SLOW  $\pi^0$ , d)  $\pi^0 \pi^0$ .

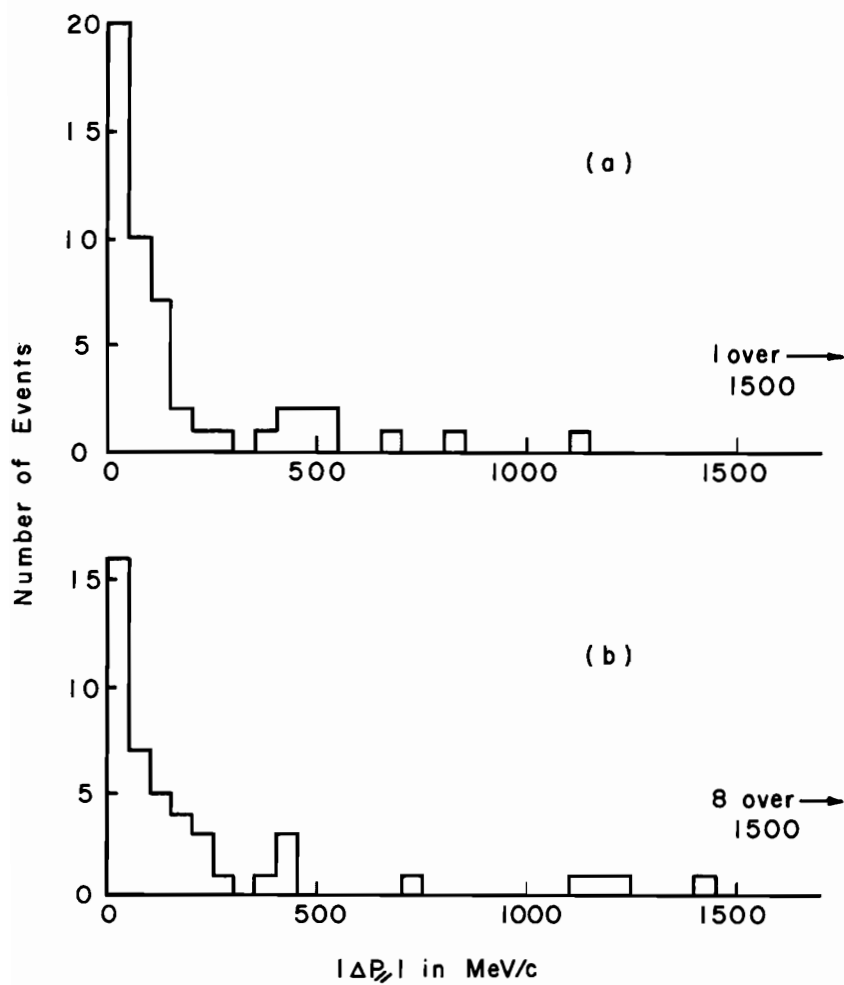


Fig. 4. Magnitude of longitudinal momentum imbalance for 4c events, a)  $\epsilon = 150\mu$ , b)  $\epsilon = 500\mu$ .

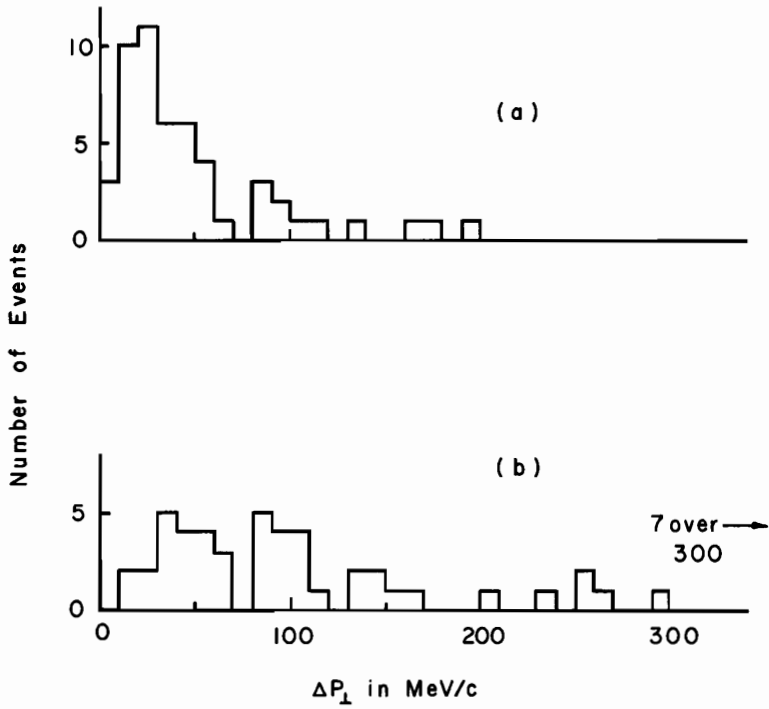


Fig. 5. Magnitude of transverse momentum imbalance for 4c events, a)  $\epsilon = 150\mu$ , b)  $\epsilon = 500\mu$ .

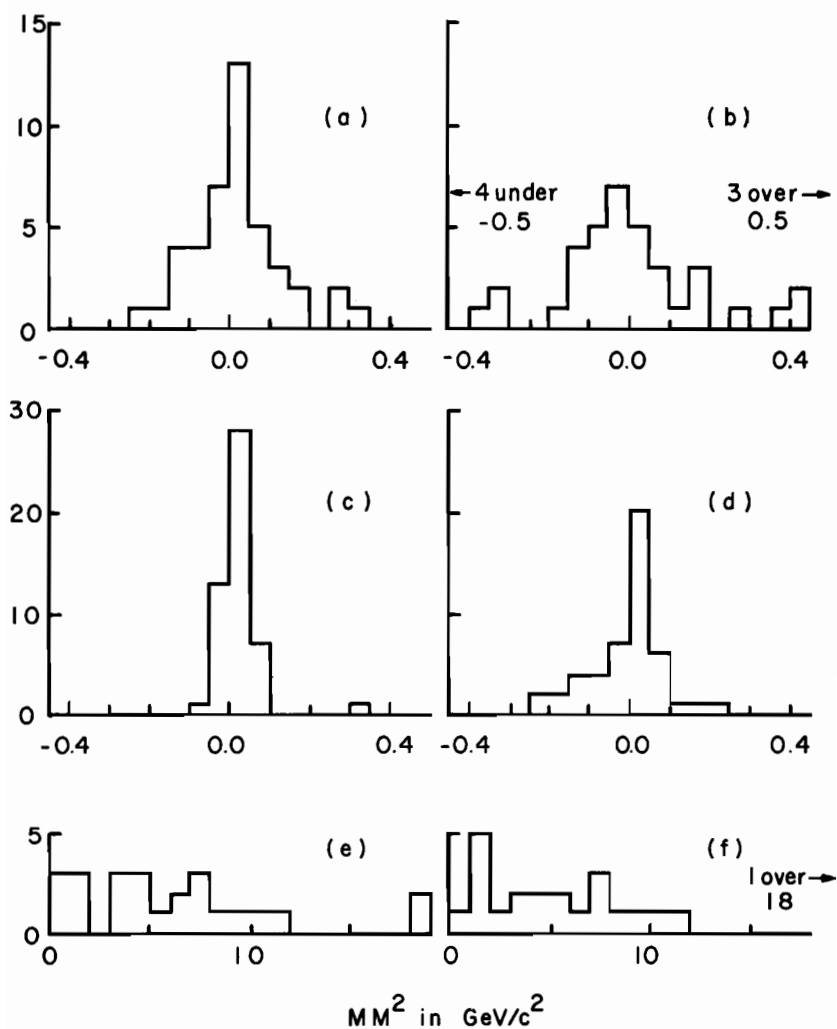


Fig. 6. Measured missing mass squared for correct track assignments, a) FAST  $\pi^0$ ,  $\epsilon = 150\mu$ , b) FAST  $\pi^0$ ,  $\epsilon = 500\mu$ , c) SLOW  $\pi^0$ ,  $\epsilon = 150\mu$ , d) SLOW  $\pi^0$ ,  $\epsilon = 500\mu$ , e)  $\pi^0 \pi^0$ ,  $\epsilon = 150\mu$ , f)  $\pi^0 \pi^0$ ,  $\epsilon = 500\mu$ .

