CONSERVED AND PARTIALLY CONSERVED CURRENTS IN THE THEORY OF WEAK INTERACTIONS

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Feynman has said that the experimental situation is not very clear in connection with the conserved vector current. He said also that if the conserved vector current hypothesis crumbles, nothing will crumble with it. It is true that there is not a vast structure of theory that will fall down, but there is some structure, and I would like to talk about that sort of structure which will crumble when the conserved vector current hypothesis is exploded.

The people who have engaged in speculation about partially conserved currents include Schwinger, Bludman, Glashow, Gürsey, Nambu, and Salam. Some remarks relevant to the subject have been made by Polkinghorne and by J. C. Taylor. I have been thinking about such matters during the last year in collaboration with Lévy, Michel, Bernstein, Fubini and Thirring.

If the hypothesis of the conserved $\Delta S = 0$ vector current is correct (and let us assume that it is), does it not suggest something about the behavior of the other currents? Is it not a little peculiar for that one current to be equal to something as important as the isotopic spin current, while there is no corresponding principle for the others? Such asymmetry is unattractive and one reason at least for the kind of research I am going to describe is to remove that assymetry.

One way to do so, which Feynman has discussed, is to adopt the idea that in the world of baryons and mesons there are no other fundamental particles except n, p, and Λ (and perhaps a neutral vector glue). Then $\bar{n}\gamma_\alpha p$ would be the isotopic spin current and if the $\Delta S = 0$ axial current were just $\bar{n}\gamma_\alpha\gamma_5 p$, the two could be quite symmetrical and no asymmetry would exist.

If we think, however, that mesons really exist and that they are present in the Lagrangian, then we find a striking asymmetry between a simple axial current of the form $\bar{n}\gamma_\alpha\gamma_5 p$ and a vector current that has additional terms pertaining to π mesons, etc.:

$$\bar{n}\gamma_\alpha p + \sqrt{2}\left(\pi^0\frac{\partial\pi^+}{\partial x_\alpha} - \pi^+\frac{\partial\pi^0}{\partial x_\alpha}\right) + \dots$$

There is another question that comes up at the same time. We like to talk about the universality of the weak interactions but what does universality mean? Presumably it means that the same $\sqrt{G}$ occurs everywhere, in the lepton currents, baryon currents, and meson currents. But do we allow a $\sqrt{2}$ sometimes? What if we re-define the fields, for example by choosing $\frac{\Lambda+\Sigma^0}{\sqrt{2}}$ and $\frac{\Lambda-\Sigma^0}{\sqrt{2}}$ instead of Λ and Λ^0? By re-expressing the currents in terms of new fields we can make factors like $\sqrt{2}$ appear and disappear at will. The concept of universality becomes very slippery unless we can define it in a representation-independent way.

If there are no meson terms in the current, then we might try to get a reasonable definition independent of the framework by making a matrix of the coefficients of the bilinear form in the fermion fields that constitutes the current (say with $\gamma_\alpha(1+\gamma_5)$ everywhere) and looking at the eigenvalues of the matrix. The definition of universality would refer to the eigenvalues.

But if there are boson terms as well as fermion terms in the weak current, then the two classes of terms are not easily comparable and the definition of universality is again a problem.

Some of us have been wondering about the possibility of a new kind of principle to determine the weak current, a principle which might help to overcome the difficulties of V-A asymmetry and of the vagueness of universality. It was first stated clearly in the literature by Bludman, I believe, and it is based on an analogy with the principle of minimal electromagnetic interaction, which is also a speculation, but a fairly well-founded one.

Consider the electromagnetic interaction to lowest order in e, just as later we will take up the weak interactions to lowest order. We have the electric charge operator Q, and universality simply requires its eigenvalues to be integral multiples of e. Using Q, we can perform an infinitesimal gauge transformation in which every field undergoes the change

$$\psi(x)\to\psi(x)-i\lambda(x)[Q,\psi(x)]$$

where $\lambda(x)$ is a gauge function depending on space and time.

Call the Lagrangian of the leptons or of the strong interactions of baryons and mesons $\mathcal{L}$. Then we have

$$\mathcal{L}\to\mathcal{L}+\frac{\delta\mathcal{L}}{\delta\lambda}\lambda+\frac{\delta\mathcal{L}}{\delta\partial_\alpha\lambda}\partial_\alpha\lambda$$

Even though λ is a gauge variable and not a field variable, you can show that Lagrange's equation holds:

$$\partial_\alpha\frac{\delta\mathcal{L}}{\delta\partial_\alpha\lambda}=\frac{\delta\mathcal{L}}{\delta\lambda}$$

We now define the minimal electric current j_α to be $i\dfrac{\delta\mathcal{L}}{\delta\partial_\alpha\lambda}$. If the Lagrangian is made invariant under gauge transformations with constant λ (transformations of the first kind), then $\dfrac{\delta\mathcal{L}}{\delta\lambda}=0$ and we have the conservation of charge $\partial_\alpha j_\alpha=0$.

Now, the corresponding proposal for the weak interactions. I shall discuss charge exchange currents only, without prejudice to the question of whether there are also neutral or charge retention currents. Let the charge exchange current (apart from a constant factor) be W_α^+ and its hermitian conjugate W_α^-. From the sum and difference of these we can define $W_{\alpha x}$ and $W_{\alpha y}$; I shall also adjoin for convenience a fictitious charge retention current $W_{\alpha z}$. We suppose that there is a weak charge operator $\mathbf{W}$ analogous to Q and that under $\mathbf{W}$ the fields undergo infinitesimal gauge transformations

$$\psi(x)\to\psi(x)-i\mathbf{t}(x).[\mathbf{W},\psi(x)]$$

while the Lagrangian changes correspondingly:

$$\mathcal{L}\to\mathcal{L}+\frac{\delta\mathcal{L}}{\delta\mathbf{t}}\cdot\mathbf{t}+\frac{\delta\mathcal{L}}{\delta\partial_\alpha\mathbf{t}}\cdot\partial_\alpha\mathbf{t}\,.$$

Again we have the Lagrange equation

$$\partial_\alpha\frac{\delta\mathcal{L}}{\delta\partial_\alpha\mathbf{t}}=\frac{\delta\mathcal{L}}{\delta\mathbf{t}}$$

and the weak current is defined as $\mathbf{W}_\alpha\equiv i\dfrac{\delta\mathcal{L}}{\delta\partial_\alpha\mathbf{t}}$. Then we have

$$\partial_\alpha\mathbf{W}_\alpha=i\frac{\delta\mathcal{L}}{\delta\mathbf{t}}\,.$$

To the extent that the Lagrangian is invariant under gauge transformations with constant $\mathbf{t}$, the weak current is conserved. The non-invariant terms in $\mathcal{L}$ generate the non-zero divergence of the weak current.

For baryons and mesons we may divide $\mathbf{W}$ into four terms that generate the four pieces of the current:

$$\mathbf{W}=\tfrac{1}{2}(\underset{\substack{\Delta s=0\\ V}}{\mathbf{C}}+\underset{\substack{\Delta s=0\\ A}}{\mathbf{D}}+\underset{\substack{\Delta s=1\\ V}}{\mathbf{F}}+\underset{\substack{\Delta s=1\\ A}}{\mathbf{G}})\,.$$

I suppose that $\mathbf{C}$ and $\mathbf{D}$ transform like $|\Delta\mathbf{I}|=1$ and $\mathbf{F}$ and $\mathbf{G}$ like $|\Delta\mathbf{I}|=\tfrac{1}{2}$.

The conserved vector $\Delta S=0$ current hypothesis is simply the notion that $\mathbf{C}=\mathbf{I}$, the isotopic spin. Since the entire strong-coupling Lagrangian is invariant under $\mathbf{I}$, the divergence of this part of the current is then zero. For the other three operators, there must be terms in the Lagrangian that violate their conservation, but there can still be a strong symmetry among the operators $\mathbf{C}$, $\mathbf{D}$, $\mathbf{F}$, and $\mathbf{G}$, and the various currents are generated in similar ways. The similarity is imperfect only in that $\mathbf{C}$ is exactly conserved and the others not.

The definition of universality no longer presents any difficulties. If $\mathbf{W}$ always has the same commutation rules, it will have only a limited set of possible eigenvalues and universality would pertain to the eigenvalues.

Glashow has examined the commutation rules of $\mathbf{W}$ for the leptons, for which only the free Lagrangians must be included, since there are no strong interactions. Suppose, for the sake of definiteness, that there are two neutrinos, one for the electron and one for the muon. We have weak currents for the pairs $(e\nu)$ and $(\mu\nu')$. Let e_L be the left-handed part of the electron field $e_L \equiv \frac{1+\gamma_5}{2}e$ and $e_R \equiv \frac{1-\gamma_5}{2}e$. For the neutrino, there is only the left-handed part ν_L. We deal then with ν_L, e_L, e_R; ν'_L, μ_L, μ_R. The anti-particles come in automatically in the usual way.

The operator $\mathbf{W}$ for the leptons can now be written simply as a matrix connecting ν_L, e_L and e_R (or the other three). W_+ takes e_L into ν_L and W_- does the reverse. We have

$$W_x = \begin{array}{c c} & \begin{array}{ccc} \nu_L & e_L & e_R \end{array} \\ \begin{array}{c} \nu_L \\ e_L \\ e_R \end{array} & \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$W_y = \begin{pmatrix} 0 & -{}^i/_2 & 0 \\ {}^i/_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

If we put

$$W_z = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

then we have the result that $\mathbf{W}$ is an angular momentum.

Glashow has pointed out another interesting feature. If we write a fourth matrix

$$W_4 = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{then } [W_4, \mathbf{W}] = 0$$

and the electric charge $Q = e(W_4 + W_z)$. Also $W_4^2 + W_x^2 + W_y^2 + W_z^2$ is unity—I do not know how important that is.

The symmetry of the lepton Lagrangian under $\mathbf{W}$ is broken only by the electron and muon mechanical mass terms, which thus give rise to the non-conservation of $\mathbf{W}_\alpha$.

If we really believe in universality, and if all our talk about gauge transformations is not (as it may easily be) total nonsense, then we should expect a $\mathbf{W}$ operator with similar properties to generate the baryon-meson weak current from the strong-interaction Lagrangian L. At least the kinetic (gradient containing) part of L should be invariant for constant $\mathbf{t}$ and give rise to the weak current when $\mathbf{t}$ depends on x.

Various systems can be constructed along these lines. For examples, there is the scheme mentioned by Feynman and favored by Okun, Marshak, and others, based on just n, p, and Λ. Of course, if that is right, we do not need the elaborate machinery I have just described. We simply draw an analogy. If there is only one neutrino we use the correspondence

$$\nu_L \sim p_L, \quad \left(\frac{e+\mu}{\sqrt{2}}\right)_L \sim \left(\frac{n+1}{\sqrt{2}}\right)_L$$

If $\nu \neq \nu'$ then we have

$$\nu_L \sim \nu'_L \sim p_L, \quad e_L \sim \mu_L \sim \left(\frac{n+\varepsilon\Lambda}{\sqrt{1+\varepsilon^2}}\right)_L, \quad \text{with } \varepsilon^2 \ll 1\,.$$

In the first case, the extra weakness of the (Λp) current has to be attributed to renormalization. In the second case, it would come from the smallness of ε.

What happens, though, if mesons are really present in the Lagrangian? One possibility is that the mesons we know are all fake and that the real mesons are vector and axial vector mesons that glue nucleons and anti-nucleons together and give rise to the known mesons as bound states. There again a simple model can be constructed.

But I should like to discuss what happens if the known mesons, π and K, are present in the Lagrangian and we have a theory in which the kinetic part of $\mathcal{L}$ conserves $\mathbf{W} = \frac{1}{2}(\mathbf{I}+\mathbf{D}+\mathbf{F}+\mathbf{G})$ and $\mathbf{W}$ is an angular momentum. (We would also like to have $Q = e(W_Z + W_4)$, where $[\mathbf{W}, W_4] = 0$.)

It cannot be done with π and K alone. We need other fields representing two new kinds of mesons. First there is the σ', proposed by Schwinger, with $I = 0$, $J = 0^+$ (in other words, the properties of the vacuum.) It is a particle capable of virtual disintegration into nothing whatever; of course that does not violate any physical principle. Then there is the K' which is like the K meson ($I = \frac{1}{2}$, $J = 0$) but with opposite parity.

These scalar fields are needed so that together with the pseudoscalar ones they can make axial currents as well as vector currents. Only then can we make $\mathbf{D} \neq 0$ and $\mathbf{G} \neq 0$ for the mesons and have $\mathbf{W}$ an angular momentum.

The parity doublet (KK') differs from the one suggested some years ago in that the terms that break the conservation of **W** also break the degeneracy of K and K', π and σ'. They all have different masses.

If σ' and K' do exist they must be heavier than their counterparts π and K and we may suppose they are heavy enough to be completely unstable under the strong interactions, decaying in $\sim 10^{-23}$ seconds:

$$\sigma' \to 2\pi \qquad (s\text{-state})$$

$$K' \to K + \pi \qquad (s\text{-state}).$$

These particles would be more or less indistinguishable in simple experiments from dynamical resonances. In order to look for them you would try to find peaks in the distribution of $m^2_{2\pi}$ when two pions are produced together or in the distribution of $m^2_{K+\pi}$ when K and π are produced together.

If the K' exists, it can have a coupling of the form $KK'\pi$ as suggested several years ago in connection with parity doublets. In the $K+N \to K+\pi+N$ reaction, there would be an important dispersion diagram of the form illustrated in Fig. 1 and in

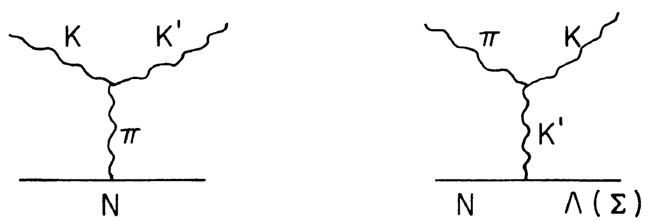

Fig. 1 Processes involving the $KK'\pi$ interaction.

associated production of strange particles a strong peaking in angle could be produced by the dispersion diagram illustrated in Fig. 1. Of course, both of these processes would happen without any K', if K' is simply replaced by $K+\pi$. But if the K' has any utility, if it is a reasonably sharp resonance in the $K+\pi$ system, then these diagrams may have an added significance; they may show up more than they would if there were no special interactions between π and K.

Let me re-emphasize that all of our discussion is completely speculative and that no one should take σ' and K' seriously unless they are actually found.

Now suppose that the new mesons are discovered and our type of speculation encouraged. Then we must ask what terms in the strong coupling Lagrangian violate the conservation of **W**, that is, of **D**, **F**, and **G**.

In the case of the leptons, it was the mechanical mass terms that violated **W** conservation. For the baryons and mesons, we may suppose that in a similar way it is the mechanical mass terms bilinear in the fields that break the symmetry.

Actually, there is an even simpler possibility, since we have a σ' meson that can appear or disappear virtually, namely a linear term in the Lagrangian proportional to σ'. In the case of **C**, in fact, we may even imagine that only the linear term breaks the conservation law.

If we try the same hypothesis for **F** and **G**, however, we find that there is too much symmetry in the system and unwanted degeneracies appear. So probably mechanical mass terms must be invoked to break the symmetries.

It is reasonable, in any case, that only linear and bilinear terms occur in $\delta\mathfrak{L}/\delta\mathbf{t}$ and thus the divergences of the various weak currents are not very singular operators, as they would be if $\delta\mathfrak{L}/\delta\mathbf{t}$ had trilinear or quadrilinear terms arising from interaction terms in the Lagrangian.

In saying that the divergence of a current, for instance the $\Delta S = 0$ axial vector current P_α, is non-singular, I mean particularly that the matrix elements of the operator emphasize low frequencies.

Now if it is really true that $\partial_\alpha P_\alpha$ is an operator that emphasizes low frequencies, if when it acts on the vacuum it emphasizes the creation of low-mass states, then we can find a simple explanation of the remarkable formula of Goldberger and Treiman (G-T).

The G-T relation connects the renormalized axial-vector coupling constant $-G_A$ in β-decay to the amplitude for pion decay into leptons. The latter can be represented by the quantity a_1, where

$$\langle 0|\partial_\alpha P_\alpha|\pi^-\rangle = \frac{ia_1}{\sqrt{2}}\langle 0|\pi_r^-|\pi^-\rangle$$

(π_r^- is the renormalized pion field.)

The rate of $\pi \to \mu + \nu'$ is

$$\Gamma_\pi = \frac{G^2}{16\pi m_\pi} \cdot \frac{m_\mu{}^2}{m_\pi{}^2}\left(1 - \frac{m_\mu{}^2}{m_\pi{}^2}\right)^2 a_1{}^2$$

in terms of a_1.

The nucleon β-decay amplitude is

$$\langle p|P_\alpha|n\rangle = \frac{-G_A}{G}\alpha(K^2)\tau_+\gamma_\alpha\gamma_5 + iK_\alpha\beta(K^2)\tau_+\gamma_5$$

where $\alpha(0) \equiv 1$. The matrix element of the divergence is

$$2m\left(\frac{-G_A}{G}\right)\alpha + K^2\beta = \frac{-a_1 g_1}{K^2 + m_\pi{}^2} + \text{higher terms},$$

where the higher terms come from the creation of three pions, five pions, $N+\bar{N}$, etc., by the operator $\partial_\alpha P_\alpha$ (Here g_1 is the renormalized π-N coupling constant.)

Now if $\partial_\alpha P_\alpha$ really emphasizes low mass states, the one-pion-pole can dominate at $K^2 = 0$ and we have

$$2m\left(\frac{-G_A}{G}\right) \sim \frac{-a_1 g_1}{m_\pi{}^2},$$

which is the G-T relation.

Another way (emphasized by Nambu) to approach the same situation is to look at the limit as $m_\pi^2 \to 0$ and $\partial_\alpha P_\alpha \to 0$, with $\frac{\partial_\alpha P_\alpha}{m_\pi^2}$ remaining fixed. We know (as pointed out by G-T and others) that β has a one-pion pole

$$\frac{a_1 g_1}{m_\pi{}^2}\frac{1}{K^2 + m_\pi{}^2}.$$

In the limit, $2m\left(\frac{-G_A}{G}\right)\alpha + K^2\beta = 0$; so β has a pole

$$\frac{-2m\left(\frac{-G_A}{G}\right)}{K^2}.$$

Evidently, we can identify these poles with each other, and so

$$-2m\left(\frac{-G_A}{G}\right) = g_1 \lim_{m_\pi{}^2 \to 0} \frac{a_1}{m_\pi{}^2}$$

If the limit is a " gentle " one, then we have once more the G-T relation.

$$2m\left(\frac{-G_A}{G}\right) \approx -\frac{a_1 g_1}{m_\pi{}^2}.$$

It is interesting to ask whether a similar approach can be applied to the other currents, with $\Delta S = 1$. There, too, there are terms analogous to β, an induced pseudoscalar term in the matrix element of the axial vector current and an induced scalar term in that of the vector current. In the limit in which the divergences of the currents approach zero, these terms acquire poles at $K^2 = 0$, like β. It is tempting to suppose that here, too, the poles are simply ones corresponding to virtual mesons, and that the masses of these mesons are approaching zero. In the case of the induced pseudoscalar term, it is presumably the K meson that is involved. If the same situation is obtained for the induced scalar term, then perhaps it is the K' meson at work! (On this last point, I am indebted to J. Bernstein for a stimulating question.)

It will be interesting to see whether there is any substance to the speculations about partially conserved currents. As far as the G-T relation is concerned, I think some real progress has been made.

DISCUSSION

SAKURAI: It is perhaps of interest to examine the effect of the σ' meson on nuclear forces. The exchange of a σ' meson between the two nucleons would lead to a short-range attraction in all angular momentum and parity states. The hypothetical meson does give a spin-orbit force of the correct sign as pointed out by Breit, Duerr and Gupta.

GELL-MANN: So you think that the hypothesis of a σ' meson is not in disagreement with experiment as far as nuclear forces are concerned?

SAKURAI: I cannot be positive because that attractive short range interaction may give some trouble. On the other hand, the gluon or a neutral vector meson can explain both the phenomenological repulsive core and the spin-orbit force.

TIOMNO: With respect to the question of the K' meson I should like to mention that I have also been making speculations about this possibility and its implications. The starting point was, however, the analysis of the angular distribution in ΛK^0 production from π^- on protons. Following Pais we used a $KK'\pi$ vertex but allowing for K'_+ (or K'_0) different from the ordinary K^+ (or K^0). Zagury, Videira and myself tried to find the best value of the mass of this K' particle both in the case of scalar and pseudoscalar coupling for the $K'p\Lambda$ vertex. We found that for the scalar coupling the best fit was for a mass of K' around the sum of the masses of K and π.

GELL-MANN: You would have a resonance decaying fairly slowly?

TIOMNO: Yes, at the energies for which reasonably good angular distributions are available the values of the K' mass ranged from 0.45 to 0.75 BeV being consistent with a unique value. Now, for the pseudoscalar coupling the mass values of K' were of the order of the π-mass. This last possibility is thus ruled out by the fact that the K mesons should then decay into π and K'. In the first case the K' mass could be higher than K plus π and thus would not lead to contradiction with experiment.

GELL-MANN: So you like the K being pseudoscalar and the K' scalar?

TIOMNO: Yes, this is the one which would lead to a stable K particle relatively to strong interactions.

WOLFENSTEIN: Am I correct that in this approach the sign of a_1 and therefore the sign of the induced pseudoscalar terms is not determined?

GELL-MANN: Not the absolute sign of a_1 but the sign of the induced pseudoscalar term relative to the axial vector coupling constant is determined in this approach. In the absorption of μ^- in carbon for example, people have always given two possible signs —one corresponding to the Goldberger effect and the other to a negative Goldberger effect. Which of these alternatives is the correct one? In this approach the second alternative is excluded. The sign is the same sign as proposed by Goldberger.

APPLICATION OF DISPERSION RELATIONS TO WEAK INTERACTIONS

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1. INTRODUCTION

With the exception of μ meson decay, all the weak reactions that we know involve at least one strongly interacting particle. It is well known that about the only tool we have for computing anything about the strong interactions is the dispersion relations, regardless of the many difficulties and disadvantages it presents.

In the past three years, starting with the work of Goldberger and Treiman, the dispersion relations have been used to study the role of strong interactions in weak processes. With varying degrees of success, almost all the known weak decays, both leptonic and non-leptonic, have been considered. The degree of the success depends, as always with the dispersion