

Hadron phenomenology in the Dyson-Schwinger approach

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Abstract. I present a brief overview of the Dyson-Schwinger approach and its application to hadron properties. Recent results for baryon and tetraquark masses as well as nucleon and Δ elastic and transition form factors are discussed.

1. Introduction

Probing hadrons with external currents reveals their basic structure properties and provides a connection with the underlying quark and gluon dynamics in Quantum Chromodynamics (QCD). Precision electron scattering and electroproduction measurements at JLab and other facilities have provided groundbreaking insight in the nucleon's structure via elastic and transition form factors, polarizabilities and generalized parton distributions [1, 2]. They have stimulated the development of tools to address questions related to quark orbital angular-momentum correlations, the transition between perturbative and non-perturbative regions, or pion-cloud rescattering effects in the chiral and low-momentum region. Future measurements at PANDA/FAIR via $p\bar{p}$ collisions will be instrumental in extending these studies towards a comprehensive understanding of hadron spectroscopy and hadron structure.

A promising tool for studying hadron phenomenology is the Dyson-Schwinger approach. The properties of QCD and hadrons are encoded in QCD's Green functions which can be determined from QCD's quantum integral equations of motion, the Dyson-Schwinger equations (DSEs) [3–5]. In combination with covariant bound-state equations (Bethe-Salpeter and Faddeev equations), they constitute an ab-initio framework for computing hadron properties. The approach is Poincaré-covariant throughout every step and provides access to all momentum scales and all quark masses without the need for extrapolations. Since one operates directly with QCD's degrees of freedom, observable phenomena at the hadron level can be systematically traced back to their microscopic origin in terms of convolutions of Green functions.

Of course, the approach comes with its own limitations: Green functions depend on the gauge, and the existence of infinitely many of them makes a truncation of the system unavoidable. This has consequences for the calculated observables: for example, at the present level of feasibility in the baryon sector, pseudoscalar-meson cloud effects and dynamical decay widths in the low quark-mass region are not yet implemented. That shortcoming can also serve as an advantage as it allows insight into the 'quark core' of a baryon. It will be imperative for future studies to introduce such effects, either from a phenomenological perspective or directly at the quark-gluon level.

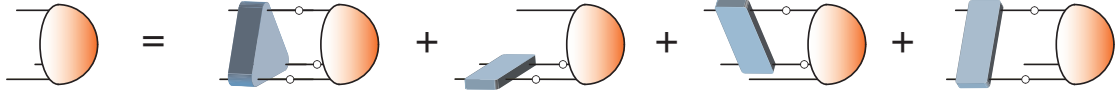


Figure 1. Covariant three-body equation for a baryon's bound-state amplitude, Eq. (1).

2. Covariant bound-state equations

The nonperturbative substructure of hadrons can be resolved through the combination of DSEs and bound-state equations. For baryons, the essential relations are derived from the properties of the quark six-point function G . Baryons correspond to poles in G , and a baryon's bound-state amplitude Ψ satisfies a self-consistent integral equation which is illustrated in Fig. 1:

$$\Psi = \mathbf{K} G_0 \Psi, \quad \text{with} \quad G_0 = S \otimes S \otimes S \quad \text{and} \quad \mathbf{K} = K_{(3)} + \left[S^{-1} \otimes K_{(2)} \right]_{\text{perm}}. \quad (1)$$

The subscript 'perm' here and in the equations below indicates three permutations with respect to the quark lines. The equation can be solved once the dressed quark propagator S and the two- and three-quark irreducible kernels $K_{(2)}$ and $K_{(3)}$, which encode the interactions at the quark-gluon level, are determined. This is also the point where a truncation becomes necessary. Omitting the term $K_{(3)}$ yields the covariant Faddeev equation that traces the binding of three quarks in a baryon to its quark-quark correlations. The simplest ansatz for $K_{(2)}$ is the rainbow-ladder kernel; it connects two quarks through a dressed gluon propagator and tree-level quark-gluon vertices. The dressed quark propagator S is then obtained as solution of the quark DSE in the same truncation, and its implementation in the Faddeev equation yields, by iteration, all dressed-gluon ladder exchanges between quark pairs.

The coupling of the baryon to external $q\bar{q}$ currents is encoded in the current matrix element J^μ and the scattering amplitude $J^{\mu\nu}$. They are given by [6]

$$J^\mu = \bar{\Psi}_f G_0 \Lambda^\mu G_0 \Psi_i, \quad J^{\mu\nu} = \bar{\Psi}_f G_0 \left[\Lambda^{\{\mu} G \Lambda^{\nu\}} - \Lambda^{\mu\nu} \right] G_0 \Psi_i, \quad (2)$$

where Ψ_i and Ψ_f correspond to the incoming and outgoing baryons and

$$\begin{aligned} \Lambda^\mu &= \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} \right]_{\text{perm}}, \\ \Lambda^{\mu\nu} &= \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} \right]_{\text{perm}}. \end{aligned} \quad (3)$$

In addition to S and $K_{(2)}$, these equations contain the quark-antiquark vertices Γ^μ and $\Gamma^{\mu\nu}$ with one or two external current legs. Their self-consistent calculation from inhomogeneous Bethe-Salpeter equations generates meson poles in the respective J^{PC} channels which dictate the timelike structure of form factors and reproduce features that are familiar from vector-meson dominance models. We have already assumed a rainbow-ladder truncation in Eq. (3); in the general case additional terms would appear in Λ^μ and $\Lambda^{\mu\nu}$. The rainbow-ladder truncation ensures that all ingredients, if calculated consistently, satisfy vector and axialvector Ward-Takahashi identities. Electromagnetic gauge invariance is therefore respected at the hadron level, and the Gell-Mann-Oakes-Renner relation for the pion mass and the Goldberger-Treiman relation between the axial and pion-nucleon coupling follow automatically.

Solving Eq. (1) yields a baryon's mass, and from Eq. (2) one can extract baryon elastic and transition form factors as well as, in principle, the nucleon's Compton scattering or pion electroproduction amplitude. The relations for mesons are simpler but obtained along the same

lines. Even in rainbow-ladder, solving these equations is complicated by their nature as multi-loop integrals and the fact that their ingredients exhibit a rich momentum, Dirac-Lorentz and color-flavor structure. Nevertheless, once the kernel $K_{(2)}$ is specified, it permits a self-consistent calculation of a wide range of hadron observables without further approximations. Virtually all pseudoscalar-meson, vector-meson, nucleon and Δ -baryon properties calculated so far exhibit only a small sensitivity to the momentum dependence of the quark-gluon coupling in $K_{(2)}$. In that way, these results effectively depend only on a single input scale.

3. Hadron spectroscopy

Considerable progress has been recently made with the covariant Faddeev equation for ground-state baryons. With a rainbow-ladder interaction $K_{(2)}$ that is fixed only to the experimental pion decay constant, the Faddeev equation yields nucleon and Δ masses of $M_N = 0.94$ GeV and $M_\Delta = 1.26$ GeV at the u/d -quark mass, and their current-mass evolution agrees well with lattice data [7–9]. These results complement a range of meson observables that are also well described within a rainbow-ladder truncation, see [10] for a review. Attempts to go beyond rainbow-ladder are underway but so far they have been limited to the meson sector. It is interesting that both N and Δ have a pronounced quark-diquark structure: under the assumption of diquark dominance, the Faddeev equation can be simplified to a quark-diquark two-body equation that implements scalar and axialvector diquarks [11]. The corresponding results in the rainbow-ladder setup agree within 5 – 10% with those from the three-body equation [7].

So far there have been only few studies of meson and baryon excitations in the Dyson-Schwinger approach [12–14]. The open questions in this respect concern the internal structure of baryon resonances, the origin of the observed level ordering, and the connection between the dynamical generation of resonances at the hadron level, where a single ‘core’ state can generate multiple resonances via meson-baryon interactions [15], versus the generation of these resonances within QCD itself. Results from a simple quark-diquark model suggest that the observed mass ordering of positive- and negative-parity nucleon excitations and the nature of the $N(1440)$ Roper resonance might be caused by diquark clustering in baryons [14]. This has long been suspected as the origin of missing nucleon resonances; nevertheless, such features have not been seen in recent lattice calculations [16].

The idea of diquark clustering has also been the guiding assumption for a recent tetraquark study in the rainbow-ladder truncation [17]. The calculation yields a light scalar four-quark state with a mass of ~ 400 MeV that is predominantly a pion-pion molecule with diquark-antidiquark admixture. It suggests an identification with the elusive $\sigma/f_0(500)$ and thereby provides further support for the identification of the light scalar-meson nonet in terms of tetraquarks.

4. Form factors

Extending these studies to baryon form factors, the rainbow-ladder setup of Eqs. (1–3) has been applied to a number of processes over the past years: nucleon electromagnetic [8], axial and pseudoscalar form factors [18] and Δ electromagnetic form factors [19] in the three-body approach, and N and Δ electromagnetic [20, 21], $N \rightarrow \Delta\gamma$ [22] and $N \rightarrow \Delta\pi$ [23] transition form factors in the quark-diquark simplification. These works build upon earlier quark-diquark studies with modeled ingredients [11, 24]. The calculations were performed in the low- to mid- Q^2 region up to a few GeV^2 and in a current-mass range roughly up to the strange-quark mass.

Without going into details, I will exemplify a few common observations that pertain to all these studies. The overall agreement with experimental data, and also with lattice results at larger quark masses, is generally quite good. Deviations are visible in the low-momentum and small quark-mass region; they are attributable to missing pion-cloud corrections which are not implemented at the present stage. A typical example is the $N \rightarrow \Delta\gamma$ magnetic dipole transition form factor $G_M^*(Q^2)$ in Fig. 2 which is comparable to quark model results and to the EBAC

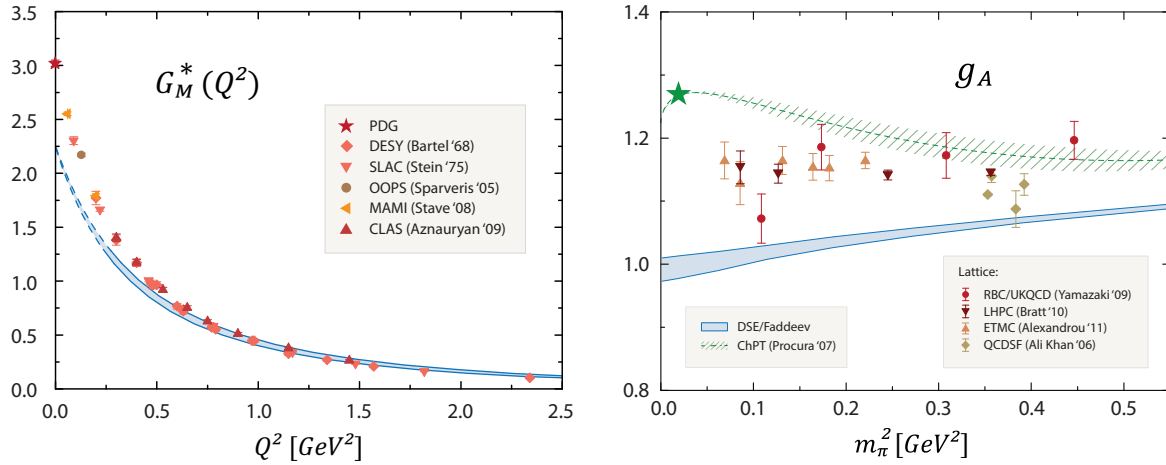


Figure 2. *Left panel:* Q^2 –dependence of the $N \rightarrow \Delta\gamma$ magnetic dipole transition form factor $G_M^*(Q^2)$ compared to experiment [22]. *Right panel:* Quark-mass dependence of the nucleon’s axial charge g_A , compared to lattice results and a chiral expansion [18].

analysis if meson-baryon rescattering effects are switched off [25]. The nucleon ‘quark core’ is also visible for various charge radii, which underestimate the experimental values, or in the nucleon axial coupling g_A in Fig. 2. An intriguing example is the nucleon’s isoscalar anomalous magnetic moment where leading-order chiral corrections cancel: there, the calculated value agrees quite precisely with the experimental number $\kappa_s = -0.12$ [8].

Via Eq. (3), the form factors inherit the properties of the quark-antiquark vertices Γ^μ . The bulk structure of electromagnetic form factors is determined by the Ball-Chiu vertex, which is the minimal construction that satisfies electromagnetic gauge invariance and depends only on the dressed quark propagator [26]. The second essential ingredient are timelike (π , ρ , a_1) meson poles in the t channel: they are selfconsistently generated in the vertices and important for the timelike and low-momentum structure of the (pseudoscalar, vector, axialvector) form factors. The properties of the quark core are mostly current-mass independent: expressed in terms of dimensionless quantities, no significant change happens for any of the calculated form factors. This means for example that, in the absence of chiral non-analyticities, the quark core of the $\Omega(1672)$ baryon is essentially identical to that of the $\Delta(1232)$ [21].

Finally, the results underline the importance of quark orbital angular momentum in the Q^2 –evolution of the form factors. Poincaré covariance entails the appearance of p waves in the (s –wave dominated) N and Δ bound-state amplitudes which are absent in non-relativistic quark models. They are, for example, responsible for the small negative value of the $N \rightarrow \Delta\gamma$ quadrupole transition ratio $R_{EM}(Q^2)$ and the falloff of the electric proton form factor $G_E^p(Q^2)$ which points toward a possible zero crossing, cf. Fig. 3.

5. Conclusions and outlook

It is imperative to develop versatile theoretical tools to accompany the experimental efforts that are currently made. Existing calculations of nucleon and Δ elastic and transition form factors in the Dyson-Schwinger approach have already offered interesting insights. These studies must be complemented by technical improvements in the future, most importantly the implementation of pion-cloud corrections and hadronic decay channels via truncations beyond rainbow-ladder. Future investigations will also aim at more sophisticated systems and processes such as baryon excitations, nucleon-to-resonance transition form factors, Compton scattering, pion electroproduction, or timelike $p\bar{p}$ annihilation processes.

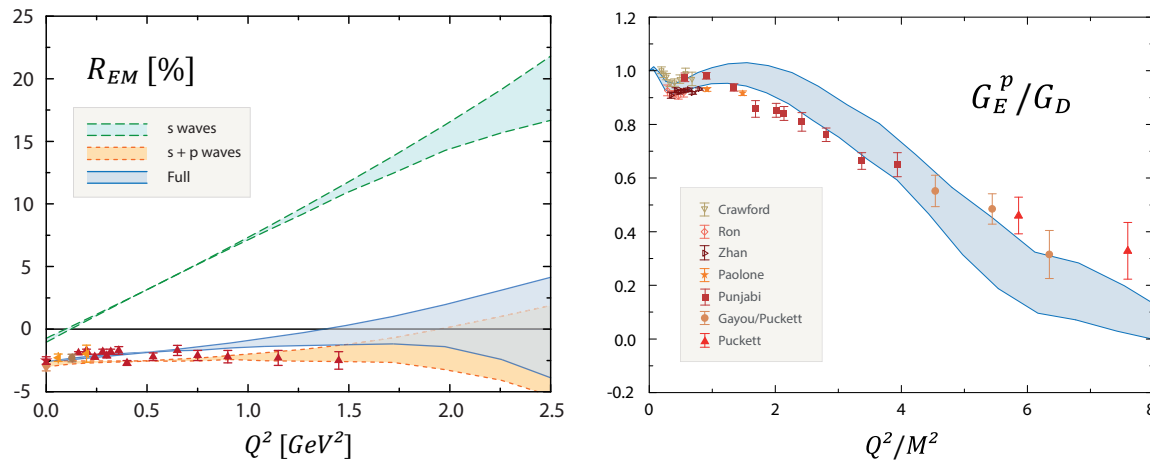


Figure 3. *Left panel:* Quark orbital angular-momentum decomposition of the $N \rightarrow \Delta\gamma$ electric quadrupole transition ratio $R_{EM}(Q^2)$ [22]. *Right panel:* Electric proton form factor $G_E^p(Q^2)$, normalized by the dipole and compared to experimental data [8].

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