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Sources of gravitational waves and their detectability

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Abstract. With the funding of the LIGO project in the US and the likely funding of VIRGO in Europe, we can now anticipate the detection and astrophysical study of gravitational waves before the end of this decade. I review the position of detector development today and the plans for the large-scale interferometers. I then survey likely sources of gravitational waves: supernovae (gravitational collapse), coalescences of compact-object binary systems, pulsars, and a stochastic background. I try to make realistic assessments of the uncertainties in our models of these sources, and in our estimates of their event rates; and I point out what information we may be able to extract from observations when they are made. I pay particular attention to the uncertainties that allow the possibility that even first-stage interferometric detectors might be able to detect gravitational waves from gravitational collapse or coalescing binaries. I also point out that, before observers will be able to extract maximum information from a coalescing binary signal, relativists will have to make more progress in understanding the two-body problem in general relativity.

1. Introduction

1.1. Gravitational wave astronomy

After many decades of developing techniques for gravitational wave detection and of studying possible sources of gravitational waves, we may finally be standing on the threshold of gravitation wave astronomy. The approval last year of the LIGO project by the US Congress, and the expected approval later this year of the VIRGO project in Europe, at last marks the beginning of the construction of instruments whose sensitivity should exceed by a large margin the targets that theoretical studies of sources have set for detecting waves that pass through the Earth a few times per year.

It will, nevertheless, be several years before these big interferometers begin operating, and it will probably be several years more before they begin returning observations with the regularity that will be needed to extract reliable astronomical information from them. In the meantime, bar detectors of increasing sensitivity will be watching for rare events of unusual strength, and it could well happen that they will turn in the first detections of gravitational waves before the interferometers get going.

In this review I will discuss our present understanding of possible sources of gravitational waves, in the context of efforts to detect them. First I will review the most important recent detector developments. Then I will focus on the most promising sources, namely supernova explosions, coalescing compact-object binary systems, and a cosmological background. Because the interferometers will be built in two stages, I will speculate as well on what they might be capable of detecting even with their lower, first-stage sensitivity.

1.2. Gravitational wave uncertainties

The most important *intrinsic* characteristics of a gravitational wave are the total energy it carries and its frequency, or its spectral bandwidth. In addition, for reliable predictions about the likelihood of detecting gravitational waves we need estimates of the distance to the source and the number of sources out to that distance. These are bound up with the sensitivity of detectors, so our review begins with a survey of the types of detectors and their expected characteristics.

Not surprisingly, the astrophysics of almost all potential gravitational wave sources is sufficiently uncertain that we have to allow for wide ranges of variability in at least some of the characteristics needed for our predictions. This is not a reason to be discouraged; on the contrary, only by making gravitational wave observations can we fill in the gaps that present observational techniques leave in our understanding of many astronomical systems.

The one exception to this "rule" of uncertainty is the coalescing binary: the radiation emitted from the orbital motion of two neutron stars or black holes as they spiral ever closer together from a tight binary orbit. We are sufficiently sure of the strength of the waves and of the number of sources in a given volume of space to have great confidence that future interferometric detectors like LIGO will see them. Interestingly, this source presents a challenge to relativists at this time: can we solve the two-body problem with sufficient accuracy to allow us to extract the wealth of information that the signal contains? I will return to this question at the end.

There is at least one aspect of our predictions about gravitational waves that is not a major source of uncertainty: relativity theory itself. The Binary Pulsar system, PSR1913+16, discovered by Hulse & Taylor[1] in 1975, has confirmed the predictions of general relativity to such an accuracy that it is now necessary to take into account the acceleration of the system caused by the Galaxy before arriving at the best value of the period change attributable to gravitational radiation.[3] This is described in much more detail in the article by Taylor in this volume.

1.3. Energy carried by a gravitational wave

The amplitude of the gravitational waves that pass the Earth regularly (once a month or more often) is likely to be well below 10^{-21}. The smallness of this number is a measure of the weakness of its interaction with the matter it passes through. This is at once a benefit and a curse: a curse because it makes the waves hard to detect, a benefit because one can be sure that the radiation has not been significantly affected by anything it may have passed through since leaving its source. Unlike photons, which are easily scattered and reprocessed, gravitational waves carry their original information intact.

The weakness of the effect of gravitational waves on our detectors should not make us overlook the fact that they carry enormous amounts of energy. Using the Isaacson stress-energy tensor (see [2]), one can easily calculate that the energy flux in a gravitational wave of amplitude h and frequency f is

$$\mathcal{F}_{gw} = 3.2 \times 10^{-3} \left[\frac{f}{1 \text{ kHz}}\right]^2 \left[\frac{h}{10^{-22}}\right]^2 \text{ W m}^{-2}. \tag{1}$$

In astronomers' language, a 1 kHz wave with amplitude 10^{-22} is as bright as a star of apparent magnitude -13, which is twice as bright as the full moon! This may last only a millisecond, but since the wave originates in a distant galaxy, it is clear that enormous energies are involved.

By integrating Eq. (1) over a sphere of radius r for a time τ, we find the relation between the amplitude h and the energy E carried by the wave during τ:

$$h = 1.4 \times 10^{-21} \left[\frac{E}{0.01\ M_\odot\ c^2}\right]^{1/2} \left[\frac{f}{1 \text{ kHz}}\right]^{-1} \left[\frac{\tau}{1 \text{ ms}}\right]^{-1/2} \left[\frac{r}{15 \text{ Mpc}}\right]^{-1}. \tag{2}$$

2. Detector developments

2.1. Types of detector

The two principal types of detector under development are bar detectors and laser interferometers. In addition, a number of other techniques have been used, such as ranging to interplanetary spacecraft and searching for irregularities in timing of millisecond pulsars.

Bar detectors use the fundamental longitudinal normal mode of a cylindrical bar to detect waves. Although bars have their greatest sensitivity near the frequency of this mode, which is usually chosen to be near 1 kHz, they do not act as resonant detectors: impulsive gravitational waves do not last long enough to excite a resonance. Instead, bars use the high Q of this mode effectively to reduce the noise due to the thermal vibrations of the bar. For this reason, they have bandwidths of some tens of Hertz, much greater than the sub-milliHertz bandwidth of the resonant mode itself. In principle they can have even larger bandwidths, but this has not yet been achieved in practice. The narrow bandwidth affects another characteristic of bars: their poor time-resolution (worse than 0.1 s) makes them unable to discriminate between the time-of-arrival of a wave at one detector and another, so that networks of bars are relatively poor at determining the direction, and therefore the intrinsic amplitude, of gravitational waves.

Bars can be made more sensitive by reducing their physical temperature, and this is the major route for progress at the moment. Novel designs of bars, such as spherical ones, may allow further improvements. But somewhere near a sensitivity of 10^{-21}, bars encounter the quantum limit, where the energy left in the bar by the gravitational wave is less than the energy of one phonon of the longitudinal mode. This can be circumvented in principle, but again there are no practical ways as yet. Bars are therefore now, and for the foreseeable future, narrow-band detectors which can in principle reach to about 10^{-21}. Despite this limitation, bars have one great advantage over interferometers: they exist and will be taking data regularly during the next few years, while interferometers are under construction.

Interferometers are described more fully in K. Danzmann's article in this volume. They do not involve any natural frequency, so they are intrinsically broad-band detectors. Their bandwidth extends from perhaps 10 Hz up to several kHz. This means that a network of 3 or more interferometers can deduce the direction to a source by measuring the time-delays among the various detectors. Optical tricks can be used to reduce their bandwidth if desired, such as for high-sensitivity observations of a particular pulsar. Interferometers are not troubled by the quantum limit at sensitivities of 10^{-22} or so, because they can take advantage of the length effect: the tidal forces in a gravitational wave increase in proportion to the size of the apparatus, so that a detector 4 km in size will experience 100 times the displacement of its end masses that a 40 m detector experiences.

In the near future, the only existing interferometers are various prototypes that are rarely used for data-taking, being required for technical development instead. In five years or so, once the kilometer-scale interferometers begin operating, they will give us our first broad-band look at the universe in gravitational waves, with a sensitivity comparable to the best that bars can achieve. Within a few years after that, they should be operating at a sensitivity that ought to guarantee a multitude of detections.

Detectors on the ground cannot hope to observe at really low frequencies, even though much of the radiation in the sub-milliHertz region is easy to predict, coming as it does from ordinary binary systems in our Galaxy. Ground vibration and even the Newtonian gravitational fluctuations produced by movements of atmospheric masses will mask signals at low frequency. Present space-based detection methods involve ranging to interplanetary spacecraft[4] and monitoring millisecond pulsars (see the article by Taylor). Both methods are improving, and will be our only access to this frequency window until dedicated interferometers are placed in space.

2.2. Current ground-based detectors

2.2.1. Cryogenic bar detectors. The best currently operating bar detectors are cryogenic, cooled to liquid helium temperatures (4.2 K). The sites currently operating include

- LSU. The cryogenic bar at Louisiana State University takes data continuously (you can even tune in to the data system on the Internet!). Its noise level is well below 10^{-18}.

- Rome. The University of Rome bar (called Explorer) is situated at CERN. It recently ran for 2 years, and will soon be back on the air. Its sensitivity is comparable to that of the LSU bar. The Rome and LSU bars have run in coincidence and the groups are currently analyzing 6 months of data.

- Perth. This cryogenic bar, made of niobium, is planned to be in operation by the end of 1992 at the University of Western Australia. Its sensitivity will be comparable to that of LSU and Rome. Three-way coincidence experiments are expected.

2.2.2. Interferometric detectors. Laser interferometers are presently being used as development tools for the larger-scale interferometers of the future. They occasionally have made observing runs. The main ones include:

- Garching. A 30 m prototype at the Max Planck Institute for Quantum Optics is currently being modified. Its broadband sensitivity at 1 kHz is a few times 10^{-18}.

- Glasgow. The 10 m prototype at the University of Glasgow has a sensitivity similar to that of Garching. The Garching and Glasgow instruments took 100 hours' data in coincidence with each other, the first such observing run between two interferometers. Analysis of these data sets is being done in my research group at Cardiff.

- Caltech. At the California Institute of Technology, the LIGO project operates a 40 m prototype, the largest in the world at present. It will soon be rebuilt, after which its sensitivity may well better that of Garching and Glasgow. There are no plans at present for long observation periods.

- MIT. The LIGO project also operates a small prototype at the Massachusetts Institute of Technology. This was one of the first to be constructed, and is used for testing interferometer techniques.

- Tokyo. There is a 10 m prototype at ISAS, near Tokyo. Its sensitivity is about 10^{-17}. It will soon to be replaced by a 100 m instrument, which is nearing completion. The 10 m detector took data over several days that partly overlapped with the Glasgow-Garching run. These data will soon be analysed for coincidences at Cardiff as well.

2.3. Future ground-based detectors

2.3.1. Ultracryogenic bars. The next generation of bars will be cooled to well below liquid helium temperatures by dilution refrigeration. The two such bars that are now under construction are:

- Rome. This bar, called Nautilus, has already been cooled (without its instrumentation) to below 90 mK. It is expected to begin observations later this year, and to approach a sensitivity of 10^{-19} within another year. With improvements in transducers, cryogenics, and other systems, it could in principle reach about 10^{-20}.

- Stanford. The group at Stanford University operated a cryogenic bar until it was destroyed by an earthquake. They are currently building an ultracryogenic detector similar to that of Rome. First tests of it are expected later this year. Coincident observing runs between these two bars offer the hope of seeing any large supernova-like event in our Galaxy.

2.3.2. Large-scale laser interferometers. There is general agreement that laser interferometers need to go to the scale of a few kilometers in order to achieve a sensitivity of 10^{-22} with foreseeable technology. Several projects have been proposed along these lines, and are at various stages of approval:

- LIGO. This project, a collaboration between the California Institute of Technology and the Massachusetts Institute of Technology, was approved last year by the US Congress. It expects to build two detectors, each 4 km long, at sites in Hanford (Washington) and in Louisiana. The Hanford installation would also contain a half-length detector. These detectors should reach a broadband sensitivity of 10^{-21} at Stage 1 (1997–8?) and can anticipate going beyond that to 10^{-22} with present designs. As technology improves, there is no fundamental barrier to further improvements beyond that.

- VIRGO. This Italian-French collaboration plans to build a detector of 3 km armlength near Pisa. It has been approved by France and is now awaiting approval by Italy. Its capability will be similar to that of each LIGO detector, but with an emphasis on reaching very low frequencies (10 Hz or below). Coincident observations by the two LIGO and the VIRGO detectors could determine the direction to a source to within a degree or so.

- GEO. This is a collaboration between Germany (Garching and the University of Hannover) and the UK (Glasgow and Cardiff). It is stalled at the moment by funding problems in both countries, after having been approved in the UK in 1989. The plan is to build a 3 km detector near Hannover, with capabilities similar to each LIGO detector. The close separation of the two European detectors is attractive for doing cross-correlation searches for a stochastic background of gravitational waves (see below).

- AIGO. This Australian project is still in the planning stage, looking for international partners. Good sites exist in Australia, and its long baseline from the other detectors makes it attractive for improving the directional resolution of the network.

3. Gravitational collapse

We turn now to a discussion of possible sources of gravitational waves. Informed by our survey of the available ground-based detectors, we shall concentrate on sources at frequencies above about 10 Hz. Historically, the most important source that drove the development of bar detectors was the supernova, or more generally gravitational collapse.

3.1. What we know and what we don't know

Gravitational collapse is attractive as a gravitational wave source because it involves a considerable mass compressed to a high density on a very short timescale. Collapses that produce neutron stars lead to supernovae of Type II, where the formation of the star creates a rebound of infalling material that then blows away the original envelope of the star. It is less clear that collapses that lead to black holes will produce bright supernovae, since these events might involve rebounds that do not succeed in blowing away the envelope; this might subsequently fall in and convert the neutron star into a black hole. Moreover, many supernovae (Type Ia, particularly) do not appear to involve gravitational collapse. So there will not be a one-to-one relation between supernovae and strong bursts of gravitational waves.

Moreover, despite decades of theoretical study of supernovae of type II, it is still difficult to assess their likely importance as sources of gravitational waves. This is because optical and gravitational wave observations of supernovae measure effectively orthogonal quantities. Optical observations see the expanding cloud of ejecta; this is material that never reached a particularly high density, and which is expanding roughly spherically. Gravitational waves do not come from spherical motions; they measure the asymmetries in the very core of the collapse. Making a link between these two has proved difficult, not least because modelling three-dimensional supernovae with a full nuclear reaction network, hydrodynamics, and radiation transport is a job that exceeds the capacity of supercomputers available today.

There is also contradictory observational evidence on the subject. If large asymmetries do develop in collapse, they are likely to be related to rotation (see below). But young pulsars, which were presumably formed in such events, do not show dynamically rapid rotation rates. The Crab pulsar, for example, rotates at 30 times per second, which is very much slower than its breakup speed of 1–2 kHz. This agues that rotation is not dynamically important, at least in the majority of collapses that produce pulsars.

On the other hand, there is general agreement that there must be some asymmetries, because pulsars seem to be given a "kick" of 100–200 km/s when they are formed, in addition to any velocity they acquire due to binary breakup. There is even a case which may show a velocity of 1600 km/s.[5] While this is again only a moderate speed (the rotational speed of the Crab pulsar at its surface is almost 2000 km/s), if it comes from an asymmetric emission of radiation (neutrinos or gravitational waves) then the energy carried away by the asymmetric radiation must be a sizeable fraction (perhaps 10% or more) of the energy released by the explosion.

In the absence of detailed modelling, the best one can do is to calculate the radiation amplitude at the Earth of a gravitational wave produced in a distant galaxy by a collapse that converts a certain amount of energy into gravitational waves. Assuming that the duration of the burst is the timescale of the rebound, *i.e.* about a millisecond, then Eq. (2) implies that the amplitude of a burst of energy E occurring at a distance r is

$$h_{\text{collapse}} = 10^{-21}\left[\frac{E}{10^{-2}M_{\odot}c^2}\right]\left[\frac{r}{15\ \text{Mpc}}\right]. \qquad (3)$$

The strongest possible burst would emit the whole binding energy of a neutron star, about 0.1 $M_{\odot}\ c^2$. This would produce an amplitude of 3×10^{-18} if it occurred in our Galaxy, and 3×10^{-21} in the Virgo cluster. A more moderate, and plausible, amount would be 0.0 $M_{\odot}\ c^2$, which would give an amplitude of about 4×10^{-22} at 40 Mpc.

These numbers are very interesting in view of our discussion of detectors. Present day bars and interferometers could in principle see a strong burst in our Galaxy. The second-stage large interferometers could see a moderate burst twice as far away as the Virgo cluster. This is a volume of space containing many thousands of galaxies, in which hundreds of supernovae occur each year. Even if a small fraction of them should produce bursts as large as this, the second- stage interferometers should see a few per year. We will return below to what the first-stage interferometers might see.

3.2. Scenarios for collapse leading to strong radiation

If even a small fraction of gravitational collapse events do lead to strong gravitational waves, they will probably be driven into asymmetric collapse by rotation. Rotation has two effects. The first is the obvious one, causing the collapsing core to form a oblate figure of rotation. The second effect develops only at sufficiently high rotation rates, and probably only for strong differential rotation: the development of non-axisymmetric "bar-mode" instabilities that cause the core to assume a tumbling tri-axial ellipsoidal shape.[6]

The radiation produced by the first effect is likely to be very small. Numerical simulations of axisymmetric gravitational collapse produce very little energy, perhaps $10^{-6}\ M_{\odot}\ c^2$.[7, 8, 9, 10]. There is also evidence that rotational effects slow the collapse and thereby lower the dominant frequency at which the radiation comes out. If this frequency turns out to be well below 1 kHz, then bar detectors will not see such events.

If there is enough rotation to lead to non-axisymmetry, then the resulting configuration must shed its excess rotational energy. Since the instability sets in when the total rotational energy is about 25% of the binding energy of the neutron star, it is reasonable to expect that this may lead to radiated energy of the order of $0.01 M_{\odot}\ c^2$ or more. The timescale and frequency of the radiation will depend in detail on the dynamics of the collapse.

If rotation does dominate collapse, then it might be possible to explain the absence of young, rapidly rotating pulsars in one of the following two ways:

- A rapidly rotating collapse eventually produces a black hole. Perhaps the outgoing shock is too weak to blow away the envelope, or perhaps the collapsed ellipsoidal core, with its rotational support, exceeds the upper mass limit of a slowly rotating neutron star.
- The collapsed core may have so much angular momentum that it fissions, expelling a small part of itself that carries off substantial angular momentum. The recoil might explain high-velocity pulsars.

Better hydrodynamical calculations even in Newtonian gravity would shed considerable light on these questions.

3.3. Supernovae and Stage-1 Interferometers

The usual assumption about supernovae is that they produce a burst of radiation in a timescale characteristic of the bounce, about 1 ms. This would be a broad-band burst at about 1 kHz. It is possible, however, that considerable radiation from a collapse event emerges at a frequency well below 1 kHz, particularly if rotation is involved. As an illustration, we construct the following simplified and optimistic scenario, in which first-stage interferometers could see a good number of events:

- Suppose the energy emerges at 100 Hz as a pulse lasting 10 ms. Then the amplitude of the signal increases by a factor of 3.
- Suppose the detector is optimized in recycling mode for detecting pulses at 100 Hz instead of 1 kHz. Then the noise goes down by a factor of 10.

The result is that the signal-to-noise ratio goes up by 30. Events that radiate $0.01 M_{\odot}\ c^2$ of energy become visible hundreds of megaparsecs away, so hundreds of thousands of supernovae per year become potential sources.

There is no reason in principle that the real situation should not be closer to this optimistic scenario than to the conventional one. Although the rotational frequency of the collapsed core must be in the kHz range for instabilities to set in, the non-axisymmetric deformation that emits the radiation may be a CFS-unstable bar mode,[11, 12] which could have a significantly lower frequency f_{CFS}. In such as case, the supernova would produce a long, low-amplitude wavetrain at a frequency $f_{\mathrm{CFS}} < 1000$ Hz rather than a structureless burst of duration $1/f_{\mathrm{CFS}}$, but *the signal-to-noise ratio will go up by the same factor of* $(1000\ \mathrm{Hz}/f_{\mathrm{CFS}})^(3/2)$ *provided we have by then good enough models of the waveform to perform matched filtering.*[13]

This scenario requires that detector builders decide to control their noise sources down to lower frequencies, and then to optimize the interferometers for the frequency f_{CFS} rather than 1 kHz. These circumstances are not part of the present plans for the first stage detectors. But the potential payoff of doing so should be kept in mind. Even if detectors are not optimized for lower frequencies, matched filters should still be applied to the low-frequency data to look for such events; in this case the potential gain in signal-to-noise is still a factor of $1000\ \mathrm{Hz}/f_{\mathrm{CFS}}$.

Unfortunately, this optimistic scenario does not help bar detectors, since a low-frequency gravitational collapse burst would be outside their bandwidth.

3.4. Astrophysical payoffs of detecting supernovae

In addition to the satisfaction of finding a supernova, there are many astrophysical reasons for wanting to detect them. If bar detectors register a collapse event in our Galaxy, then:

- a coincident observation of neutrinos associated with the event would confirm it and test models of collapse, and may help give the direction to the event if it is not seen optically;
- this would define at least one type of supernova that can be a strong gravitational wave emitter.

If two interferometers detect a collapse event in a distant galaxy, then:

- they will give waveform information that will greatly constrain collapse models and the nuclear physics of neutron stars (the waveform information will always be available because the detection threshold will be at a signal-to-noise ratio of 7 or so, which means that any detection will yield extra information too);
- they may, from the characteristic frequencies of the waveform, be able to measure the mass of the compact object and therefore identify it as a neutron star or black hole.

Finally, if three or more interferometers detect the event, then they can do the above plus:

- they will fix the direction to the event (to better than a degree) and be able to measure its intrinsic amplitude h and polarization;

- the direction information may lead to an identification of the galaxy or cluster of galaxies in which the event occurred, from which one could infer a distance and thence, from the amplitude h, estimate the total energy released in gravitational waves;

- if the detection can be confirmed and its direction analyzed quickly, then optical astronomers can be alerted to look in that direction for a new supernova — it is rare for astronomers to have the opportunity to see a supernova before it reaches its maximum brightness.

4. Coalescing compact-object binaries

The first suggestion that the orbital radiation emitted just before two binary neutron stars coalesce would be an interesting source of gravitational radiation was, remarkably, made by Freeman Dyson[14] before the discovery of pulsars, at a time when the existence of neutron stars was speculative. This paper correctly estimated the principal features of the radiation and its sources — the timescale of a few seconds, the waveform of increasing frequency and amplitude that we now call a "chirp", even that the distance to a typical source would be 100 Mpc — but erred in considering that these events would be detectable by the first bar detectors that were then under construction by Joseph Weber. In fact, no bar detectors so far designed have the frequency bandwidth that would be necessary to detect a chirp.

Subsequent papers[15, 16, 17] developed the basic theory of such systems, with the view that the actual coalescence event could be detectable by bars, a possibility which still exists. But the present intensive interest in coalescing binaries arose from the realization[18] that the planned interferometers would have a more reliable chance of seeing these events than of seeing supernovae. In this section I review the main characteristics of these sources, the prospects of seeing them with first-stage and second-stage detectors, and the considerable astrophysical information that we can expect to extract from the waveforms, provided relativists can make progress on the 2-body problem in the next few years.

4.1. Basics

The Binary Pulsar PSR 1913+16 referred to earlier will, in about 10^8 years, evolve to a point when the stars are separated by about 250 km. By that time the orbit will have circularized and the system will be emitting radiation with a frequency of 70 Hz. From then until the stars begin to merge, the system will evolve in a predictable way, the frequency of the radiation increasing as the stars gradually spiral together. Over a period of some 5–6 s, the frequency will increase from 70 Hz to nearly 1 kHz. Theoretical calculations by Clark and Eardley[16] and more recent numerical simulations by Oohara and Nakamura[19] have shown us that the stars do not begin to merge until they are so close together that the radiation has a frequency near 1 kHz. Although LIGO-type detectors may be able to detect this radiation from about 10 Hz, their best sensitivity will be above 70–100 Hz, so estimates of detection range are most conservatively made by assuming the detection begins at about this frequency.

The general character of the orbit is well represented if we simply take the radiation to be governed by the quadrupole formula. This tells us that there are of order $N \sim 630$ periods of gravitational waves until the stars begin to coalesce. If we can construct a detailed waveform that follows the radiation over this whole period, then we should be able to use it as a matched filter to improve the signal-to-noise ratio of a detection by about $\sqrt{N} \sim 25$ over what it would be for a broad-band burst of the same amplitude and central frequency. Put another way, if we know the waveform we can obtain the same signal-to-noise ratio as we would have for a broadband burst of the same total energy (integrated over the whole signal) at the same frequency.[13] Since the orbital motion radiates some $5 \times 10^{-3} M_{\odot}\, c^2$ of energy, these sources will be as detectable as a moderate supernova burst *at a low frequency, say 100 Hz.*

Our discussion above of low-frequency bursts from supernovae shows that this makes coalescing binaries some 30 times more detectable than a conventional kilo-Hertz supernova burst of the same energy. It therefore becomes possible to contemplate detecting these events out as far as 800 Mpc, which is a redshift of $z = 0.26(H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

There are also likely to be a certain number of binaries containing a neutron star and a black hole of, say, $10M_{\odot}$. These radiate with an amplitude approximately 5 times as large as that from two neutron stars. Although they execute fewer orbits before coalescence, such systems are still visible almost 3 times further away. And systems consisting of two black holes can be seen more than 10 times further away, which means that they are visible essentially everywhere in the Universe.

This great distance is the key to the interest in these sources. Although the coalescence events are rare in any galaxy, the volume of space we can observe is very large, and the number of events turns out to be very significant. Moreover, as we shall see, the observations themselves can carry very interesting cosmological information.

4.2. Event rate

Since coalescence events are rare and have never been observed, we can only infer a likely event rate from observations of their progenitors, binary systems containing two neutron stars in an orbit close enough to decay in a Hubble timescale. Such systems may be observable in our Galaxy if one of the neutron stars is a pulsar. The Binary Pulsar PSR1913+16 was our first example, and two others have been discovered since: PSR2127+11C (in the globular cluster M15) and PSR1534+12. Searches currently underway are very likely to turn up more.

Based on a careful analysis of the selection effects in the pulsar surveys that have taken place so far, two independent studies[20, 21] have reached broadly similar conclusions about the rate of coalescence in our Galaxy. Its most probable value is about one neutron-star coalescence per 10^7 years. When we extrapolate this to a distance of 800 Mpc, containing some 2×10^9 galaxies, we arrive at about 200 coalescences per year. Perhaps 10% of these will give strong enough signals to be detected[22], so the detection rate should be some tens per year. This estimate is to be regarded as an observational lower bound to the event rate, and it is uncertain by a factor of 10 either way. (This is a considerable improvement on the best earlier estimate, by Clark, *et al*,[17], which I estimated to be uncertain by a factor of 100.[24]) It could be increased by any of the following:

- If new searches turn up new progenitor systems, the estimated fraction of pulsars in binaries will increase and so will the predicted event rate.

- The event rate may also increase because of another remarkable conclusion of the two studies just cited. Because the formation of a black hole in a supernova explosion retains more of the mass of the original star, it has a smaller probability of disrupting a binary system. This means that the number of binaries containing a black hole and a neutron star may be much larger than one would at first suppose: it may be that one compact-object binary in three contains a black hole. If so, then pulsar surveys now underway should find such a system in our Galaxy. Since such systems are detectable at greater distances, they may even dominate the detection rate, raising it by a factor of 2 or more above the estimate we have just made.

- We have considered only the sensitivity of detectors above about 70 Hz. Depending on the noise performance at low frequencies, it may be possible to gain at least a factor of 2 in signal-to- noise ratio by going down to 40 Hz or lower.[23] This would double the range and multiply the detection rate by 8. Improving sensitivity at low frequency also offers more information about stars from the waveform, but only if the two-body problem in relativity can be solved more completely than it has been so far. I will return to this challenge below.

If one is optimistic about all these possibilities, then the expected detection rate in stage-two detectors could be pushed up from two per month to as many as 40 per day!

4.3. Coalescing binaries and first-stage detectors

More interesting, perhaps, is the possibility that stage-one detectors could see coalescing binaries. This possibility is usually discounted, because these detectors are not expected to be optimized for observing as low as 70 Hz, and this, coupled with a 10-times poorer sensitivity, leaves them with an expected event rate of about one per 10^5 years!

But if a stage-one detector is built with noise controlled down to 40 Hz, and is optimized for observing at 70 Hz, then its range is only a factor of 5 less than the conservative assumption we made for the stage-two detector above. Then if the event rate is actually 10 times higher, and if black holes contribute a further factor of 2, the expected detection rate even with a stage-one detector could be a few per year. Again, to exploit this possibility, detector designers and builders would have to give priority to low-frequency performance at the first stage of construction.

4.4. Information to be learned from observing them

The amount of information that one can in principle extract from coalescing binary observations is enormous. As for supernovae, we consider what different arrays of detectors can do. Coalescing binaries are essentially invisible to bar detectors because they emit very little of their total energy in the bandwidth of realistic detectors. If two laser interferometers were to observe a coalescing binary, they could do the following.

- They would automatically measure what we call the "chirp mass" of the binary, which is the following combination of the reduced mass μ and total mass M of the system:

$$\mathcal{M} = \mu^{3/5} M^{2/5}. \tag{4}$$

This is the only parameter that governs the quadrupolar emission of radiation, and so it determines the overall acceleration of the frequency of the chirp signal. Determining the value of this parameter is essential to extracting the signal from the interferometer noise.

- Provided sensitivity at or below 70 Hz is adequate, or if the system is near and the signal is sufficiently strong, they would be able to measure the individual component masses from post- Newtonian effects in the orbit. The dominant effect is a slowly accumulating phase error in the signal, as post-Newtonian corrections to the orbit grow secularly alongside the quadrupolar secular terms. With good low-frequency sensitivity, a stage-two detector should be able to measure the masses of hundreds of neutron stars and black holes over a few years' observing. The improvement in our understanding of neutron star formation, evolution, and equation of state will be enormous.
- Again, for the strongest systems they should be able to measure other parameters, such as the spins of the neutron stars. These have a marginal but accumulating effect on the orbital phase.
- They should observe black holes coalescing. Provided such events occur more than once a year out to quasar distances, second-stage detectors should see them. If numerical calculations of black-hole coalescence can be performed by the time observations occur, the observations will provide a unique test of strong-field gravity theory.

If three or more interferometric detectors detect a coalescing binary signal, then they can determine its intrinsic amplitude and direction, as well as secondary parameters such as the orientation of the plane of the binary system. Compact-object binaries are so easy to model that a knowledge of the chirp mass $\mathcal{M}$, the intrinsic amplitude, and the orientation of the binary are enough to tell us exactly how far away the system is. Such simple and reliable "standard candles" are rare in astronomy, and offer a wealth of new possibilities. Two such possibilities are:

- The determination of the Hubble constant.[25] This can happen in two ways.
  - We may be lucky enough to observe an event whose position can be determined precisely enough to locate it in a particular galaxy. This requires more precision than the $\pm 1°$ accuracy we expect for typical events. This may happen if the event is much closer and therefore stronger, or if it is also observed with other instruments (such as gamma-ray detectors, as described in the next section). Then the redshift of the galaxy can be measured, and its gravitational-wave distance then determines the Hubble constant.
  - More likely, individual events that occur within 100–150 Mpc will fall within error boxes containing a few clusters of galaxies. Since about 50% of galaxies are members of identifiable clusters, there is a good chance that, by measuring the redshifts of all the clusters and obtaining *candidate* values of the Hubble constant from each, one will find the correct value of H_0 among the candidates. Then if a number of different events are treated in the same way,

the correct value will repeat often, while the incorrect candidates will be distributed randomly. After a few (ten or so) such events, the real value of the Hubble constant will emerge. Depending on the event rate, this could take anywhere from 1 year to a decade to accomplish.

- Test the cosmological mass distribution for super-clustering on the 200+ Mpc scale. Given positions and distances to these systems, and given that they probably sample the visible mass distribution fairly well, one can expect to accumulate statistics on the homogeneity of the mass distribution on scale that we have little information about at present.

4.5. Gamma-ray bursts and coalescing binaries

Among the most puzzling astronomical phenomena at present are gamma-ray bursts. Originally these were thought to originate in neutron stars in the Galaxy, as fairly low-energy and relatively benign events. But the recent announcement[26] that the BATSE instrument on the Compton Observatory has detected hundreds of bursts, and that they are distributed perfectly isotropically on the celestial sky, has cast grave doubt on such models, which would have predicted a concentration towards the galactic plane. A currently popular model is that they are associated with the coalescence event of two neutron stars or a neutron star and a black hole.

Although all models are uncertain at present, it this one turns out to be correct, then gamma ray bursts should also be accompanied by gravitational wave signals. Nicholson and I[27] have studied the consequences of joint observations between a gamma-ray burst detector and two or three gravitational wave detectors. The principal advantage is that the gravitational wave detectors can lower their threshold for detection, since they need only discriminate between noise and real events in a narrow window of time (perhaps a second or less) before the gamma burst. This improves the range of the detectors and of course markedly affects the detected event rate. The more distant events that are now detectable have, of course, lower signal-to-noise, so it is harder to extract information from them. In particular, distance determinations become very inaccurate outside of perhaps 500 Mpc. Nevertheless, the added statistics on the chirp mass, and the ability to correlate gravitational waveforms with the individual characteristics of the gamma bursts is likely to be of great value in making models.

4.6. Challenge to relativity

When I discussed above the information that can be extracted from the signals from coalescing binaries, I qualified the discussion with the proviso that the two-body problem should have been adequately solved by the time observations are made. The reason is that, to identify a signal buried in detector noise that is of much higher amplitude, and to measure its parameters, one must perform matched filtering of the data stream: one must match the pattern expected from a source to the actual incoming wave. To do this, one needs good theoretical predictions of the waveform over a long period of time, and the prediction must keep in phase with the actual wave all the way.

The problem is that, although we have solutions of the two-body problem including terms of post- Newtonian order, including radiation reaction, there is doubt[28] that the post- Newtonian hierarchy of approximations will produce a convergent, or at least well-behaved, approximation for the evolution of the orbital phase as a function of time over the several minutes that systems can be observed if they are picked up at a few tens of Hz. In fact, the problem gets worse for systems that can be observed first at lower frequencies, that is when they are more nearly Newtonian. This counter-intuitive problem is worth describing.

The post-Newtonian hierarchy of approximations is an asymptotic approximation in a small parameter (essentially $\Phi \sim GM/rc^2$) that is uniform for a fixed number of orbits.[29] But observing coalescing binaries means following them as they decay, on a timescale proportional to $(v/c)^{-4} \sim \Phi^{-2}$. As the system becomes more Newtonian, so that $\Phi \to 0$, any fixed number of orbits occupies a time proportional to $1/v \sim \Phi^{-1/2}$, and therefore represents a fraction of the overall decay time that gets smaller as $\Phi^{3/2} \sim (v/c)^3$. It follows that, the more closely Newtonian the system is when it is first observed, the *worse* the post-Newtonian approximation will be for predicting what the orbit does all the way to coalescence.

If, despite this, the post-Newtonian approximations do provide a convergent description of the orbit, then it is just a matter of hard work to develop them out to post-post-Newtonian order, which might be sufficient for most observations. But if, as seems likely, they do not, then relativists have a challenge: find another way to give a reasonably complete solution. Numerical calculations for widely separated bodies seem prohibitively expensive. Is there another analytic method, involving expansions in a new small parameter, that will produce computationally feasible approximations that converge?

The reward will be signal templates that can extract the maximum information from the observations. It should be emphasized that the mere detection of the signal, and the measurement of the simple chirp mass, does not necessarily require a solution of the whole approximation problem: filters can be designed to pick up most signals even if they do not fit the whole wavetrain with a single analytic expression. But to measure physically interesting things requires filters that translate physical parameters into the observed waveforms.

5. Pulsars and the stochastic background

I will briefly discuss two other possible sources of gravitational waves: pulsars and a cosmological background. These are not as strongly emphasized in discussions of gravitational wave sources because they are more speculative, but they would nevertheless be very interesting if they are observed. It is arguable, in fact, that a positive detection of a cosmological background would be the most important observation that interferometers could make.

5.1. Pulsars old and new

A number of pulsars spin fast enough for any gravitational waves emitted by them to be of a high enough frequency (twice the pulsar frequency) to be detected by ground-based detectors. Indeed, searches for radiation from the Crab pulsar have been performed with special bar detectors built to resonate at 60 Hz.[30] Such radiation would arise in non-axisymmetric mass distributions, small lumps or irregularities in the crust of the star.

Irregularities are certainly present, if only because the stars possess non-axisymmetric magnetic fields, but they need not be large enough to produce observable amplitudes. The only observational limits we have on most pulsars come from spindown: gravitational waves must not carry away more energy than can be accounted for by the spindown of the pulsar. In fact, the gravitational wave luminosity of pulsars is likely to be much smaller than this, since the dominant loss of energy is almost certainly in the form of electromagnetic and particle fluxes.

The bounds that spindown places on the radiation from the Crab and Vela pulsars still leaves plenty of room for detectable amplitudes, and second-stage interferometers will certainly look for it. Millisecond pulsars, on the other hand, tend to be older and to be spinning down less rapidly, so their radiation limits are more stringent. A recently-discovered nearby millisecond pulsar[31] might offer some hope of eventual detection.

5.2. All-sky search for pulsars

There are many more radio-quiet ("dead") pulsars than there are active ones: perhaps one star in 10 in our Galaxy is a neutron star. So the nearest is only a few parsecs away, and it might well be a source of gravitational waves even though its radio emission has ceased.

Finding it, however, means performing an all-sky, all-frequency survey of the sky in gravitational waves. The sensitivity of such a search, it turns out, will be limited by available computing power in the foreseeable future.[32] The reason is that, provided a data set longer than a few hours is used, it will be necessary to remove the Doppler effects of the Earth's motion from the signal, and these effects depend on the pulsar's position on the sky. The data set must therefore be searched separately for each of a large number of small patches on the sky. This will limit data sets to a couple of weeks in the near future.

5.3. Stochastic background and the early Universe

While pulsars are relatively well-known objects, sources of a stochastic cosmological background of gravitational waves are a good deal more exotic. The leading candidates are cosmic strings. If they are massive enough to act as seeds for galaxy formation, then they must also produce gravitational waves as they oscillate and decay.[33] Other candidates are bubble collisions in extended inflation scenarios and quantum fluctuations in the fields that drive inflation.[34]

A stochastic background looks just like noise in a single detector. The only way to detect it is to look for correlated noise between two different detectors. The detectors must be separated enough so that other sources of noise — ground vibration, for example — are uncorrelated. But if the detectors are separated by too much, they will also not be correlated in their response to the background: if the time it takes a random wave to travel from one detector to the other is a good fraction of the period of the wave, then the responses of the detectors will not be correlated at a given time. For observing at 50 Hz, separations less than 1500 km are ideal. The two European detectors are well placed for this, while the two LIGO sites are rather further apart.

Bar detectors can also look for background radiation, and they have done so.[35] But their narrow bandwidth and limited sensitivity allows only very weak limits to be set at present.

The strength of the background is conveniently expressed by a quantity called Ω_{gw}, which is the fraction of the closure density contributed by the energy of the background per decade of frequency. In a bandwidth of about 50 Hz about 50 Hz, stage-two LIGO-type detectors could in principle reach below $\Omega_{gw} = 10^{-9}$, which would be enough to eliminate the cosmic string model or detect its background. Current limits from pulsar timing are around 10^7 at very low frequencies (periods of several years) — see the article by Taylor in this volume.

Although the existence of such a background is very speculative, a detection would open up an entirely new window on the very early Universe; the radiation would be coming to us directly from an epoch to which we have no other possible direct access. Performing a correlation experiment is arguably one of the most important goals of interferometer development, and it has a high priority in the detector groups.

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