COLOR GAUGE INVARIANCE OF T-ODD FRAGMENTATION FUNCTIONS *

ANDREAS METZ
Institut fuer Theoretische Physik II
Ruhr-Universitaet Bochum
44780 Bochum, Germany
E-mail: metza@tp2.ruhr-uni-bochum.de

The T-odd fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ particle is studied in the framework of a simple model. Special attention is payed to the gluon exchange which is incorporated in the gauge link of the fragmentation function. Comparing the transverse single spin asymmetries in e^+e^- annihilation and semi-inclusive deep-inelastic scattering it is found that, in the one-loop approximation, the corresponding fragmentation function is universal.

1 Introduction

For inclusive DIS the importance of gluon exchange between the struck quark and target spectators has been emphasized recently [1]. It has been demonstrated that this rescattering effect causes (additional) on-shell intermediate states in the Compton amplitude, resulting in a shadowing contribution to the DIS cross section. In Feynman gauge, this shadowing effect is described by the gauge link appearing in the definiton of parton distributions.

Subsequently, the effect of rescattering has also been investigated in the case of semi-inclusive DIS [2]. Using a simple model, it has been shown that a transverse single target-spin asymmetry arises from the interference between the tree-level amplitude of the fragmentation process and the imaginary part of the one-loop amplitude, where the latter describes the gluon exchange between the struck quark and the target system. This asymmetry has been interpreted as model for the time-reversal odd (T-odd) and transverse momentum dependent Sivers function [3] including its gauge link [4].

*This work is supported by the Sofia Kovalevskaya Programme of the Alexander von Humboldt Foundation

Figure 1: Tree-level diagrams of fragmentation in e^+e^- annihilation and semi-inclusive DIS. In both cases a quark fragments into a spin-$\frac{1}{2}$ hadron and a scalar remnant (dashed line).

By considering the behaviour of the path-ordered exponential under time-reversal a very interesting observation has been made in Ref. [4]: the Sivers asymmetry in semi-inlcusive DIS has the opposite sign compared to the one in Drell-Yan, i.e., the Sivers function is non-universal.

Here we study the universality of T-odd spin-dependent fragmentation functions, focussing on the fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ hadron [5]. To this end we investigate the gluon exchange incorporated in the gauge link of the fragmentation functions for both e^+e^- annihilation and semi-inclusive DIS. In a one-loop model calculation we find universality of T-odd fragmentation [5]. (Note that, in contrast to T-odd parton densities, the corresponding fragmentation functions obtain a universal non-vanishing contribution, even if their gauge link is neglected [6].) We point out that universality holds for the T-odd Collins function [7] (fragmentation of a transversely polarized quark into an unpolarized hadron) as well.

2 Rescattering in T-odd fragmentation to one-loop

In Fig. 1, the tree-level diagrams of the two fragmentation processes are displayed. For e^+e^- annihilation we consider the decay of a timelike virtual photon into a $q\bar{q}$ pair, where the quark fragments into a spin-$\frac{1}{2}$ hadron (e.g. a proton) and a scalar remnant (we use the model of Ref. [2]), i.e.,

$$\gamma^*(q) \to \bar{q}(p_1, \lambda') + p(p, \lambda) + s(p_2) \, . \tag{1}$$

Figure 2: One-loop diagrams of fragmentation in e^+e^- annihilation and semi-inclusive DIS. The possible on-shell intermediate states are indicated by thin lines.

The one-loop corrections are shown in Fig. 2. For e^+e^- annihilation (semi-inclusive DIS) a single photon is exchanged between the remnant and the anti-quark (initial quark). These diagrams provide a simple Abelian model for the lowest order contribution of the gauge link of the fragmentation function. Two cuts (on-shell quark and antiquark, as well as on-shell antiquark and remnant) for e^+e^- annihilation have no counterpart in semi-inclusive DIS. It turns out that these two on-shell states contribute to the transverse spin asymmetry, but their contributions cancel against each other [5]. The quark-photon cut in e^+e^- annihilation corresponds to the cut in DIS.

We consider e^+e^- annihilation in the rest frame of the timelike photon. The proton in the final state has no transverse momentum, and its minus-momentum is given by zq^-, where q^- is the minus-momentum of the virtual photon. We fix the plus-momentum of the antiquark according to $p_1^+ \approx q^+$. The antiquark also carries a soft transverse momentum $-\vec{\Delta}_\perp$, implying that the fragmenting quark and the outgoing proton have a relative transverse momentum, which is necessary for the transverse spin asymmetry. These requirements specify the kinematics:

$$
\begin{aligned}
q &= \Big(Q\,, Q\,, \vec{0}_\perp\Big)\,, & p_1 &= \left(Q\,, \frac{\vec{\Delta}_\perp^2 + m_q^2}{Q}\,, -\vec{\Delta}_\perp\right), \\
p &= \left(\frac{M^2}{zQ}\,, zQ\,, \vec{0}_\perp\right), & p_2 &= \left(\frac{\vec{\Delta}_\perp^2 + m_s^2}{(1-z)Q}\,, (1-z)Q\,, \vec{\Delta}_\perp\right). \qquad (2)
\end{aligned}
$$

For simplicity, in (2) only the leading terms have been listed.

In the model of Ref. [2] the proton carries no electromagnetic charge. Therefore, the charge of the fragmenting quark (denoted by e_1) and the one of the remnant are equal. The interaction between the quark, the proton, and the remnant is described by a scalar vertex with the coupling constant g. The x-component of the current for the diagram on the *lhs* in Fig. 1 reads

$$\begin{aligned} J^1_{(0)}(\lambda,\lambda') &= e_1\, g\, \frac{1}{s-m_q^2}\, \bar{u}(p,\lambda)\,(\not{q}-\not{p}_1+m_q)\,\gamma^1\, v(p_1,\lambda') \\ &= e_1\, g\, \frac{1-z}{\sqrt{z}}\, \frac{Q}{\vec{\Delta}_\perp^2+\tilde{m}^2}\left[(\Delta^1 - i\lambda\Delta^2)\delta_{\lambda,-\lambda'} - \lambda\left(\frac{M}{z}+m_q\right)\delta_{\lambda,\lambda'}\right], \quad (3) \\ \text{with}\quad \tilde{m}^2 &= \frac{1}{z}\left(M^2\,\frac{1-z}{z} + m_s^2 - m_q^2\,(1-z)\right), \end{aligned}$$

where use has been made of the relation

$$s - m_q^2 = (q-p_1)^2 - m_q^2 = \frac{z}{1-z}\left(\vec{\Delta}_\perp^2 + \tilde{m}^2\right), \tag{4}$$

which connects the total energy $\sqrt{s}$ in the *cm*-frame of the outgoing proton and scalar remnant with the variables z and $\vec{\Delta}_\perp^2$.

The transverse spin asymmetry $\mathcal{A}_x$ (polarization of the proton along x-axes) is given by $\sigma_{pol}/\sigma_{unp}$, with the cross sections evaluated according to

$$\begin{aligned} \sigma_{unp} &\propto \frac{1}{2}\sum_{\lambda,\lambda'} J^1(\lambda,\lambda')\left(J^1(\lambda,\lambda')\right)^*, \\ \sigma_{pol} &\propto \frac{1}{2}\sum_{\lambda'}\left[J^1(s_x=\uparrow,\lambda')\left(J^1(s_x=\uparrow,\lambda')\right)^*\right. \qquad (5) \\ &\qquad \left. - J^1(s_x=\downarrow,\lambda')\left(J^1(s_x=\downarrow,\lambda')\right)^*\right]. \end{aligned}$$

While σ_{unp} is computed using the tree level current, in σ_{pol} the imaginary part of the one-loop amplitude is inlcuded to obtain a non-zero asymmetry.

Taking into consideration the imaginary part due to the on-shell $q\gamma$ intermediate state, one obtains the following asymmetry [5]:

$$\begin{aligned} \mathcal{A}_{x,q\gamma} &= -\frac{(e_1)^2}{8\pi}\,\frac{2\,\frac{M}{z}\,\Delta^2}{\left(\frac{M}{z}\right)^2+\vec{\Delta}_\perp^2}\,\frac{\vec{\Delta}_\perp^2+\tilde{m}^2}{\vec{\Delta}_\perp^2}\left[\ln\frac{p_{20}-|\vec{p}_2|\cos\alpha}{m_s}\right. \\ &\quad \left. +\cos\alpha\ln\frac{p_{20}+|\vec{p}_2|}{m_s} + \frac{1-\cos^2\alpha}{4(1-z)}\left(1-\frac{p_{20}}{|\vec{p}_2|}\ln\frac{p_{20}+|\vec{p}_2|}{m_s}\right)\right], \quad (6) \end{aligned}$$

where the result holds for $m_q = 0$. In Eq. (6) the energy and the three-momentum of the remnant, as well as the scattering angle (angle between the antiquark and the remnant) in the *cm*-frame of the proton and the remnant appear. These variables can be expressed in terms of z and $\vec{\Delta}_\perp^2$ (see Ref. [5]).

By explicit calculation we have shown that for semi-inclusive DIS the spin-asymmetry caused by the on-shell $q\gamma$ state coincides with the result in (6), i.e., the asymmetry in both processes has even the same sign. As a consequence, the total transverse spin asymmetries in e^+e^- annihilation (i.e. including all three on-shell intermediate states) and in semi-inclusive DIS are equal, showing that the T-odd fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ hadron is universal in a one-loop model. As a final remark we note that universality of fragmentation functions should hold in general [8].

Acknowledgements

I am very grateful to John Collins for drawing my attention to the fact that in e^+e^- annihilation, compared to semi-inclusive DIS, two additional on-shell intermediate states may contribute to the transverse spin asymmetry.

References

[1] S.J. Brodsky, P. Hoyer, N. Marchal, S. Peigné, and F. Sannino, Phys. Rev. D **65**, 114025 (2002).

[2] S.J. Brodsky, D.S. Hwang, and I. Schmidt, Phys. Lett. B **530**, 99 (2002).

[3] D.W. Sivers, Phys. Rev. D **41**, 83 (1990); Phys. Rev. D **43**, 261 (1991).

[4] J.C. Collins, Phys. Lett. B **536**, 43 (2002).

[5] A. Metz, Phys. Lett. B **549**, 139 (2002).

[6] A. Bacchetta, R. Kundu, A. Metz, and P.J. Mulders, Phys. Lett. B **506**, 155 (2001).

[7] J.C. Collins, Nucl. Phys. **B396**, 161 (1993).

[8] J.C. Collins, A. Metz, in preparation.