

# Neutrino mass hierarchy and CP phase measurement using atmospheric neutrino flux

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**Abstract.** Multi-megaton scale ice or water Cherenkov detectors with relatively low (sub-GeV) threshold energy can accumulate huge statistics of atmospheric neutrino data. With reasonable energy and angular reconstruction efficiency for the neutrino events, these data can be used to establish yet unknown neutrino mass hierarchy with high confidence. Leptonic CP phase can also be measured using atmospheric neutrino flux, once hierarchy is established and uncertainty on the flux and other neutrino parameters are better understood. Following up on previous work on this topic we will present the latest calculation results in light of recent developments.

## 1. Introduction

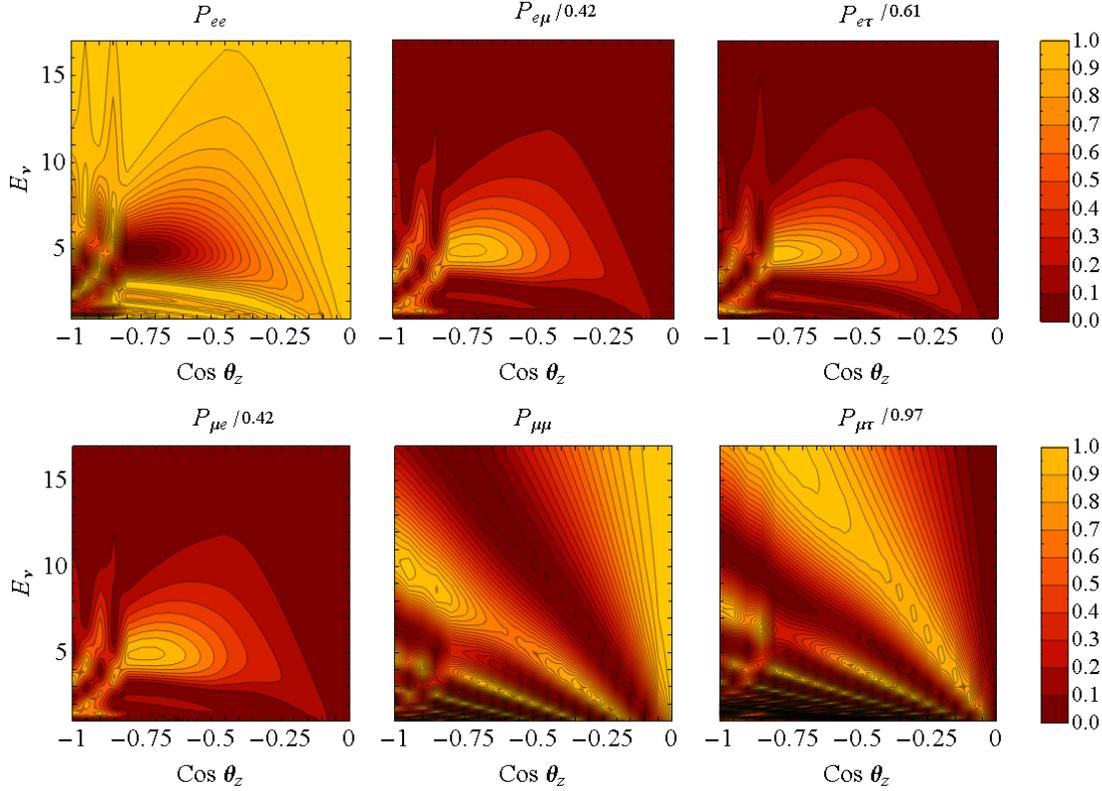
Neutrino mass hierarchy and the value of the leptonic CP phase are two outstanding problems in particle physics [1, 2]. Atmospheric neutrino flux measurements in large water/ice detectors can be used to determine the mass hierarchy, i.e. the sign of the atmospheric mass-square-difference  $\pm\Delta m_{31}^2$ , and the Dirac CP phase  $\delta$ . Information on hierarchy and different  $\delta$  values are encoded in the neutrino oscillation probabilities after propagation inside the earth (matter effect) [3, 4].

It has been found recently that future detectors such as the *Precision IceCube Next Generation Upgrade* (PINGU) and *Oscillation Research with Cosmics in the Abyss* (ORCA) with  $\sim 3$  GeV threshold will have good sensitivity to determining the neutrino mass hierarchy [5, 6, 7]. However, measurement of  $\delta$  would require a detector with larger effective volume and improved characteristics in the  $< 2$  GeV range and in this context a future upgrade of PINGU (and also of ORCA), called Super-PINGU, was proposed in Ref. [8] with detailed estimate of sensitivity. Here we present latest development in determining mass hierarchy and measuring leptonic CP phase using atmospheric neutrino flux.

## 2. Neutrino Oscillograms

Atmospheric neutrinos, produced isotropically by cosmic-ray interactions in the upper atmosphere, penetrate through the earth to reach a neutrino detector. The amount of earth's interior mantle and core structure the neutrinos pass through depends on the arrival direction at the detector (zenith angle  $\theta_z$ ). This also determines how much oscillation is induced by the matter effect [9, 10]. Figure 1 shows oscillation probabilities of different neutrino flavors passing through the earth with different trajectories and energy. We use the earth's interior model from ref. [11]. These 2-D plots, also called the oscillograms of the earth [3] are useful to understand

effects of various neutrino oscillation parameters (and eventually determine them), such as the neutrino mass hierarchy and CP phase, on the probabilities.



**Figure 1.** Neutrino oscillation probabilities in the  $E_\nu$ - $\cos\theta_z$  plane for different oscillation channels with the contours representing lines of equal probabilities. The probabilities are normalized by their maximal values in the parameter space of the panels:  $P_{\alpha\beta}/P_{\alpha\beta}^{max}$ , with  $P_{ee}^{max} = P_{\mu\mu}^{max} = 1$ .  $E_\nu$  is in GeV. Such plots are called neutrino oscillograms of the Earth. From Ref. [5]

### 3. PINGU and Super-PINGU

We compute atmospheric neutrino event rates for different flavors in the proposed PINGU and Super-PINGU detectors. The PINGU detector [6] will have 40 strings additional to the IceCube DeepCore strings with 60 Digital Optical Modules (DOMs) at 5 m spacing in each string. The effective mass, i.e. density times the effective volume, of PINGU can be parametrized as [8]

$$\begin{aligned}\rho V_{\text{eff},\mu}(E_\nu) &= 3.0 [\log(E_\nu/\text{GeV})]^{0.61} \text{ Mt} \\ \rho V_{\text{eff},e}(E_\nu) &= 3.1 [\log(E_\nu/\text{GeV})]^{0.60} \text{ Mt},\end{aligned}\quad (1)$$

respectively for  $\nu_\mu$  and  $\nu_e$ . For the Super-PINGU detector we use an effective mass (both for  $\nu_\mu$  and  $\nu_e$ ) parameterized as [8]

$$\rho V_{\text{eff}}(E_\nu) = 2.6 [\log(E_\nu/\text{GeV}) + 1]^{1.32} \text{ Mt},\quad (2)$$

which can be realized for a total of 126 strings and 60 DOMs per string. This gives an effective mass of  $\sim 2.8$  Mt at  $\sim 1$ -2 GeV, which is 4 times larger than PINGU.

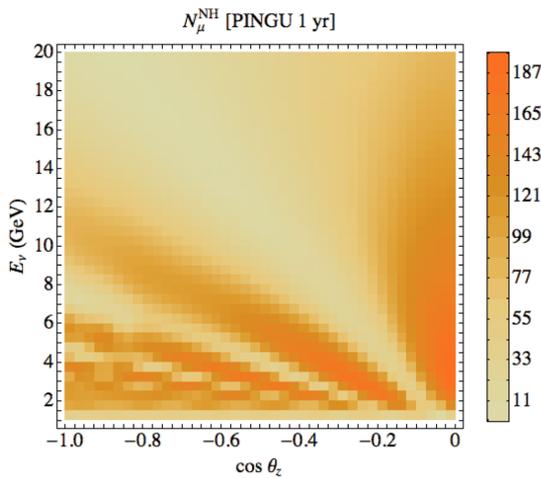
Since the most sensitive energy range for hierarchy determination is  $\sim 4\text{--}14$  GeV [5], PINGU can determine hierarchy rather efficiently but with little sensitivity to the CP phase. The energy range most sensitive to the CP phase is below  $\sim 2$  GeV [8], where PINGU has very little sensitivity. A detector such as Super-PINGU with large volume at low energy will be required to measure the CP phase.

#### 4. Distinguishability between hierarchies and CP phases

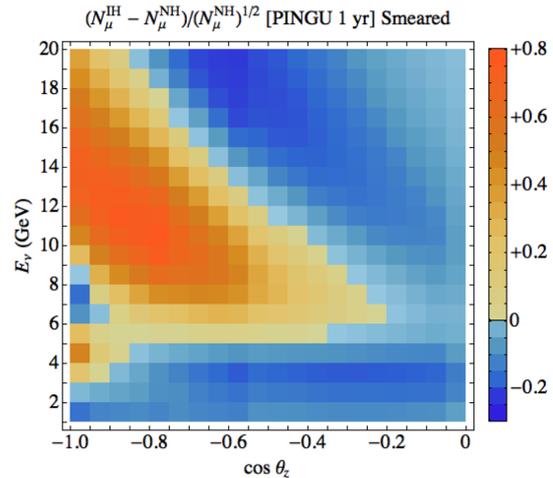
The number of neutrino events for a particular flavor  $\alpha = e, \mu$  with energies and zenith angles in small bins  $\Delta(E_\nu)$  and  $\Delta(\cos\theta_z)$  marked by subscript  $ij$  can be calculated as

$$N_{ij,\alpha} = 2\pi N_A \rho T \int_{\Delta_i \cos\theta_z} d\cos\theta_z \int_{\Delta_j E_\nu} dE_\nu V_{\text{eff},\alpha}(E_\nu) d_\alpha(E_\nu, \theta_z). \quad (3)$$

Here  $T$  is the exposure time,  $N_A$  is the Avogadro's number. The density of events of type  $\alpha$ ,  $d_\alpha$  (the number of events per unit time per target nucleon), is given by  $d_\alpha(E_\nu, \theta_z) = \sigma_\alpha \Phi_\alpha + \bar{\sigma}_\alpha \bar{\Phi}_\alpha$  in terms of fluxes at the detector, and  $\Phi_\alpha = \Phi_\mu^0 P_{\mu\alpha} + \Phi_e^0 P_{e\alpha}$  with corresponding oscillation probabilities  $P_{\mu\alpha}$  and  $P_{e\alpha}$ , and  $\nu N$  interaction cross sections  $\sigma_\alpha$  and  $\bar{\sigma}_\alpha$ . The original muon and electron neutrino fluxes at the production are  $\Phi_\mu^0 = \Phi_\mu^0(E_\nu, \theta_z)$  and  $\Phi_e^0 = \Phi_e^0(E_\nu, \theta_z)$ . We use the atmospheric flux model in ref. [12] which fits available data rather well.



**Figure 2.** Distribution of  $\nu_\mu + \bar{\nu}_\mu$  events/year in PINGU for NH in the  $E_\nu$ - $\cos\theta_z$  plane. From ref. [5]



**Figure 3.** Distribution of  $S_{ij}$  for  $\nu_\mu + \bar{\nu}_\mu$  events in PINGU between the NH and IH in the  $E_\nu$ - $\cos\theta_z$  plane. From ref. [5]

Figure 2 shows the distributions of expected  $\nu_\mu + \bar{\nu}_\mu$  events/year in PINGU in the  $E_\nu$ - $\cos\theta_z$  plane. Accumulation huge statistics ( $\sim 70,000$ /year for PINGU and  $\sim 90,000$ /year for Super-PINGU) can be used to discriminate between the mass hierarchies and different CP phases.

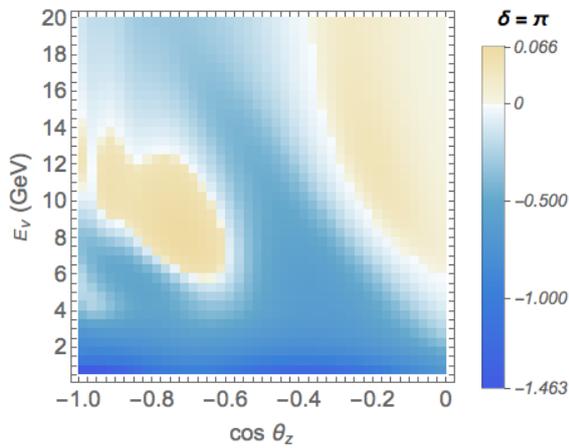
To determine mass hierarchy with PINGU we compute the distributions of  $\nu_\mu + \bar{\nu}_\mu$  and  $\nu_e + \bar{\nu}_e$  events for the normal hierarchy (NH) and inverted hierarchy (IH), keeping all other oscillation parameters fixed, and take difference between the distributions in the  $E_\nu$ - $\cos\theta_z$  plane. Since there are errors associated with reconstructing the true neutrino energy and directions, we smear the ideal distributions with the energy and angular resolution functions of the detector to mimic the real situation. To estimate the sensitivity to IH from NH (which is assumed true) we employ a distinguishability parameter defined as [5]

$$S_{ij}(f) = (N_{ij}^{\text{IH}} - N_{ij}^{\text{NH}})/\sigma_{ij}(f), \quad (4)$$

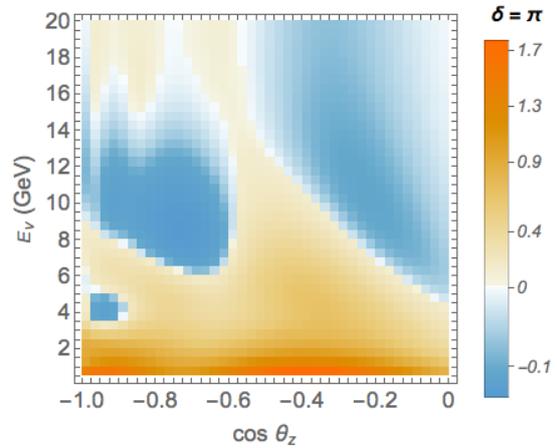
where  $N_{ij}^{\text{NH}}$  and  $N_{ij}^{\text{IH}}$  are the reconstructed number of events in the  $ij$ -th bin in the  $E_\nu$ - $\cos\theta_z$  plane for the NH and IH, respectively, and  $\sigma_{ij}^2(f) = N_{ij}^{\text{NH}} + (fN_{ij}^{\text{NH}})^2$  is the total error in the  $ij$ -th bin. Parameter  $f$  is a measure of uncorrelated systematic errors [5]. Figure 3 shows the expected distribution of  $S_{ij}$ , with  $f = 0$  for 1-year PINGU data. The total distinguishability

$$S_\sigma = \sqrt{\sum_{ij} S_{ij}^2} \quad (5)$$

is a quick measure of statistical significance. We found that hierarchies can be determined in this approach with  $S_\sigma \sim 3$ –4 within 3 years of PINGU operation [5].



**Figure 4.** Distribution of  $S_{ij}$  for  $\nu_\mu + \bar{\nu}_\mu$  events in Super-PINGU between  $\delta = \pi$  and  $\delta = 0$  in the  $E_\nu$ - $\cos\theta_z$  plane. From ref. [8]



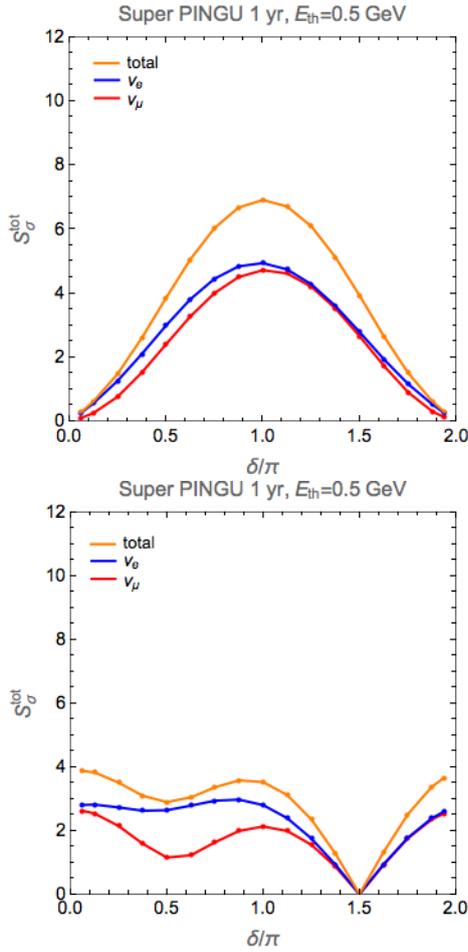
**Figure 5.** Distribution of  $S_{ij}$  for  $\nu_e + \bar{\nu}_e$  events in Super-PINGU between  $\delta = \pi$  and  $\delta = 0$  in the  $E_\nu$ - $\cos\theta_z$  plane. From ref. [8]

In order to measure the CP phase with atmospheric neutrinos, we calculate neutrino events in Super-PINGU by varying  $\delta$  and compare with  $\delta = 0$ , keeping all other oscillation parameters fixed. As we did for hierarchy, we compute the distributions of  $\nu_\mu + \bar{\nu}_\mu$  and  $\nu_e + \bar{\nu}_e$  events for  $\delta = 0$  and  $\delta \neq 0$  and take difference between the distributions in the  $E_\nu$ - $\cos\theta_z$  plane. To estimate the sensitivity of measuring a CP phase different from zero we employ a distinguishability parameter, similar to the hierarchy case, defined as

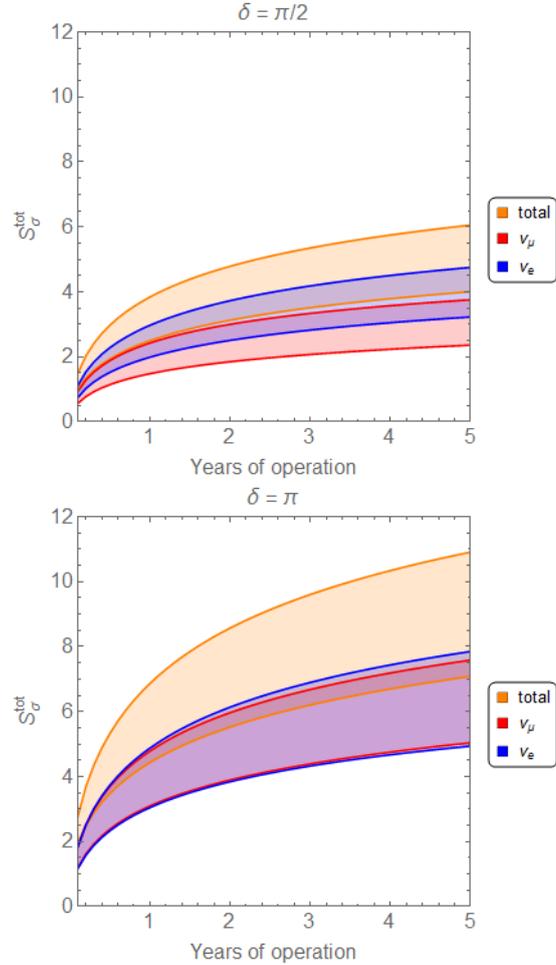
$$S_{ij}(f) = (N_{ij}^\delta - N_{ij}^0)/\sigma_{ij}(f), \quad (6)$$

where  $N_{ij}^\delta$  and  $N_{ij}^0$  are the reconstructed number of events in the  $ij$ -th bin in the  $E_\nu$ - $\cos\theta_z$  plane for  $\delta$  and  $\delta = 0$ , respectively, and  $\sigma_{ij}^2(f) = N_{ij}^0 + (fN_{ij}^0)^2$  is the total error in the  $ij$ -th bin. The total distinguishability is calculated using the same formula, Eq. (5). Figure 4 shows the distinguishability  $S_{ij}$  for  $\nu_\mu + \bar{\nu}_\mu$  events with  $f = 0$  for  $\delta = \pi$  and  $\delta = 0$  using 1-year of Super-PINGU data. Normal mass hierarchy is assumed. The shape of the distributions, specially their domain structures, is largely explained as due to grids of solar, atmospheric and interference magic lines in the  $E_\nu$ - $\cos\theta_z$  plane. The oscillation probabilities are independent of  $\delta$  along these lines, thus separating regions of same sign distinguishability. Figure 5 shows  $S_{ij}$  distributions for  $\nu_e + \bar{\nu}_e$  events.

The uncertainties associated with atmospheric neutrino flux,  $\nu N$  cross section, effective volume, etc. affect neutrino event distributions across bins in the  $E_\nu$ - $\cos\theta_z$  plane. We include



**Figure 6.** Total distinguishability to measure the CP phase with Super-PINGU 1-year data in the  $\nu_\mu$  and  $\nu_e$  channels. The top panel is for true  $\delta = 0$  and the bottom panel is for true  $\delta = 3\pi/2$ .



**Figure 7.** Time to distinguish CP phase  $\delta = \pi/2$  (top panel) and  $\delta = \pi$  (bottom panel) from  $\delta = 0$  using Super-PINGU data, with given level of the total distinguishability  $S_\sigma^{\text{tot}}$ . The shaded bands correspond to different levels of uncertainties.

effects of these correlated uncertainties in our calculation with analogy to the pull method in  $\chi^2$  analysis. In particular we minimize the following distinguishability parameter

$$S_\sigma^{\text{tot}}(\xi_k) = \left[ \sum_{l=e,\mu} \sum_{ij} \frac{[N_{ij,l}^\delta(\xi_k) - N_{ij,l}^0(\xi_k^{\text{st}})]^2}{\sigma_{ij,l}^2} + \sum_k \frac{(\xi_k - \xi_k^{\text{st}})^2}{\sigma_k^2} \right]^{1/2}, \quad (7)$$

where  $\xi_k$  are the pull variables and  $\xi_k^{\text{st}}$  are their standard values. The event distributions with varying  $\xi_k$  are calculated as

$$N_{ij,l}^\delta(\xi_k) = \alpha z_l (E/2 \text{ GeV})^\eta [1 + \beta(0.5 + \cos \theta_z)] N_{ij,l}^\delta(\xi_k^{\text{st}}), \quad (8)$$

where  $\alpha$  is the overall normalization factor with the error  $\sigma_\alpha = 0.2$ ,  $z_l$  is the flux (flavor) ratio uncertainty ( $z_e \equiv 1$  for  $\nu_e$  events), with the error  $\sigma_z = 0.05$ ;  $\eta$  is the energy tilt parameter with

$\sigma_\eta = 0.1$ ;  $\beta$  is the zenith angle tilt with  $\sigma_\beta = 0.04$ . Figure 6 shows the  $S_\sigma^{tot}$  minimized over  $(\xi_k)$  for different correlated uncertainties. The upper (lower) panel corresponds to the case when the true CP is  $\delta = 0$  ( $3\pi/2$ ). A threshold energy of 0.5 GeV,  $f = 2.5\%$  as well as all correlated uncertainties have been assumed. Note that the contributions of  $\nu_e$  and  $\nu_\mu$  channels to  $S_\sigma^{tot}$  are comparable.

Figure 7 shows the years of Super-PINGU data required to achieve different levels of distinguishability in case  $\delta = \pi/2$  (top panel) and  $\delta = \pi$  (bottom panel). The shaded bands correspond to different levels of flavor misidentifications in the detector, i.e., what fraction of  $\nu_m u$  events are reconstructed as  $\nu_e$  events and vice versa. The upper edge of the bands correspond to no flavor misidentification and the lower edge of the bands correspond to 20% flavor misidentification. Preliminary PINGU simulations show that flavor misidentification can be kept at 10%–30% level [6].

## 5. Summary

In summary, we find that the proposed future PINGU upgrade of the IceCube neutrino observatory will be able to determine the neutrino mass hierarchy using atmospheric neutrino flux, either NH or IH, within  $\sim 3$  years of its operation. The hypothetical Super-PINGU detector will be sensitive to sub-GeV neutrinos and can distinguish the CP phase  $\delta \geq \pi/2$  from zero within  $\sim 5$  years of its operation. We took into account various systematic effects in our computation as close as possible to the realistic scenario. Systematic effects dominate in both PINGU and Super-PINGU. Determination of hierarchy with PINGU can be very quick compared to the reactor/accelerator experiments [13]. Measurement of CP phase with Super-PINGU using atmospheric neutrinos can be competitive with long baseline neutrino experiments [14].

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## References

- [1] Branco G C, Felipe, R G and Joaquim F R 2012 *Rev. Mod. Phys.* **84** 515
- [2] Nunokawa H, Parke, S J and Valle J W F 2008 *Prog. Part. Nucl. Phys.* **60** 338
- [3] Akhmedov E Kh, Maltoni M and Smirnov A Yu 2008 *JHEP* **0806** 072
- [4] Mena O, Mocioiu I and Razzaque S 2008 *Phys. Rev. D* **78** 093003
- [5] Akhmedov E Kh, Razzaque S and Smirnov A Yu 2013 *JHEP* **1302** 082; *erratum ibid* 2013 *JHEP* **1307** 026
- [6] Aartsen M G *et al.* [IceCube-PINGU Collaboration] 2014 Letter of Intent: The Precision IceCube Next Generation Upgrade (PINGU) *Preprint* arXiv:1401.2046 [physics.ins-det].
- [7] Katz U F [KM3NeT Collaboration] 2014 The ORCA Option for KM3NeT *Preprint* arXiv:1402.1022 [astro-ph.IM].
- [8] Razzaque S and Smirnov A Yu 2015 *JHEP* **1505** 139
- [9] Wolfenstein L 1978 *Phys. Rev. D* **17** 2369
- [10] Mikheev S P and Smirnov A Yu 1985 *Sov. J. Nucl. Phys.* **42** 913 [*Yad. Fiz.* **42** 1441]
- [11] Dziewonski A M and Anderson D L 1981 *Phys. Earth Planet. Interiors* **25** 297
- [12] Honda M, Kajita, T, Kasahara, K and Midorikawa, S 1995 *Phys. Rev. D* **52** 4985
- [13] Blennow M and Schwetz T 2013 *JHEP* **1309** 089
- [14] Adams C *et al.* [LBNE Collaboration] 2013 The Long-Baseline Neutrino Experiment: Exploring Fundamental Symmetries of the Universe *Preprint* arXiv:1307.7335 [hep-ex]