ON MULTI-QUARK GREEN FUNCTIONS IN NAMBU–JONA-LASINIO MODEL

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Introduction and Summary

Nambu-Jona-Lasinio (NJL) model with quark content is one of the most successful effective models of quantum chromodynamics of light hadrons in the non-perturbative region(see review [1]). In overwhelming majority of the investigations, the NJL model has been considered in the mean-field approximation or in the leading order of $1/n_c-$expansion. However, a number of perspective physical applications of NJL model is connected with multi-quark functions (for example, meson decays, pion-pion scattering, baryons, pentaquarks etc.). These multi-quark functions arise in higher orders of the mean-field expansion (MFE) for NJL model.

In present report we review some preliminary results of investigation of higher orders of MFE for NJL model. To formulate MFE we have used an iteration scheme of solution of Schwinger-Dyson equation with fermion bilocal source, which has been developed in works [2]. We have considered equations for Green functions of NJL model in MFE up to third order. The leading approximation and the first order of MFE maintain equations for the quark propagator and the two-particle function and also the first-order correction to the quark propagator. A consideration of these equations is the usual field of investigations of NJL model. The second order of MFE maintains the equations for four-particle and three-particle functions, and the third order maintains the equations for six-particle and five-particle functions. (Note, that the construction of the five-particle functions gives us a possibility to investigate the pentaquark states in NJL model.) Here we discuss first results of investigation of the second-order equations for four-particle and three-particle Green functions.

1 Mean-field expansion in bilocal-source formalism

We consider NJL model with the Lagrangian

$$\mathcal{L} = \bar\psi i\hat\partial\psi + \frac{g}{2}\Big[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\tau^a\psi)^2\Big].$$

The Lagrangian is invariant under transformations of chiral group $SU_V(2)\times SU_A(2)$ and corresponds to u-d quark sector. A generating functional of Green functions (vacuum expectation values of T-products of fields) can be represented as the functional integral with bilocal source:

$$G(\eta) = \int D(\psi,\bar\psi)\exp i\Big\{\int dx\mathcal{L} - \int dxdy\bar\psi(y)\eta(y,x)\psi(x)\Big\}.$$

Here $\eta(y,x)$ is the bilocal source of the quark field.

The n-th functional derivative of G over source η is the n-particle (2n-point) Green function:

$$\frac{\delta^n G}{\delta\eta(y_1,x_1)\cdots\delta\eta(y_n,x_n)}\bigg|_{\eta=0} = i^n < 0 \mid T\Big\{\psi(x_1)\bar\psi(y_1)\cdots\psi(x_n)\bar\psi(y_n)\Big\} \mid 0 >\equiv S_n\begin{pmatrix} x_1 & y_1 \\ \cdots & \cdots \\ x_n & y_n\end{pmatrix}.$$

Translational invariance of the functional-integration measure gives us the functional-differential Schwinger-Dyson equation for generating functional G :

$$\delta(x-y)G + i\hat{\partial}_x \frac{\delta G}{\delta\eta(y,x)} + ig\Big\{\frac{\delta}{\delta\eta(y,x)}\,\mathrm{tr}\left[\frac{\delta G}{\delta\eta(x,x)}\right] - \gamma_5\tau^a \frac{\delta}{\delta\eta(y,x)}\,\mathrm{tr}\left[\gamma_5\tau^a \frac{\delta G}{\delta\eta(x,x)}\right]\Big\} =$$

$$= \int dx_1 \eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)}.$$

We shall solve this equation employing the method which proposed in work [3]. A leading approximation is the functional

$$G^{(0)} = \exp\Big\{\mathrm{Tr}\left(S^{(0)} * \eta\right)\Big\}.$$

The leading approximation generates the linear iteration scheme:

$$G = G^{(0)} + G^{(1)} + \cdots + G^{(n)} + \cdots ,$$

Functional $G^{(n)}$ is

$$G^{(n)} = P^{(n)} G^{(0)},$$

where $P^{(n)}$ is a polynomial of $2n$ -th order over the bilocal source η .

The unique connected Green function of the leading approximation is the quark propagator. Other connected Green functions appear in the following iteration steps. The quark propagator in the chiral limit is

$$S^{(0)} = (m - \hat{p})^{-1},$$

where m is the dynamical quark mass, which is a solution of gap equation.

A first-order functional is

$$G^{(1)} = \Big\{\frac{1}{2}\mathrm{Tr}\Big(S_2^{(1)} * \eta^2\Big) + \mathrm{Tr}\Big(S^{(1)} * \eta\Big)\Big\}G^{(0)} .$$

The iteration-scheme equations give us the equation for first-order two-particle function $S_2^{(1)}$:

$$S_2^{(1)}\begin{pmatrix} x & y \\ x' & y' \end{pmatrix} = -S_0(x-y')S_0(x'-y) +$$

$$+ig\int dx_1\Big\{(S_0(x-x_1)S_0(x_1-y))\,\mathrm{tr}\left[S_2^{(1)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \end{pmatrix}\right] -$$

$$-(S_0(x-x_1)\gamma_5\tau^a\tau^{a_1}S_0(x_1-y))\,\mathrm{tr}\left[\gamma_5\tau^a\tau^{a_1}S_2^{(1)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \end{pmatrix}\right]\Big\} \qquad (1)$$

and the first-order correction to quark propagator $S^{(1)}$:

$$S^{(1)}(x-y) = ig\int dx_1 S^{(0)}(x-x_1)\Big\{S_2^{(1)}\begin{pmatrix} x_1 & y \\ x_1 & x_1 \end{pmatrix} - \gamma_5\tau^a S_2^{(1)}\begin{pmatrix} x_1 & y \\ x_1 & x_1 \end{pmatrix}\gamma_5\tau^a\Big\} +$$

$$+ig\int dx_1 S^{(0)}(x-x_1)S^{(0)}(x_1-y)\,\mathrm{tr}\,S^{(1)}(0).$$

The graphical representations of these equations see on Figs. 1 and 2.

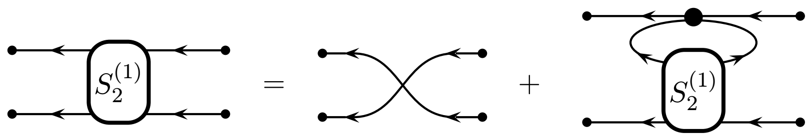

Fig.1. The equation for first-order two-particle function.

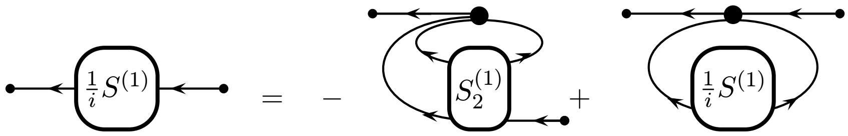

Fig. 2. The of equation for first-order correction to quark mass.

Here the graphical notations of Fig. 3 are used.

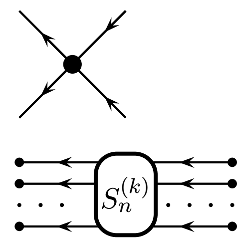

Fig. 3. Diagram rules.

To describe the solution of the first-order equation for two-particle function and for future purposes we introduce the composite meson propagators by following way:

a) Let us define scalar-scalar function

$$S_\sigma(x - x') \equiv tr\left[S_2^{(1)}\begin{pmatrix} x & x \\ x' & x' \end{pmatrix}\right] \sim < \bar{\psi}\psi(x)\bar{\psi}\psi(x') > . \tag{2}$$

From the equation (1) for two-particle function we obtain (in momentum space)

$$S_\sigma(p) = \frac{1}{ig}(1 - i\Delta_\sigma(p)). \tag{3}$$

Here we define the function, which we call the $\sigma-$meson propagator

$$\Delta_\sigma(p) = \frac{Z(p)}{4m^2 - p^2}, \tag{4}$$

where $Z_\sigma(p) = \frac{I_0(4m^2)}{I_0(p^2)}$ and $I_0(p) = \int d\tilde{q}\frac{1}{(m^2-(p+q)^2)(m^2-q^2)}$.

b) Pseudoscalar-pseudoscalar function is defined as

$$S_{\pi}^{ab}(x-x') \equiv tr\left[S_2^{(1)}\begin{pmatrix} x & x \\ x' & x' \end{pmatrix}\gamma_5\frac{\tau^a}{2}\gamma_5\frac{\tau^b}{2}\right] \sim < \bar{\psi}\gamma_5\frac{\tau^a}{2}\psi(x)\bar{\psi}\gamma_5\frac{\tau^b}{2}\psi(x') > . \tag{5}$$

From the equation (1) for two-particle function we obtain (in momentum space):

$$S_{\pi}^{ab}(p) = -\frac{1}{ig}(\delta^{ab} - i\Delta_{\pi}^{ab}(p)). \tag{6}$$

Here we define the pion propagator

$$\Delta_{\pi}^{ab}(p) = -\frac{\delta^{ab}Z(p)}{p^2}, \tag{7}$$

where $Z_{\pi}(p) = \frac{I_0(0)}{I_0(p^2)}$.

2 Second-order equations

Second-order generating functional is

$$G^{(2)}[\eta] = \left\{\frac{1}{4!}\mathrm{Tr}\Big(S_4^{(2)} * \eta^4\Big) + \frac{1}{3!}\mathrm{Tr}\Big(S_3^{(2)} * \eta^3\Big) + \frac{1}{2}\mathrm{Tr}\Big(S_2^{(2)} * \eta^2\Big) + \mathrm{Tr}\Big(S^{(2)} * \eta\Big)\right\}G^{(0)}.$$

The equations for four-quark and three-quark functions see on Figs. 4 and 5

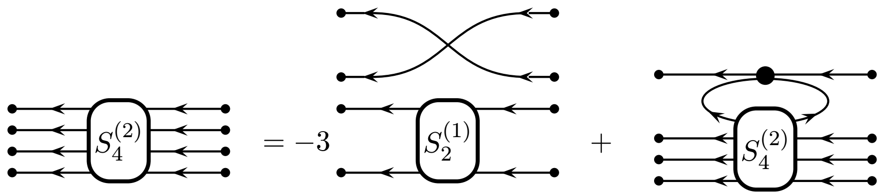

Fig. 4. The equation for four-quark function.

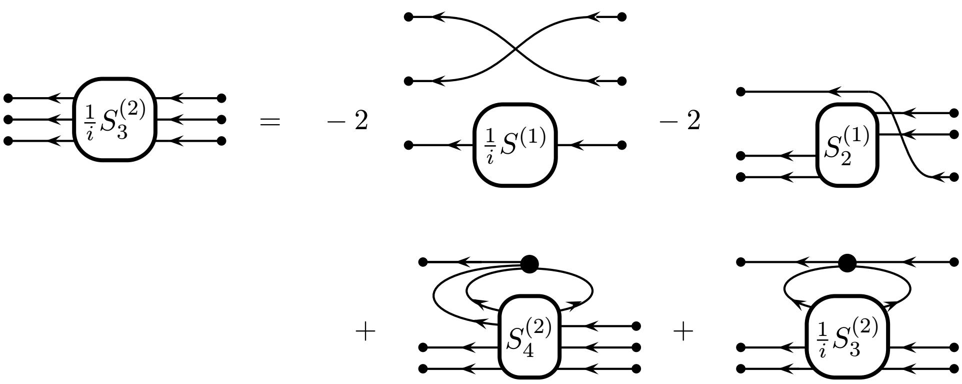

Fig. 5. The equation for three-quark function.

The equations for the four-quark function $S_4^{(2)}$ and for the three-quark functions $S_3^{(2)}$ are new, and the equations for two-particle function $S_2^{(2)}$ and propagator $S^{(2)}$ have the same form as the corresponding first-order equation except of the inhomogeneous terms. For second-order equations these terms contain four-quark function $S_4^{(2)}$ and three-quark $S_3^{(2)}$function.

The equation for the four-quark function has a simple exact solution which is the product of first-order two-quark functions (see Fig. 6).

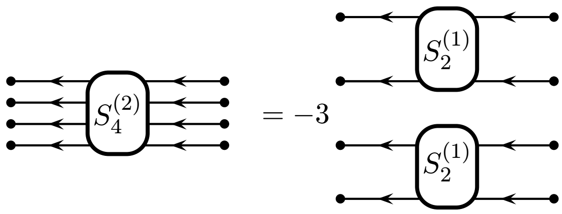

Fig. 6. The solution of equation for four-quark function.

As it seen from this solution, the $\pi\pi-$scattering in NJL model is suppressed, i.e. in the second order of MFE this scattering is absent, and it can arise in the third order only.

3 Vertex $\sigma\pi\pi$

The existence of above exact solution for the four-quark function gives us a possibility to obtain a closed equation for the three-quark function. As a first step in an investigation of this rather complicated equation we shall solve a problem of definition of $\sigma\pi\pi-$vertex with composite sigma-meson and pions. Let us introduce a function

$$W^{ab}_{\sigma\pi\pi}(xx'x'') \equiv tr\left[S_3^{(2)}\begin{pmatrix} x & x \\ x' & x' \\ x'' & x'' \end{pmatrix}\gamma_5\frac{\tau^a}{2}\gamma_5\frac{\tau^b}{2}\right] \sim <\bar{\psi}\psi(x)\bar{\psi}\gamma_5\frac{\tau^a}{2}\psi(x')\bar{\psi}\gamma_5\frac{\tau^b}{2}\psi(x'')>$$

and define:

a) scalar vertex

$$V_\sigma(xx'x'') \equiv tr\left[S_0(x-x')S_2^{(1)}\begin{pmatrix} x' & x \\ x'' & x'' \end{pmatrix}\right] = 2in_c\int dx_1 v_S(xx'x_1)\Delta_\sigma(x_1-x''). \qquad (8)$$

Here $v_S(xx'x'') = \mathrm{tr}_\alpha[S_0(x-x')S_0(x'-x'')S_0(x''-x)]$ is the triangle diagram.

b) pseudoscalar vertex

$$V^{ab}_\pi(xx'x'') \equiv tr\left[S_0(x-x')\gamma_5\frac{\tau^a}{2}S_2^{(1)}\begin{pmatrix} x' & x \\ x'' & x'' \end{pmatrix}\gamma_5\frac{\tau^b}{2}\right] = 2in_c\int dx_1 v_P(xx'x_1)\Delta^{ab}_\pi(x_1-x''). \quad (9)$$

Here $v_P(xx'x'') = \mathrm{tr}_\alpha[S_0(x-x')\gamma_5 S_0(x'-x'')\gamma_5 S_0(x''-x))]$.

With definitions (2)-(9) we obtain for vertex function W^{ab} the following equation:

$$W^{ab}_{\sigma\pi\pi}(xx'x'') = W^{ab}_0(xx'x'') + 2ign_c\int dx_1 l_S(x-x_1)W^{ab}_{\sigma\pi\pi}(x_1x'x''),$$

where $l_S(x) \equiv \mathrm{tr}_\alpha[S_0(x)S_0(-x)]$ is the scalar quark loop and inhomogeneous term W^{ab}_0 is

$$W^{ab}_0(xx'x'') = V^{ab}_\pi(xx'x'') + V^{ab}_\pi(xx''x')+$$

$$+4ig\int dx_1 V_\pi^{a_1a}(xx_1x')S_\pi^{a_1b}(x_1-x'')+4ig\int dx_1 V_\pi^{a_1b}(xx_1x'')S_\pi^{a_1a}(x_1-x')+$$

$$-ig\int dx_1(V_\sigma(xx_1x_1)-4V_\pi^{a_1a_1}(xx_1x_1))S_\pi^{ab}(x'-x'')\ .$$

Using definitions (2)–(9) we have:

$$[W_0^{ab}(xx'x'')]^{con}=-2n_c\int dx_1dx_2 v_P(xx_1x_2)[\Delta_\pi^{a_1a}(x_2-x')\Delta_\pi^{a_1b}(x_1-x'')+\Delta_\pi^{a_1ab}(x_2-x'')\Delta_\pi^{a_1a}(x_1-x')].$$

The equation for W^{ab} can be easy solved in the momentum space and a solution is

$$W_{\sigma\pi\pi}^{ab}(pp'p'')=i\Delta_\sigma(p)W_0^{ab}(pp'p'')$$

where p is $\sigma-$meson momentum, and p', p'' are pion momenta: $p=p'+p''$.

The connected part of W^{ab} is an amplitude of decay $\sigma\to\pi\pi$. It has a following form:

$$[W_{\sigma\pi\pi}^{ab}(pp'p'')]^{con}=\frac{2n_c}{i}\Delta_\sigma(p)[v_P(pp'p'')+v_P(pp''p')]\Delta_\pi^{aa_1}(p')\Delta_\pi^{a_1b}(p''). \qquad (10)$$

(See also Fig 7.)

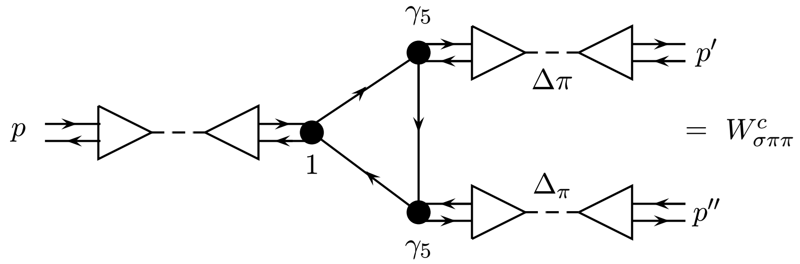

Fig. 7. The connected part of sigma-pion-pion-vertex.

4 Third-order equations

The third-order generating functional is

$$G^{(3)}[\eta]=\left\{\frac{1}{6!}\mathrm{Tr}\left(S_6^{(3)}*\eta^6\right)+\frac{1}{5!}\mathrm{Tr}\left(S_5^{(3)}*\eta^5\right)+\frac{1}{4!}\mathrm{Tr}\left(S_4^{(3)}*\eta^4\right)+\right.$$

$$\left.+\frac{1}{3!}\mathrm{Tr}\left(S_3^{(3)}*\eta^3\right)+\frac{1}{2}\mathrm{Tr}\left(S_2^{(3)}*\eta^2\right)+\mathrm{Tr}\left(S^{(3)}*\eta\right)\right\}G^{(0)}.$$

The equation for six-quark functions and equation for five-quark function see on Figs. 8 and 9.

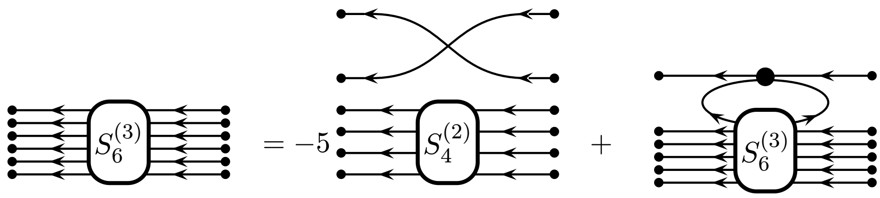

Fig. 8. The equation for six-quark function.

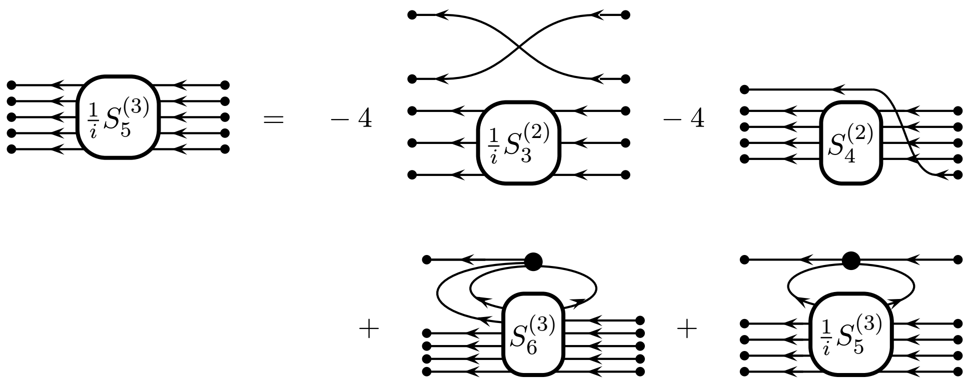

Fig. 9. The equation for five-quark function.

The equations for the six-quark function and for the five-quark function in our iteration scheme are new, and the equations for four-quark function $S_4^{(3)}$, three-quark function $S_3^{(3)}$, two-quark function $S_2^{(3)}$ and quark propagator $S^{(3)}$ have the same form as the second-order equations exept of the inhomogeneous term, which contains the six-quark function and the five-quark function.

Acknowledgement

One of the authors (R.J.) would like to express his sincere gratitude to the Theory Division of IHEP and Organizing Committee for warm hospitality.

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