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Inclusive Reactions, Finite-Energy Sum Rules

and

Reggeon-Particle Scattering

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ABSTRACT

Finite-energy sum rules are applied to inclusive reactions in the context of current data for $p + p \rightarrow p$ + anything and $p\pi \rightarrow p$ + anything. It is found that the finite-energy sum rules are indeed satisfied. Further results are interpreted in terms of Reggeon-particle scattering amplitudes. In particular, the questions of the validity of duality, the presence of fixed poles, and the possibilities of extracting specific triple-Regge couplings are discussed. Representative numerical values for these couplings are also given.

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In the limit $s/M^2 \rightarrow \infty$ and t fixed for the inclusive reaction $a + b \rightarrow c + X$ ($s = (p_a + p_b)^2$, $M^2 = (p_a + p_b - p_c)^2$, ($t = p_a - p_c)^2$, and X stands for hadronic "anything"), the expected behavior of the differential cross section can be pictorially represented as in Fig. 1. It is tempting to interpret Fig. 1 as the optical theorem for Reggeon-particle scattering. More generally one wishes to know whether the concept of Reggeon-particle scattering is a viable one and, if so, how the properties of the Reggeon-particle scattering amplitude compare to those for particle-particle scattering. Of course, in the presence of complete factorization for the Reggeons, the Reggeon-particle amplitude is certainly a sensible topic of discussion. This conclusion is not so clear cut for the, as yet, poorly understood Pomeron exchange which we shall concentrate on here. However, since the simplest interpretation of the present work is in terms of Pomeron (Reggeon)-particle scattering amplitudes, we shall proceed using this language. In what follows we shall focus our attention on the properties of Pomeron(Reggeon)-particle amplitudes as they can be studied in inclusive experiments both present and future. In particular we shall attempt to show that current data can be summarized by the statement that the behavior of Reggeon-particle amplitudes is identical to particle-particle amplitudes in many important respects.

In the limit $s/M^2 \rightarrow \infty$, $M^2 \rightarrow \infty$ one would expect, by anology with particle-particle amplitudes, that the Reggeon-particle amplitude is described by single Regge pole exchange. This leads to the usual triple-Regge (TR) behavior for the inclusive cross section, i.e.,[1] ($m_o^2 = 1$ GeV^2)

$$\frac{d^2\sigma}{dt\,dM^2} \underset{\substack{s/M^2\to\infty\\ M^2\to\infty}}{\cong} \frac{m_0^2}{16\pi s^2}\sum_{ijk} G_{ijk}(t)\left(s/M^2\right)^{\alpha_i(t)+\alpha_j(t)}\left(M^2/m_0^2\right)^{\alpha_k(0)} \tag{1}$$

The coupling G is the product $G = \beta_i(t)\,\beta_j(t)\,\tilde{\beta}_k(o)\,g_{ijk}(t)$ where the β's are the usual Reggeon-particle-particle couplings and $g_{ijk}(t)$ is the three Reggeon coupling. Recent studies[2] indicate that such a description is already sufficient at present energies with very few (2 or 3) terms on the right hand side. Further, if the Reggeon-particle amplitude has the single cut plane analyticity in M^2 implied by unitarity, and if it satisfies crossing, one can write a finite-energy sum rule for the inclusive reaction (FESRIR).[3] If we specialize to the case a = c (i.e., Pomeron exchange where there is the most data) and keep only the terms discussed in Ref. 2,[4] the first moment (right signature) sum rule has the form

$$\int^{\bar{M}_0^2} d\bar{M}^2\,\bar{M}^2\,\frac{d\sigma}{dt\,dM^2}(a+b\to a+X) \underset{\substack{s/\bar{M}_0^2\to\infty\\ \bar{M}_0^2\ \text{Large}}}{\cong} \frac{m_0^2}{16\pi s^2}\left\{\frac{G_{PPP}(t)\left(s/\bar{M}_0^2\right)^{2\alpha_P(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_P(0)+2}}{2+\alpha_P(0)-2\alpha_P(t)}\right.$$

$$\left. + \frac{G_{PPf}(t)\left(s/\bar{M}_0^2\right)^{2\alpha_P(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_f(0)+2}}{2+\alpha_f(0)-2\alpha_P(t)} + G_{ffP}(t)\,\frac{\left(s/\bar{M}_0^2\right)^{2\alpha_f(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_P(0)+2}}{2+\alpha_P(0)-2\alpha_f(t)}\right\} \tag{2}$$

We have introduced the crossing symmetric variable $\overline{M}^2 = M^2 - m_b^2 - t$.

First consider the case where $\overline{M}_0^2$ is large enough that the inclusive cross section exhibits TR behavior. Then by checking the phenomenological validity of the FESRIR, Eq. (2), we are testing the assumed analyticity of the Regge-particle scattering amplitude. Next we consider how well Eq. (2) is satisfied when $\overline{M}_0^2$ is below the usual TR region in order to investigate the role of duality in Reggeon-particle scattering. This is to be done in the same way that one proceeds to study duality in the particle-particle scattering amplitude[5] where one finds a simple relationship between leading Regge behavior and "average" low energy behavior. Thus a study of Eq. (2) will allow us to test: (1) analyticity of the Reggeon-particle amplitude, (2) duality in the Reggeon-particle amplitude.

In Fig. 2 we plot data[6] for $p + p \to p + X$ at 29 GeV/c for the integrand on the LHS of Eq. (2) (i.e., $\overline{M}^2\, d\sigma/dtdM^2$). Note the important feature that the elastic peak is multiplied by $\overline{M}_{e\ell}^2 = -t$ which yields a contribution of size comparable to the larger M^2 contributions. Also plotted (smooth curve) is the triple-Regge curve for $\overline{M}^2\, d\sigma/dtdM^2$ from Eq. (1) where we used $\alpha_P(t) = 1.0 + 0.5t$ and $\alpha_f = 0.5 + t$. We have considered the two extreme cases: $G_{PPP} \equiv 0$ and $G_{PPf} \equiv 0$. The actual values of the coupling constants used to generate the smooth curves are indicated in the figure and were chosen by studying $d\sigma/dtdM^2$ for $M^2 \gtrsim 6$ GeV^2. The agreement of the data and the smooth curve at large M^2 again verifies the validity of TR behavior (Eq. 1). Equality of the area under the data and the area under the smooth curve implies that Eq. (2) is satisfied. Specific values are given for the case $\overline{M}_0^2 = 8\ \mathrm{GeV}^2$ in Fig. 2. Simple inspection should readily convince the reader that Eq. (2) is quite well satisfied, "on the average", for any $\overline{M}_0^2$ above $2\ \mathrm{GeV}^2$. Unfortunately it is also apparent that the present data are not sufficient to clearly distinguish the relative contributions of PPP and PPf. The two extreme cases are dramatically different only for $\overline{M}^2 \approx -t$ (i.e., the elastic peak). Only a very literal application of "local duality" will allow a separation. However by extrapolating the curves of Fig. 2a to NAL energies, as in Fig. 3., we see a striking and easily distinguishable difference in shape.[7] One can expect the actual data to lie somewhere between these two curves and thus specify the ratio G_{PPf}/G_{PPP}. Note that the difference is most marked in the intermediate region ($10\ \mathrm{GeV}^2 \leq M^2 \leq 30\ \mathrm{GeV}^2$) rather than at "large" M^2.

We have performed a similar analysis on recent $\pi^- p \to X^- p$ data.[8] We have allowed for an uncertainty in the relative normalization of the pp and πp data,[9] but we have used the t dependence implied by Fig. 2a and 2b to do the t averaging required in the πp data. The results are shown in Fig. 4[10] where the couplings shown are for $t = -.17\ \mathrm{GeV}^2$.

The most reasonable conclusion which one is able to draw from the above analysis is that current data are certainly consistent with the validity of the FESRIR. Further, the data are consistent with the FESRIR being satisfied, on the average, down to such low M^2 as to imply the usual sort of duality. The next logical step is to check the extension of the Freund-Harari conjecture[11] to Reggeon-particle scattering. This requires a good determination of the ratio G_{PPf}/G_{PPP} and of the ratio of diffractively produced resonances to diffractively produced background. The strongest statement which follows from Figs. 2 and 4 is that probably $G_{PPf}/G_{PPP} \gtrsim 1$ which is in general agreement with the usual statement concerning dominance[6] of diffractively produced resonances over background.[12] Representative values for the case $G_{PPP} = G_{PPf}$ are $G_{PPP} = G_{PPf} =$ 88 mb/GeV^2 and $G_{ffP} = 1.2 \times 10^3$ mb/GeV^2 which give a curve intermediate between the two shown in Fig. 2 for the pp data.

It is also interesting to consider the zeroth moment sum rule[13]

$$\int^{\bar{M}_0^2} dM^2 \frac{d\sigma^2}{dt\,dM^2}(a+b \to a+X) \underset{\substack{s/M^2 \to \infty \\ \bar{M}^0 \text{ Large}}}{\cong} \frac{m_0^2}{16\pi s^2} \left\{ \frac{G_{PPP}(t)\left(s/\bar{M}_0^2\right)^{2\alpha_P(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_P(0)+1}}{1+\alpha_P(0)-2\alpha_P(t)} \right.$$

$$\left. + \frac{G_{PPf}(t)\left(s/\bar{M}_0^2\right)^{2\alpha_P(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_f(0)+1}}{1+\alpha_f(0)-2\alpha_P(t)} + \frac{G_{ffP}(t)\left(s/\bar{M}_0^2\right)^{2\alpha_f(t)}\left(\bar{M}_0^2/m_0^2\right)^{\alpha_P(0)+1}}{1+\alpha_P(0)-2\alpha_f(t)} \right\} + R(s,t) \tag{3}$$

The new feature is the presence of possible fixed pole contributions. The general residue, $R(s,t)$, is of the form

$$R(s,t) = \sum_{i,j} \left(s/m_0^2\right)^{\alpha_i(t)+\alpha_j(t)-2} R_{ij}(t) \tag{4}$$

The important point is that the fixed pole residue R_{ij} is believed to set the scale for the 2 Reggeon cut contribution to particle-particle scattering arising from the exchange of α_i and α_j.[14] Hence by studying inclusive reactions, in particular Eq.(3), one may hope to understand the role cuts play in 2 particle scattering. Then, for self consistency, one must verify that cut contributions in the inclusive process are indeed small, since we have explicitly ignored them in arriving at Eq.(3).

If we apply Eq. (3) to the data already discussed we find evidence for a finite contribution from R. Typical numbers are illustrated in the Table where the couplings are those of Fig. 2 and 4. The theoretical and systematic uncertainties of these numbers are, of course, large but the possibility that $R \equiv 0$ seems highly unlikely. The most striking phenomenological evidence for the presence of a fixed pole is the zero in the denominator of the PPf term in the neighborhood of $t = -.5\ \mathrm{GeV}^2$.[15] If the coupling $G_{PPf'}$ is finite at this point, one is led to conclude that there is a multiplicative fixed pole present whose contribution, by definition, is just such as to cancel the PPf contribution at the singular point. Since the above analysis is consistent with a nonzero, smoothly behaved $G_{PPf'}$, it seems very likely that such a multiplicative fixed role is indeed present. Note also that if we accept a simple Freund-Harari picture then we have

$$\int^{\bar{M}_0^2} d\bar{M}^2 \frac{d^2\sigma}{dt\,dM^2}\Big(\underset{b^* \equiv \text{Diff. Res.}}{a+b \to a+b^*}\Big) \underset{\substack{s/M^2 \to \infty \\ \bar{M}^{*2}\ \text{Large}}}{\cong} \frac{m_0^2}{16\pi s^2} \frac{G_{PPf}\left(s/\bar{M}_0^2\right)^{2\alpha_P(t)}}{\alpha_f(0)+1-2\alpha_P(t)} \left(\frac{\bar{M}_0^2}{m_0^2}\right)^{\alpha_f(0)+1} + R \qquad (5)$$

Since $\alpha_f(0) + 1 - 2\alpha_p(t) < 0$ for $t > -0.5$ and the LHS is positive definite, a finite positive value for R is clearly required.

The above analysis can be summarized by stating that present inclusive reaction data indicates that Pomeron (Reggeon) - particle scattering amplitudes behave just like ordinary particle-particle amplitudes in many important respects. In particular, they seem to exhibit ordinary Regge behavior and cut plane analyticity, to satisfy the FESRIR, and they are consistent with the usual ideas about duality. We have further noted (Fig. 3) that the data soon to be obtained at the new accelerators should serve to clarify the situation to the point of specifying individual couplings and even fixed pole residues. This should greatly improve our understanding not only of inclusive reactions but also of the general properties of Regge poles and two Reggeon cuts.

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[1] C. E. DeTar, et al., Phys. Rev. Letters 26, 675 (1971); D. Silvermann and C-I Tan, Nuovo Cimento, 2A, 489 (1971); and Phys. Rev. D3, 991 (1971); H. Abarbanel, et al., Phys. Rev. Letters 26, 937 (1971); N. F. Bali, et al., Phys. Rev. D3, 1167 (1971); G. Chew and A. Pignotti, NAL Summer Study (1968).

[2] S. D. Ellis and A. I. Sanda, NAL-THY-30, 1971 (to be published in Phys. Rev.). For an example with charge exchange see P. Chliapnikov, et al, Phys. Letters 35B, 581 (1971).

[3] A. I. Sanda, NAL-THY-25 (1971), (to be published in Phys. Rev.); M. B. Einhorn, J. Ellis, and J. Finkelstein, SLAC-PUB-1006 (1972). In particular to write Eq. (2), we have assumed that Pomeron-particle amplitudes do not have right signature fixed poles as ruled out by the usual analyticity and unitarity constraints. Note that there is a subtle point concerning where the data in the LHS of Eq. (2) should be evaluated. In principle half the data (direct channel) should be at energy s whereas the other half (cross channel) should be at $s + \mathcal{O}(M^2/s)$. In the present analysis this can be interpreted as a 10% ambiguity in the ffP contribution.

[4] In Ref. (2) the PPP term was not included in most of the discussion. This was primarily for reasons of simplicity as it was noted that the data itself allowed little distinction between PPP and PPf in the large M^2 region.

[5]See e.g., R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[6]E. W. Anderson, et al., Phys. Rev. Letters 19, 198 (1967), Phys. Rev. Letters 16 855 (1966).

[7]The authors have also considered the available data at 19 GeV/c, Allaby, et al., CERN preprint 70-16 (1970) but the smallness of the change in energy coupled with the uncertainty in relative normalization makes it difficult to reach any conclusions.

[8]A. Antipov, et al., preprint 1972, (submitted to Phys. Letters).

[9]In particular, the elastic differential cross section appearing in the data of Ref. 6 has the value ≈ 10 mb/GeV^2 at $t = -.17$. This is to be compared with a world average closer to 15 mb/GeV^2. If we change the normalization of the pp data by a factor of 1.5 we find reasonable agreement with the πp data via factorization using $\frac{\beta_{fpp}}{\beta_{f_{\pi\pi}}} \approx \frac{\beta_{Ppp}}{\beta_{P_{\pi\pi}}} \approx 3/2$ (see Ref. 2). This also brings Ref. 6 into better agreement with the data mentioned in Ref. 7.

[10]Note that the various ratios, e.g., G_{PPf}/G_{fPP}, have changed somewhat from their values in the pp data. However, the uncertainties are such that it is premature to draw any conclusions about what this says about factorization.

[11]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[12] Although we consider the current data to be consistent with a naive extension of Freund-Harari, we cannot rule out other possibilities. See e.g., M. B. Einhorn, H. B. Green, and M. Virasoro (to be published).

[13] To write such a sum rule one is asserting that the amplitude splits into two terms, one with the L. H. cut and one with the R. H. cut. Each term is taken to have appropriate Regge asymptotic behavior, plus possible non Regge contributions which cancel in the signatured amplitude. For previous applications see J. H. Schwartz, Phys. Rev. 159, 1269 (1967) and Ref. 5.

[14] H. D. I. Abarbanel, NAL-THY-28, (to be published in Phys. Rev.); C. Lovelace, Phys. Letters 36B, 127 (1971).

[15] Note that the coefficient of the PPP term is singular at $t=0$. But $G_{PPP}(t)$ is known to vanish here for other reasons. See for example, H. D. I. Abarbanel, et al., Phys. Rev. Letters 26, 937 (1971).

TABLE

		$R = \int \frac{d\sigma}{dt dM^2} dM^2$			- PPP	- PPf	- ffP
p - data	(a)	16.3	=	14.7	0	+ 3.2	- 1.6
$t = -.17\ GeV^2$	(b)	2.8	=	14.7	- 10.9	0	- 1.0
$t = -.46$	(c)	13.4	=	2.3	0	+11.4	- .3
	(d)	.7	=	2.3	- 1.5	0	- .1
π-data	(e)	8.8	=	6.6	0	+ 2.9	- .7
$P_L = 25$ GeV/c	(f)	.8	=	6.6	- 5.3	0	- .5
$P_L = 40$ GeV/c	(e)	8.3	=	6.1	0	+ 2.5	- .3
	(f)	1.2	=	6.1	- 4.7	0	- .2

Numerical evaluation of the fixed pole contribution R(s,t) from Eq. (3). From figures 2 and 4, we consider the coupling (a) $G_{PPP}=0$, $G_{PPf}=2\times10^2$, $G_{ffP}=1.5\times10^3$ mb/GeV^2; (b) $G_{PPP}=1.3\times10^2$, $G_{PPf}=0$, $G_{ffP}=10^3$ mb/GeV^2; (c) $G_{PPP}=0$, $G_{PPf}=1.6\times10^2$, $G_{ffP}=10^3$ mb/GeV^2; (d) $G_{PPP}=8.5\times10^1$, $G_{PPf}=0$, $G_{ffP}=6.0\times10^2$ mb/GeV^2; (e) $G_{PPP}=0$, $G_{PPf}=1.6\times10^2$, $G_{ffP}=1.8\times10^3$ mb/GeV^2; (f) $G_{PPP}=1.1\times10^2$, $G_{PPf}=0$, $G_{ffP}=1.3\times10^3$ mb/GeV^2 p-data corresponds to $p+p\to p+$Anything (Ref. 6) and π-data corresponds to π-$p\to p+$Anything (Ref. 8). All units are mb/GeV^2.

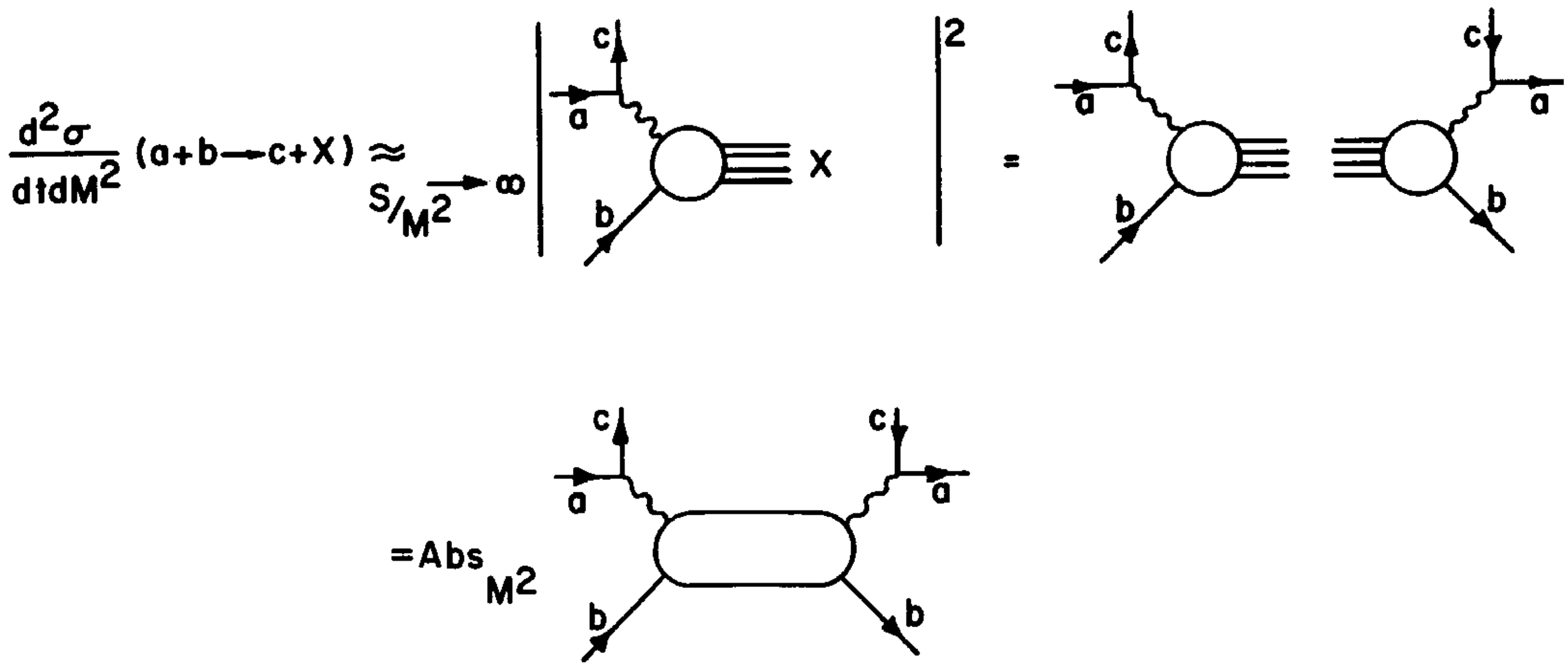
FIGURE CAPTIONS

Figure 1 : Relation between inclusive cross section and Reggeon-particle scattering amplitude.

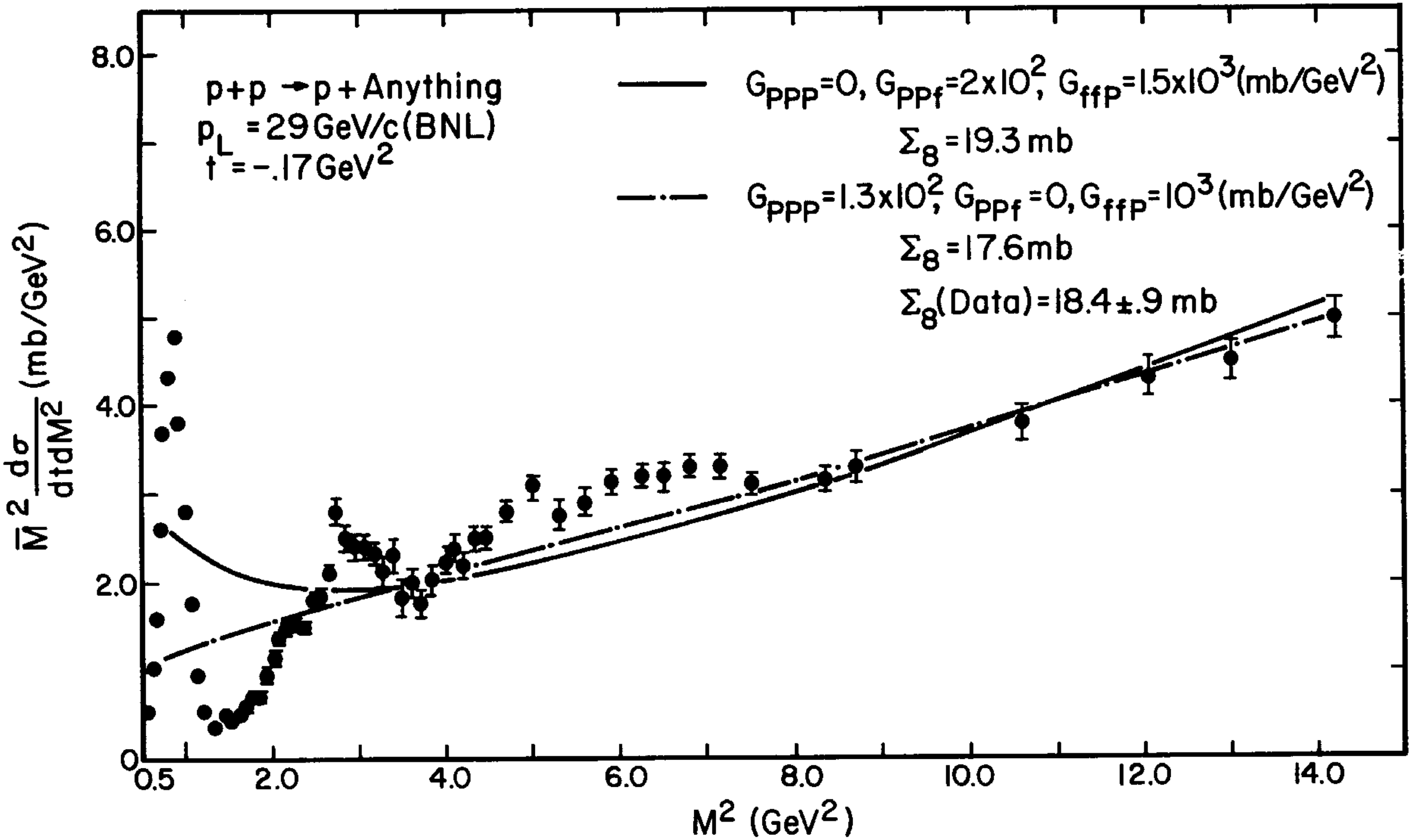
Figure 2: $p+p \rightarrow p + X$ data from Ref. (5) (only a fraction of the data is shown) presented in the form $\overline{M}^2 \times \frac{d\sigma}{dtdM^2}$. The solid line is the two term triple-Regge description with PPf+ ffP only. The broken line is with PPP + ffP only. Numbers are given for the area under the various curves up to $M^2 = 8$ GeV^2. Part (a) is for $t = -.17$ and (b) for $t = -.46$.

Figure 3: Predicted shape of cross section for $p+p \rightarrow p + X$ at NAL energies, normalized to data from Ref. (5). The solid line is for PPf+ ffP only and the broken line is for PPP+ffP only.

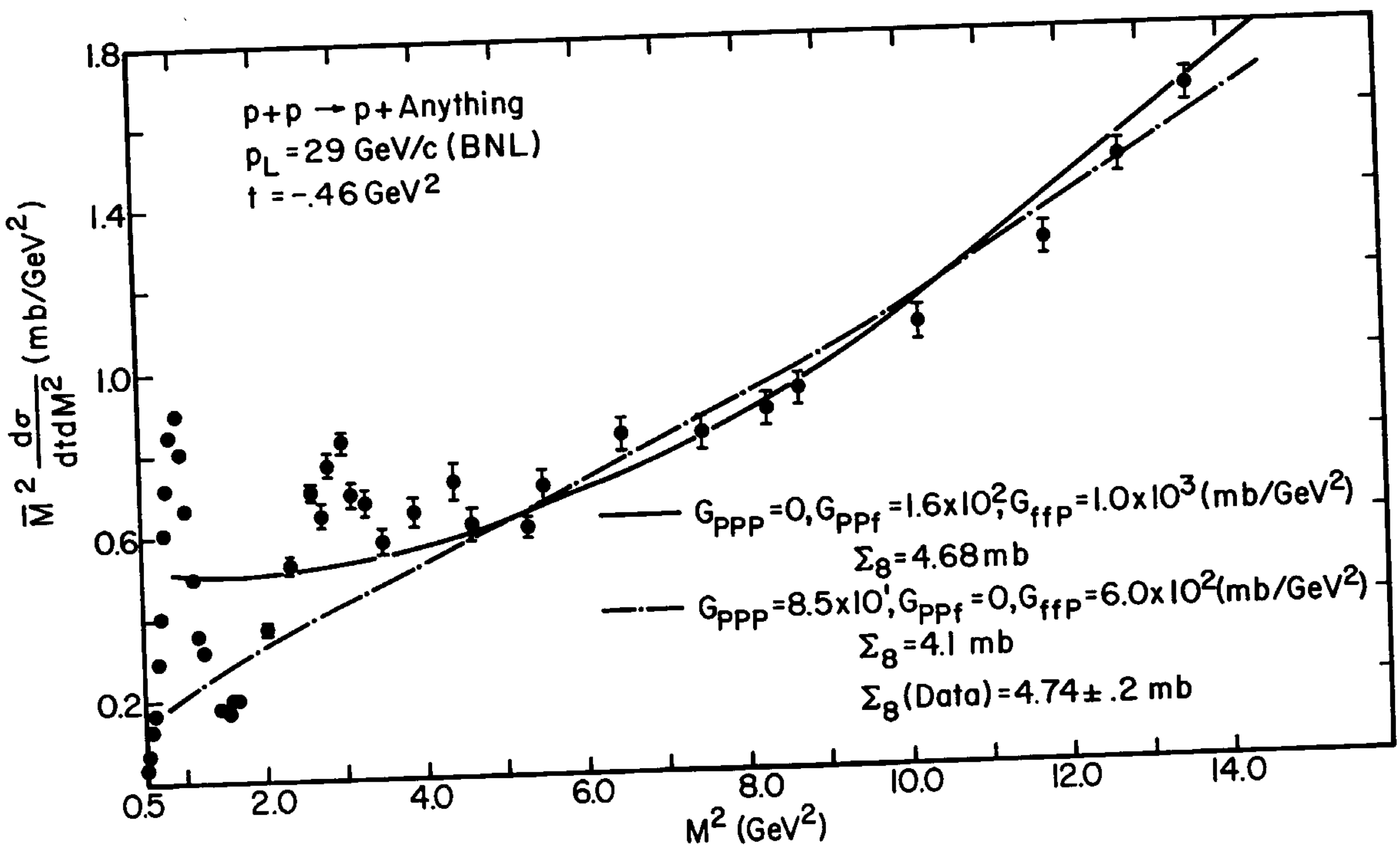
Figure 4: $p + \pi^- \rightarrow p + X^-$ data from Ref. (7) presented in the form $\overline{M}^2 \times d\sigma/dtdM^2$. Only a fraction of the total data is shown and the error bars represent the statistical uncertainty of the individual data points. The solid line is for PPf+ff P only and the broken line is for PPP+ffP only. The areas under the various curves are also given for an upper limit of 8 GeV^2. Part (a) is for $P_L = 25$ GeV/c and (b) for $P_L = 40$ GeV/c.



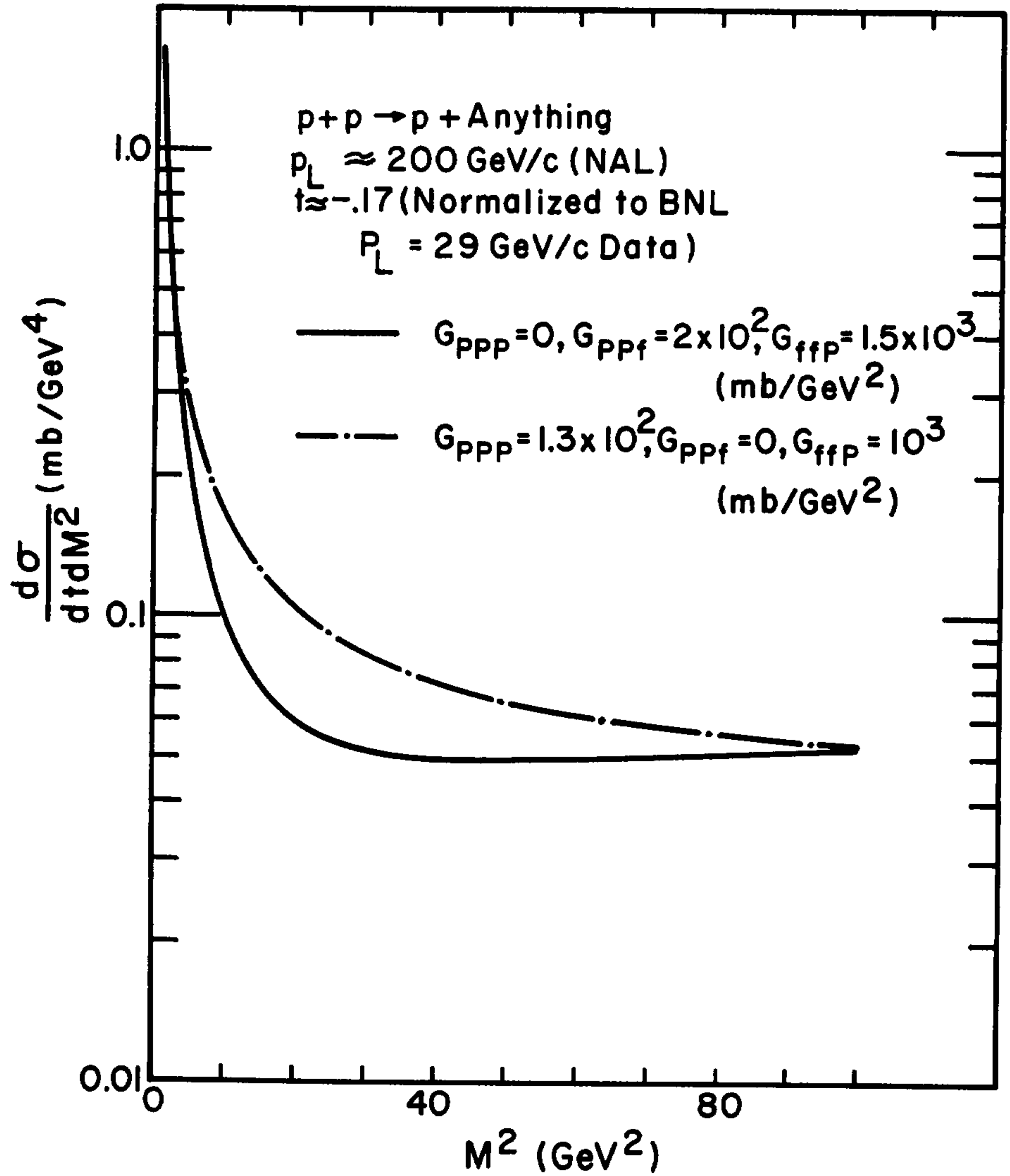
$\frac{d^2\sigma}{dt\,dM^2}(a+b \rightarrow c+X) \approx$
$S/M^2 \rightarrow \infty$
c
a
b
X
2
=
c
a
b
c
a
b
=Abs
M^2
c
a
b
c
a
b

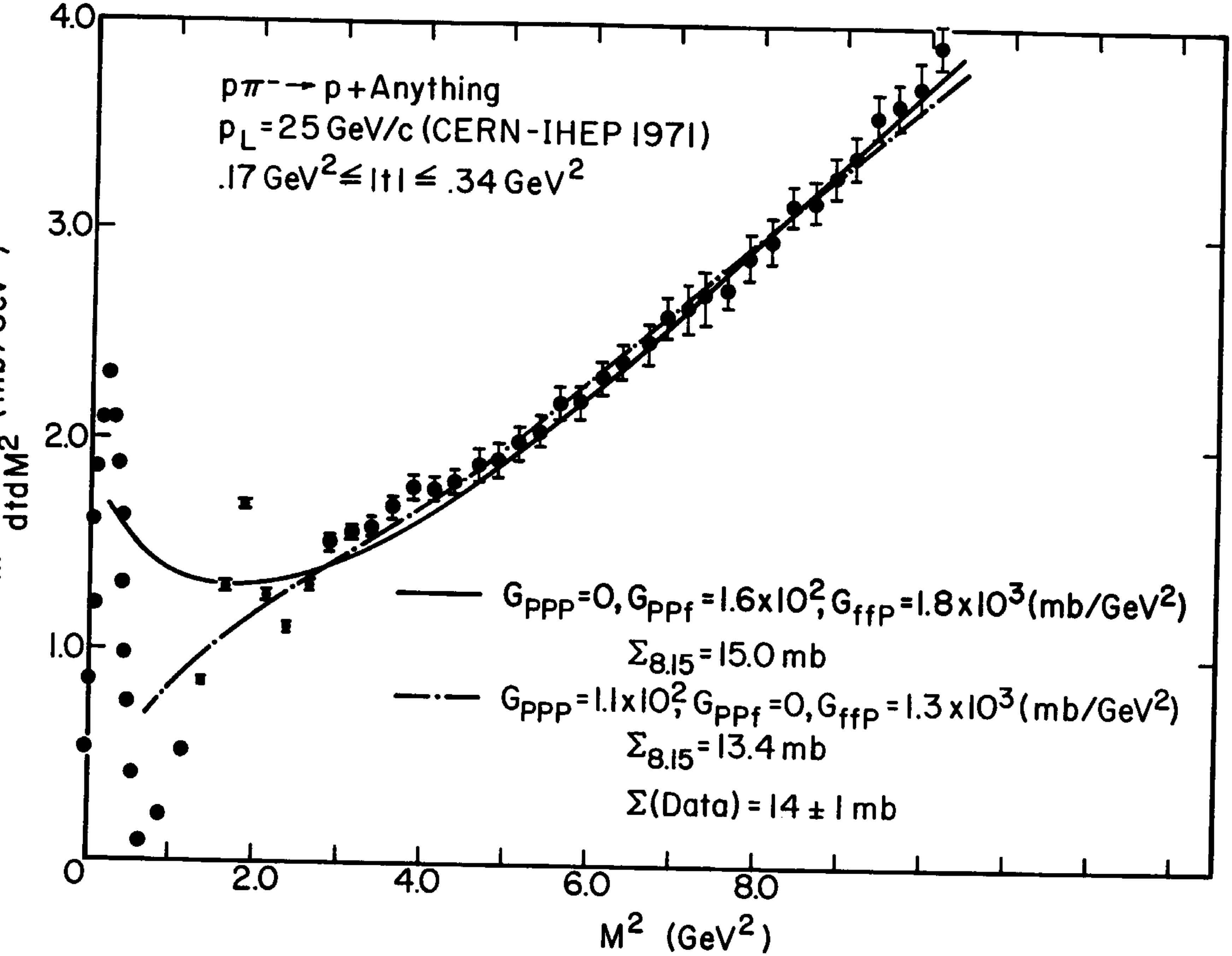


p+p → p+Anything
p_L = 29 GeV/c (BNL)
t = −.17 GeV²
$G_{PPP}=0,\ G_{PPf}=2\times10^2,\ G_{ffP}=1.5\times10^3\ (mb/GeV^2)$
$\Sigma_8 = 19.3$ mb
$G_{PPP}=1.3\times10^2,\ G_{PPf}=0,\ G_{ffP}=10^3\ (mb/GeV^2)$
$\Sigma_8 = 17.6$ mb
Σ_8(Data) = 18.4 ± .9 mb
$\bar{M}^2 \frac{d\sigma}{dt\,dM^2}$ (mb/GeV²)
M² (GeV²)
0
2.0
4.0
6.0
8.0
0.5
10.0
12.0
14.0

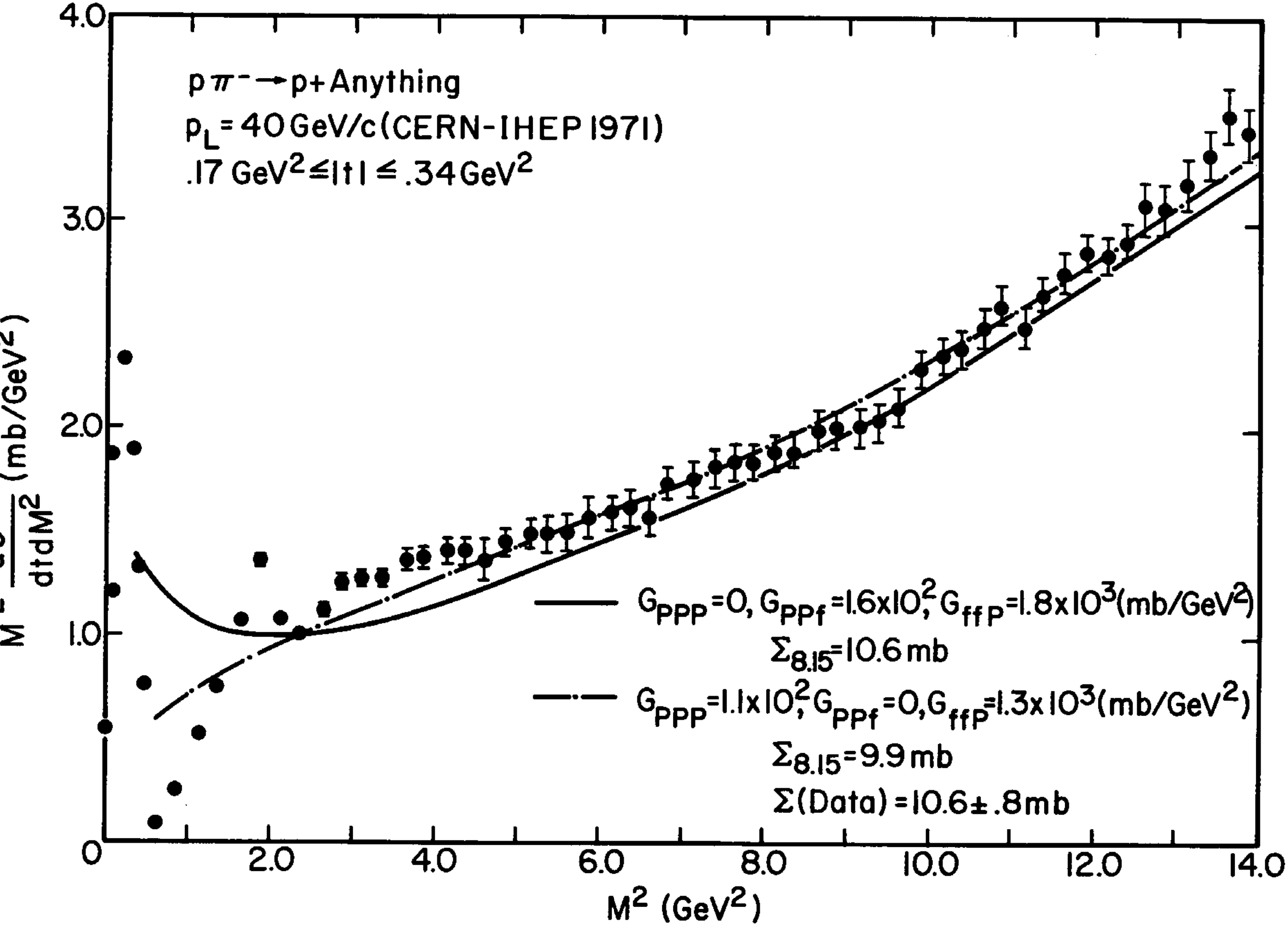


p+p → p+Anything
pL = 29 GeV/c (BNL)
t = -.46 GeV2
GPPP = 0, GPPf = 1.6x102, GffP = 1.0x103 (mb/GeV2)
Σ8 = 4.68 mb
GPPP = 8.5x101, GPPf = 0, GffP = 6.0x102 (mb/GeV2)
Σ8 = 4.1 mb
Σ8 (Data) = 4.74 ± .2 mb
M2 dσ/dtdM2 (mb/GeV2)
M2 (GeV2)
1.8
1.4
1.0
0.6
0.2
0.5
2.0
4.0
6.0
8.0
10.0
12.0
14.0


p+p → p + Anything
p_L ≈ 200 GeV/c (NAL)
t≈−.17 (Normalized to BNL
P_L = 29 GeV/c Data)
G_{PPP}=0, G_{PPf}=2x10^2, G_{ffP}=1.5x10^3
(mb/GeV^2)
G_{PPP}=1.3x10^2, G_{PPf}=0, G_{ffP}=10^3
(mb/GeV^2)
$\frac{d\sigma}{dtdM^2}$ (mb/GeV^4)
1.0
0.1
0.01
0
40
80
M^2 (GeV^2)



$p\pi^- \rightarrow p + \text{Anything}$
$p_L = 25$ GeV/c (CERN-IHEP 1971)
$.17\ \text{GeV}^2 \leq |t| \leq .34\ \text{GeV}^2$
$G_{PPP} = 0, G_{PPf} = 1.6 \times 10^2, G_{ffP} = 1.8 \times 10^3$ (mb/GeV^2)
$\Sigma_{8.15} = 15.0$ mb
$G_{PPP} = 1.1 \times 10^2, G_{PPf} = 0, G_{ffP} = 1.3 \times 10^3$ (mb/GeV^2)
$\Sigma_{8.15} = 13.4$ mb
$\Sigma(\text{Data}) = 14 \pm 1$ mb
$\bar{M}^2 \frac{d\sigma}{dt dM^2}$ (mb/GeV^2)
M^2 (GeV^2)
0
1.0
2.0
3.0
4.0
2.0
4.0
6.0
8.0



$p\pi^- \to p$+Anything
p_L = 40 GeV/c (CERN-IHEP 1971)
.17 $GeV^2 \leq |t| \leq$ .34 GeV^2
$G_{PPP}=0, G_{PPf}=1.6\times10^2, G_{ffP}=1.8\times10^3$ (mb/GeV^2)
$\Sigma_{8.15}$=10.6 mb
$G_{PPP}=1.1\times10^2, G_{PPf}=0, G_{ffP}=1.3\times10^3$ (mb/GeV^2)
$\Sigma_{8.15}$=9.9 mb
Σ(Data) = 10.6 ± .8 mb
$\bar{M}^2 \frac{d\sigma}{dt\,dM^2}$ (mb/GeV^2)
M^2 (GeV^2)
0
1.0
2.0
3.0
4.0
4.0
6.0
8.0
10.0
12.0
14.0