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QCD AMPLITUDES WITH THE GLUON EXCHANGE AT HIGH ENERGIES (AND GLUON REGGEIZATION PROOF)

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Abstract

We demonstrate that the multi-Regge form of QCD amplitudes with gluon exchanges is proved in the next-to-leading approximation. The proof is based on the bootstrap relations, which are required for the compatibility of this form with the s-channel unitarity. It was shown that the fulfillment of all these relations ensures the Reggeized form of energy dependent radiative corrections order by order in perturbation theory. Then we prove that all these relations are fulfilled if several bootstrap conditions on the Reggeon vertices and trajectory hold true. All these conditions are checked and proved to be satisfied for all possible t-channel color representations. That finally completes the proof of the gluon Reggeization in the next-to-leading approximation and provides the firm basis for BFKL approach therein.

1 Introduction

Reggeization of gluons as well as quarks is one of remarkable properties of Quantum Chromodynamics (QCD). The gluon Reggeization is especially important since cross sections non vanishing in the high energy limit are related to gluon exchanges in cross channels. A primary Reggeon in QCD turns out to be the Reggeized gluon.

The gluon Reggeization gives the most common basis for the description of high energy processes. In particular, the famous BFKL equation [1)] was derived supposing the Reggeization. The most general approach to the unitarization problem is the reformulation of QCD in terms of a gauge-invariant effective field theory for the Reggeized gluon interactions.

The gluon Reggeization was proved in the leading logarithmic approximation (LLA), i.e. in the case of summation of the terms $(\alpha_S \ln s)^n$ in cross-sections of processes at energy $\sqrt{s}$ in the c.m.s., but till now remains a hypothesis in the next-to-leading approximation (NLA), when the terms $\alpha_S(\alpha_S \ln s)^n$ are also kept. Now the BFKL approach [1)], based on the gluon Reggeization, is intensively developed in the NLA.

We present the proof of the gluon Reggeization in the NLA. Substantially in our consideration we follow the paper [2)]. All references are presented therein. First we show that the fulfillment of the bootstrap relations guarantees the multi–Regge form of QCD amplitudes. Then we demonstrate that an infinite set of these bootstrap relations are fulfilled if several conditions imposed on the Reggeon vertices and the trajectory (bootstrap conditions) hold true. Now almost all these conditions are proved to be satisfied. In our consideration we hold the following terminology:

Multi-Regge kinematics (MRK). Let us consider the amplitude (see fig.1) $\mathcal{A}_{2\to n+2}$ of the process $A + B \to A' + J_1 + \ldots + J_n + B'$: see the figure. We use light-cone momenta n_1 and n_2, with $n_1^2 = n_2^2 = 0$, $(n_1 n_2) = 1$, and denote $(pn_2) \equiv p^+$, $(pn_1) \equiv p^-$. Let assume that initial momenta p_A and p_B have predominant components p_A^+ and p_B^-. MRK supposes that rapidities of final jets J_i with momenta k_i $y_i = \frac{1}{2}\ln\left(k_i^+/k_i^-\right)$ decrease with i: $y_0 > y_1 > \ldots > y_n > y_{n+1}$; as for y_0 and y_{n+1}, it is convenient to define them as $y_0 = y_A \equiv \ln\left(\sqrt{2}p_A^+/|q_{1\perp}|\right)$ and $y_{n+1} = y_B \equiv \ln\left(|q_{(n+1)\perp}|/\sqrt{2}p_B^-\right)$. Notice that q_i indicate the Reggeon momenta and $q_1 = p_{A'} - p_A \equiv q_A$, $q_{n+1} = p_B - p_{B'} \equiv q_B$.

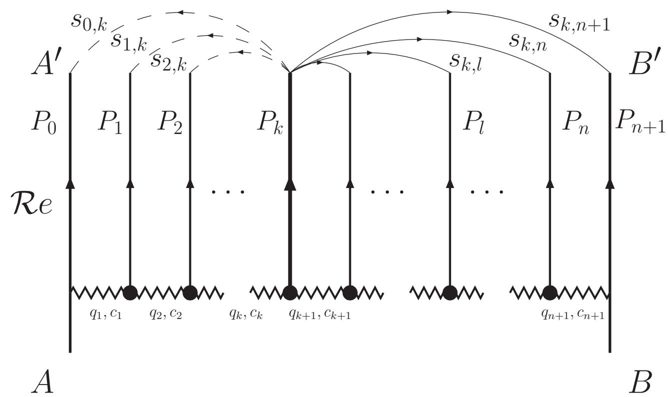

Figure 1: *The amplitude for the process* $2 \to n+2$.

Signature in the channel t_l for multi-particle production means the symmetrization (or antisymmetrization) with respect to the substitution $s_{i,j} \leftrightarrow -s_{i,j}$, for $i < l \le j$. Here $s_{i,j} = (k_i + k_j)^2$.

Hypothesis of the gluon reggeization claims that in MRK the real part of the NLA-amplitude $2 \to n+2$ with negative signature has universal form, where all energy dependence is exponentiated:

$$\Re\,\mathcal{A}_{AB}^{A'B'+n} = \bar{\Gamma}_{A'A}^{R_1} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i^2)(y_{i-1}-y_i)}}{q_{i\perp}^2} \gamma_{R_i R_{i+1}}^{J_i} \right) \frac{e^{\omega(q_{n+1}^2)(y_n - y_{n+1})}}{q_{(n+1)\perp}^2} \Gamma_{B'B}^{R_{n+1}}, \quad (1)$$

where $y_i = \frac{1}{2}\ln(\frac{k_i^+}{k_i^-})$ — particle (P_i) rapidities, and $\gamma_{R_i R_{i+1}}^{J_i}$, $\Gamma_{P'P}^{R}$ — known effective vertices, and $\omega(q_i^2)$ — (perturbatively) known gluon trajectory.

2 The concept of the gluon Reggeization proof

Using the elementary properties of the signaturized NLO amplitude we can obtain the following bootstrap relations:

$$\Re\Big(\sum_{l=k+1}^{n+1} \mathrm{disc}_{s_{k,l}} - \sum_{l=0}^{k-1} \mathrm{disc}_{s_{l,k}} \Big) \frac{\mathcal{A}_{AB}^{A'B'+n}}{-2\pi i} = \frac{1}{2}(\omega(t_{k+1}) - \omega(t_k))\Re\,\mathcal{A}_{AB}^{A'B'+n}. \quad (2)$$

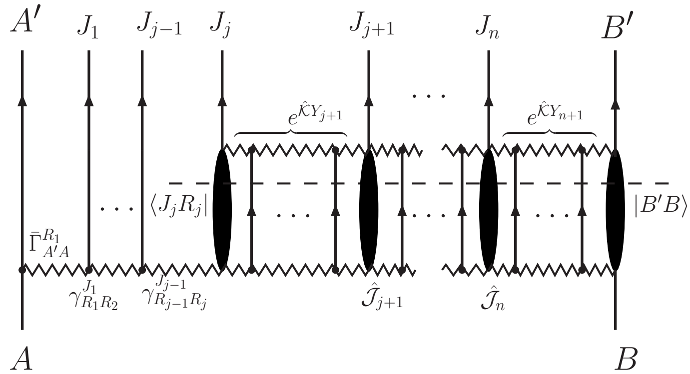

Figure 2: $s_{j,n+1}$*-channel discontinuity calculation via unitarity relation.*

These relations constitute an infinite number of necessary and sufficient conditions for compatibility of the Regge amplitude form (eq.1) with unitarity: the discontinuities (see fig.2) in these relations must be calculated using the s–channel unitarity and the multi-Regge form of the amplitudes (eq.1). Evidently, there is an infinite number of the bootstrap relations, because there is an infinite number of amplitudes $\mathcal{A}_{2\to n+2}$.

3 Bootstrap conditions

An infinite number of the bootstrap relations (eq.2) are satisfied if the finite number of following bootstrap conditions are fulfilled. In [2)] we show that the fulfilment of the bootstrap conditions guarantees the implementation of all the infinite set of the bootstrap relations (eq.2). These conditions can be divided into two sorts: elastic and inelastic ones.

Elastic bootstrap conditions describe the properties of the transition of the initial particle B to B' with the two Reggeon emission in the t channel: see the rightmost blob in fig2. In Reggeon operator formalism [2)] the impact factors for scattering particles satisfy equations

$$|\bar{B}'B\rangle = g\Gamma^{R_{n+1}}_{B'B}|R_\omega(q_{B\perp})\rangle, \quad \langle A'\bar{A}| = g\bar{\Gamma}^{R_1}_{A'A}\langle R_\omega(q_{A\perp})|, \tag{3}$$

where $\langle R_\omega(q_\perp)|$ and $|R_\omega(q_\perp)\rangle$ are the *bra*– and *ket*– vectors of the universal (process independent) eigenstate of the BFKL kernel $\hat{\mathcal{K}}$ with the eigenvalue $\omega(q_\perp)$,

$$\hat{\mathcal{K}}|R_\omega(q_\perp^2)\rangle = \omega(q_\perp^2)|R_\omega(q_\perp)\rangle, \;\; \langle R_\omega(q_\perp^2)|\hat{\mathcal{K}} = \langle R_\omega(q_\perp)|\,\omega(q_\perp^2), \tag{4}$$

The last equations give us elastic bootstrap conditions. The bootstrap conditions (eq.3) and (eq.4) are known since a long time and have been proved to be satisfied about ten years ago.

Inelastic bootstrap conditions connect the Reggeon-gluon impact factors (the leftmost blob in fig.2) and the gluon production operator (the centre blobs in fig.2). In our formalism these conditions can be written in the following universal form:

$$\hat{\mathcal{J}}_i\,|R_\omega(q_{(i+1)\perp})\rangle\, g\, q^2_{(i+1)\perp} + |\bar{J}_i R_{i+1}\rangle = |R_\omega(q_{i\perp})\rangle\, g\, \gamma^{J_i}_{R_i R_{i+1}}. \tag{5}$$

The first term is referred to as the operator of the jet J_i production. In our approximation (NLA) the jet J_i is either one gluon, or quark-antiquark pair (or two gluons) with close rapidities. The operator of the jet production $\langle \mathcal{G}_1'\mathcal{G}_2'|\hat{\mathcal{J}}_i\,|R_\omega(q_{(i+1)\perp})\rangle$ (i.e. operator $\hat{\mathcal{J}}_i$ projected onto the two-Reggeon t-channel state $\langle \mathcal{G}_1'\mathcal{G}_2'|$ and onto kernel eigenstate $|R_\omega(q_{i\perp})\rangle$) can be explicitly viewed [2)] through known in NLO effective vertices and the gluon trajectory. The second element $\langle \mathcal{G}_1'\mathcal{G}_2'|\bar{J}_i R_{i+1}\rangle$ of (eq.5) is the impact-factor of the jet production. It describes the transition of the t-channel Reggeon R_{i+1} into the final jet J_i and two-Reggeon t-channel state. The analytical form through the effective vertices and the trajectory one can find in [2)]. For the case when the jet J_i is quark-antiquark pair or two gluons the bootstrap condition (eq.5) was proved several years ago [3)]. The last unproved bootstrap condition reproduces the case when J_i is one gluon.

From the explicit form of the effective vertices it is easy to see that for the gluon contribution there are only three independent colour structures that lead to the nontrivial bootstrap condition. The optimal choice is the “trace-based”:

$$\mathrm{Tr}[T^{c_2}T^aT^{c_1}T^i],\;\; \mathrm{Tr}[T^aT^{c_2}T^{c_1}T^i],\;\; \mathrm{Tr}[T^aT^{c_1}T^{c_2}T^i], \tag{6}$$

where a is a colour index of the external gluon; i is a colour index of one-Reggeon t-channel state, and c_1, c_2 are colour indices of the two-Reggeon t-channel state. Two years ago by the direct loop calculation we demonstrated that the inelastic bootstrap condition was fulfilled being projected onto the colour octet in the

t-channel. For the quark contribution all bootstrap conditions can be obtained from the octet one and thereby are fulfilled.

The bootstrap conditions for the second and third colour structures in (eq.6) can be obtained from the octet one in a simple way. The first colour structure (symmetric with respect to c_1 and c_2) is essentially new.

Up to date by the direct loop calculation in the dimensional regularization we found both impact-factor and operator of the gluon production, and checked the cancellation of all singular (collinear and infrared singularities), logarithmic, and rational terms within the bootstrap condition for this structure. The matter of the nearest future is to cancel all dilogarithmic and double logarithmic terms. That will accomplish the NLA gluon reggeization proof irreversibly.

4 Conclusion

We presented the basic steps of the proof that in the multi–Regge kinematics real parts of QCD amplitudes for processes with gluon exchanges have the simple multi-Regge form.

The proof is based on the bootstrap relations required by the compatibility of the multi–Regge form (eq.1) of inelastic QCD amplitudes with the s–channel unitarity.

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