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Limit of Sensitivity of Low-Background Counting Equipment

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In most articles on counting statistics associated with radiation detectors, the counting time is assumed to be sufficiently long to permit the probability distribution of the observed counts to be adequately approximated by the normal distribution.[1,2] In such a distribution, the mean and variance are equal to the expected number of counts. However, as the expected counts approach zero, the assumption becomes invalid. This problem is significant when determining the limit of sensitivity for a counter with a very low background, e.g., 0.05 counts/min, and a counting time that results in fewer than 30 counts.

The Hazards Control Department's Radiological Measurements Laboratory (RML) analyzes many types of sample media in support of the Laboratory's health and safety program. The Department has determined that the equation for the minimum limit of sensitivity,

$$\mathrm{MDC}(\alpha,\beta) = 2.71 + 3.29\,(r_b t_s)^{1/2}. \qquad (1)$$

is also adequate for RML counting systems with very-low-background levels.

This paper reviews the normal distribution case and addresses the special case of determining the limit of sensitivity of a counting system when the background count rate is well known and small. In the latter case, we must use an exact test procedure based on the binomial distribution. However, the error in using the normal distribution for calculating a detection system's limit of sensitivity is not significant even as the total observed number of counts approaches or equals zero.

Notation

In this study, we are concerned with two types of counting errors. The Type I error occurs when we state that the observed count is greater than background when, in fact, it is background. The Type II error occurs when we state that the observed count is less than background when, in fact, it is greater than background.

In this report, we also use the following notation:

r_b = background counting rate (counts/min)
r_{sb} = sample plus background counting rate (counts/min).
r_s = net sample counting rate (counts/min)
t_b = background counting time (min).
t_s = sample counting time (min).
N_b = total background counts
= $r_b t_s$.
N_{sb} = total sample plus background counts
= $r_{sb} t_s$.

Case I: $r_b t_s \geq 30$

In Case I, where $r_b t_s \geq 30$, the normal distribution is applicable. Thus, the standard deviation, sdev, of the background count is:

$$\mathrm{sdev}(N_b) = (r_b t_s)^{1/2}.$$

The standard deviation of the sample plus background count is:

$$\mathrm{sdev}(N_{sb}) = (r_{sb} t_s)^{1/2}.$$

The net sample count is:

$$N_s = N_{sb} - N_b.$$

The standard deviation of the net count is:

$$\begin{aligned}\mathrm{sdev}(N_s) &= [\mathrm{sdev}(N_{sb})^2 + \mathrm{sdev}(N_b)^2]^{1/2} \\ &= [(r_{sb} t_s) + (r_b t_s)]^{1/2}.\end{aligned}$$

Limit of Sensitivity When Background Is Well Known

The statement that the background is "well known" implies that the mean of the backgrou count rate, r_b, has been accurately measured, e. via a very long background count. Figure 1 show an example of a normal distribution, which represents the distribution of the expected background counts, N_b, in the specified sample counting period, t_s. In this example, the background is 40 counts ($r_b t_s$), and the standard deviation is 6.3 counts.

If the background count is repeated, Eq. (6) defines the probability of the repeat count exceeding the "well-known" count by a specific multiple of the standard deviation:

$$P\{N'_b > [N_b + MSC(\alpha)]\} = \alpha, \quad (6)$$

where:

$P\{x\}$ = probability of x.
$MSC(\alpha)$ = minimum significant count
= $k(\alpha) \times sdev(N_b)$.
N_b = "well-known" background count.
N'_b = observed background count for the repeat trial.
$k(\alpha)$ = abscissa of the standardized normal distribution corresponding to the probability level $1 - \alpha$.

Values for $k(\alpha)$ are given in tables of the normal distribution. For $\alpha = 0.05$, $k(\alpha) = 1.645$. Consequently, in the example shown in Figure 1, the observed background count will be ≥ 51 counts in less than 5% of the counting trials (a single observed count can only be an integer). Table 1 shows the $k(\alpha)$ values for several other probability intervals.

The MSC(α) can be used to characterize the limit of sensitivity for counting equipment. However, as shown in Figure 2, if the true net count is equal to the MSC(α), the observed net count will be less than the MSC(α) 50% of the time. That is, 50% of the normal distribution lies below N_{sb}. Thus, if the MSC(α) is used to characterize a counter's limit of sensitivity, the probability of a Type I error (stating that the observed count is greater than background when, in fact, it is background) is 5%. The probability of a Type II error (stating that the observed count is less than background when, in fact, it is greater than background) is 50%. As the true count (the mean of the distribution) increases, the probability of a Type II error is reduced because the probability of observed net counts falling below the MSC(α) value decreases.

Just as the probability of a Type I error, α, can be selected, the probability of a Type II error, β, can also be selected. In the previous example, $\alpha = 0.05$ and $\beta = 0.50$. Although the 5% probability of

Table 1. Values of $k(\alpha)$ for several probability intervals.[a]

α	$k(\alpha)$
0.005	2.576
0.010	2.326
0.025	1.960
0.050	1.645
0.100	1.282

[a] $k(\alpha)$ = abscissa of the standardized normal distribution corresponding to the probability level $1 - \alpha$.

a Type I error is acceptable, the 50% probability of a Type II error is not. In other words, we want to further reduce the probability of stating that the observed count is background when, in fact, it is greater than background. To do this, we can define a new limit, called the minimum detectable count, that is a function of both α and β, MDC(α,β), as follows:

$$N_{sb} - k(\beta) \times sdev(N_{sb}) = N_b + MSC(\alpha),$$

where:

$k(\beta)$ = abscissa of the standardized normal distribution corresponding to the probability level $1 - \beta$.

Figure 3 shows an example of this new limit. Substituting the previously discussed expression into Eq. (2), we obtain:

$$r_{sb}t_s - k(\beta)(r_{sb}t_s)^{1/2} = r_b t_s + k(\alpha)(r_b t_s)^{1/2}.$$

If we let $k(\alpha) = k(\beta) = k$, then:

$$r_{sb}t_s - k(r_{sb}t_s)^{1/2} = r_b t_s + k(r_b t_s)^{1/2}.$$

Because r_b is well known:

$$r_{sb}t_s = r_s t_s + r_b t_s.$$

Therefore:

$$r_s t_s - k(r_{sb}t_s)^{1/2} = k(r_b t_s)^{1/2}$$

$$(r_{sb}t_s)^{1/2} = \frac{r_s t_s}{k} - (r_b t_s)^{1/2}.$$

Squaring both sides, we obtain:

$$r_{sb}t_s = \frac{r_s^2 t_s^2}{k^2} + r_b t_s - \frac{2r_s t_s}{k}(r_b t_s)^{1/2}. \quad (13)$$

Again, using Eq. (10):

$$r_s t_s = k^2 + 2k(r_b t_s)^{1/2} \quad (14)$$

or

$$\text{MDC}(\alpha,\beta) = k^2 + 2k(r_b t_s)^{1/2}, \quad (15)$$

where:

MDC(α,β) = the minimum detectable count, above background, for the counting interval, t_s.

In Figure 3, the MDC(α,β) for $\alpha = \beta = 0.05$ ($k = 1.645$) is MDC(0.05, 0.05) = $1.645^2 + 2 \times 1.645\,(40)^{1/2} = 23.5$ counts above background. Once we determine the MDC(α,β) of a particular counting system, we can calculate the limit of sensitivity as follows:

$$\text{LOS} = \frac{\text{MDC}(\alpha,\beta)}{(e)(t_s)}, \quad (16)$$

where:

LOS = limit of sensitivity (disintegrations/min).
MDC(α,β) = minimum significant count, above background, for t_s.
e = system counting efficiency (counts/disintegration).
t_s = sample counting time used to calculate the MDC(α,β) (min).

Alternatively:

$$\text{LOS} = \frac{\text{MDC}(\alpha,\beta)}{2.22(e)(y)(t_s)}, \quad (17)$$

where:

LOS = limit of sensitivity (pCi).
2.22 = number of disintegrations per minute per pCi.
y = fractional sample yield.

Figure 4 shows the MDC(α,β) as a function of background, $r_b t_s$, for $\alpha = \beta = 0.05$ using Eq. (15).

Case II: $r_b t_s < 30$

In the previous discussion of the MDC(α,β), w assumed that the observed counts were adequately represented by the normal distribution. Strictly speaking, this is only true when the mean of the observed counts is greater than approximately 30 counts because the normal distribution is merely an approximation of the underlying binomial distribution when the observed count is large (>30). Therefore, for observed counts <30, the binomial distribution should be used to determine the limit of sensitivity. Because the probability of detecting an individual count is very small and constant, we can use the computationally less cumbersome Poisson distribution to adequately represent th binomial distribution.

Figure 4 shows the MDC(α,β) using the Poiss distribution. For example, if a particular alph counter has a well-known background of 0.05 counts/min, the expected number of counts in a 20-min counting period is 1.0. From Figure 4, th MDC is 6.4 counts using the Poisson distribution and 6.0 using the normal distribution. Even if a counter is blessed with no background, the MD 2.9 counts and 2.7, using the Poisson and normal distributions, respectively. In other words, if t α and β errors are both set at 5%, the minimum detectable count is approximately 3, even if th counter has "no" background. Then the counting system's limit of sensitivity is finally determined by how long samples are counted, e.g., 3 counts/20 min, 3 counts/100 min.

Because the Poisson is a nonsymmetric discr distribution, the integral of the curve associat with the desired probability interval (α or β) replaced by a summation. In Figure 4, the summation area is 0.05, i.e., $\alpha = \beta = 0.05$, and the difference between the Poisson-generated MDC(α,β) and the normal-generated MDC(α, never exceeds 30%. Consequently, Eq. (15) is m than adequate for determining the MDC(α,β) Although the Poisson distribution is nonsymmetric, the error associated with the Type I e (long tail toward the right of the Poisson dis bution) is effectively cancelled by the error associated with the Type II error (short tail toward the left).

Limits of Sensitivity for the Radiological Measurements Laboratory

The Hazard's Control Department has determined that, for counting systems within the Radiological Measurements Laboratory (RML), the limit of sensitivity will be determined using the following equation for the minimum detectable count:

$$\text{MDC}(\alpha,\beta) = k^2 + 2k(r_b t_s)^{1/2}, \quad (18)$$

where:

$k(\alpha)$ = abscissa of the standardized normal distribution corresponding to the probability level $1 - \alpha$.

$k(\beta)$ = abscissa of the standardized normal distribution corresponding to the probability level $1 - \beta$.

For RML reporting, $\alpha = \beta = 0.05$. Therefore:

$$\text{MDC}(\alpha,\beta) = 2.71 + 3.29\,(r_b t_s)^{1/2}, \quad (19)$$

where:

r_b = well-known background counting rate.

t_s = sample counting time.

An Example

Assume that an alpha counter has a well-known background of 0.05 counts/min. What is the MDC(α,β) for samples counted for 100 min?

$$\begin{aligned}\text{MDC}(\alpha,\beta) &= 2.71 + 3.29\,(0.05 \times 100)^{1/2} \\ &= 10.0 \text{ counts.}\end{aligned} \quad (20)$$

If you observed 15 counts during the 100-min counting interval, the net count, 10 (15 – 5), is exactly at the minimum detectable count. In other words, the probability of stating that the observed count is *greater* than background when, in fact, it *is* background is 5%. The probability of stating that the observed count is less than background when, in fact, it is greater than background is also 5%. All observed net counts that are ≤10 should be reported as MDC(α,β), which is 10 counts.

If the efficiency of the alpha counter is 0.3 counts/disintegration, the limit of sensitivity fc the 100-min sample count is:

$$\begin{aligned}\text{LOS} &= \frac{10 \text{ counts}}{\left(0.3\,\frac{\text{counts}}{\text{dis}}\right)(100 \text{ min})} \\ &= 0.33 \text{ disintegrations/min.}\end{aligned} \quad (2$$

The Hazards Control Department has determined the limit of sensitivity of low-backgroun counting equipment used at the Radiological Measurements Laboratory. Although the nonsymmetric Poisson distribution is more appropriate for approximating the observed count distribution at low levels, e.g., <20 counts the error in using the normal distribution for calculating a system's limit of sensitivity is insignificant even as the total observed counts approach or equal zero.

References

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2. L. A. Currie, "Limits for Qualitative Detection and Quantitative Determinatic Applications to Radiochemistry," *Analytical Chemistry* 40, 586 (1968).

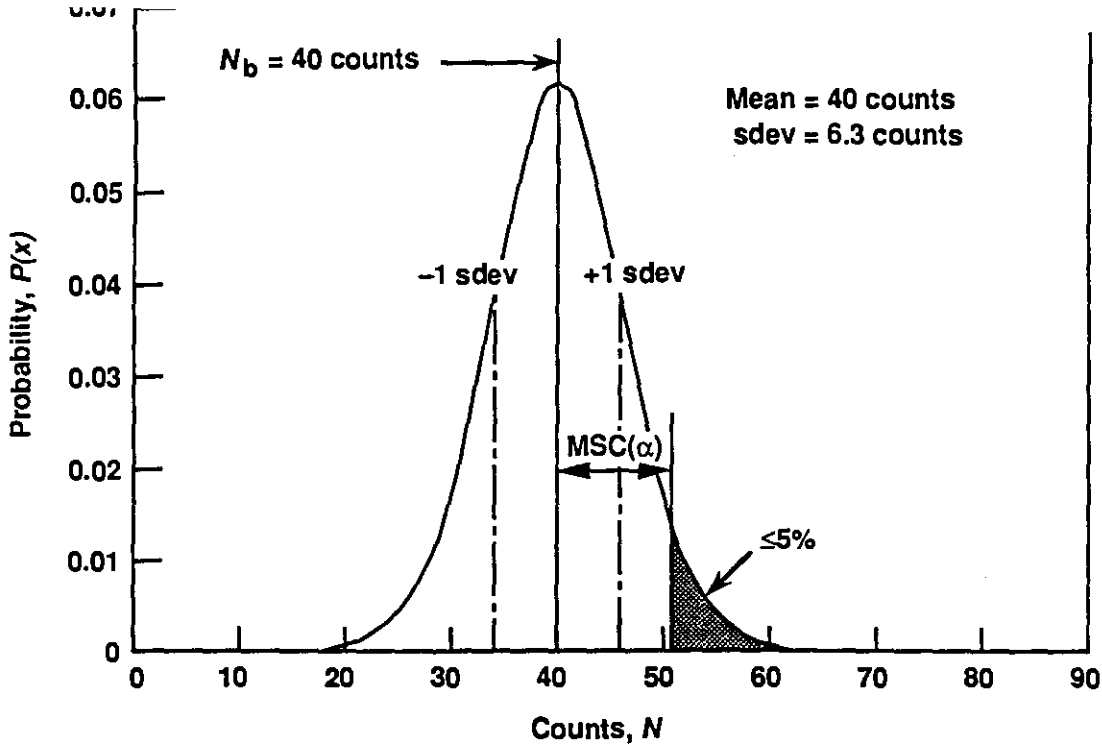

Figure 1. Example of a normal distribution for the expected background counts, N_b, in the specified sample counting period, t_s.

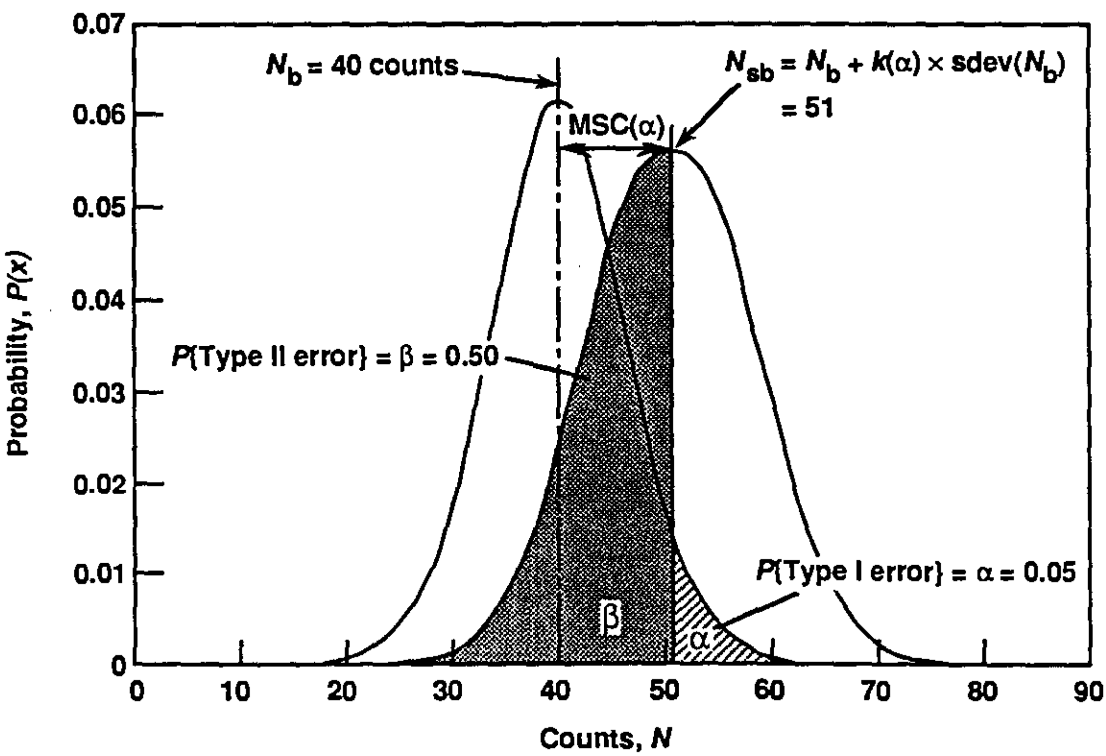

Figure 2. Example of a normal distribution where the true net count is equal to the MSC(α). In this distribution, the observed net count will be less than the MSC(α) 50% of the time. That is, 50% of t

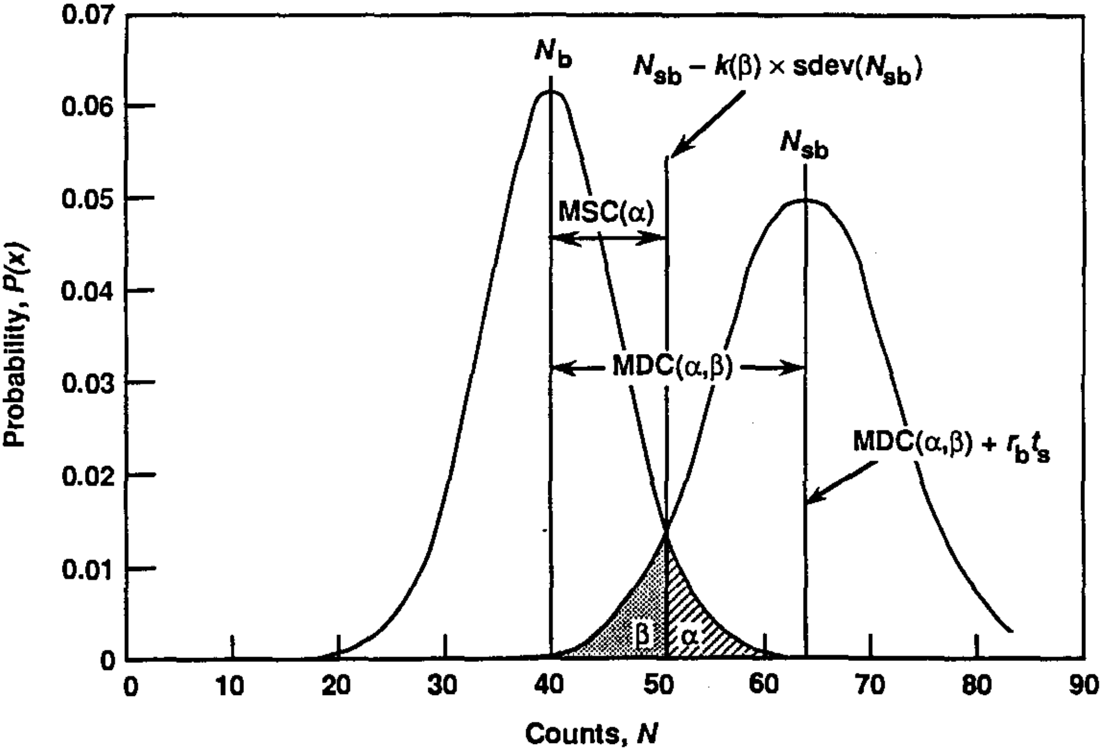

Figure 3. Example of a new limit that is a function of both α and β, MDC(α,β).

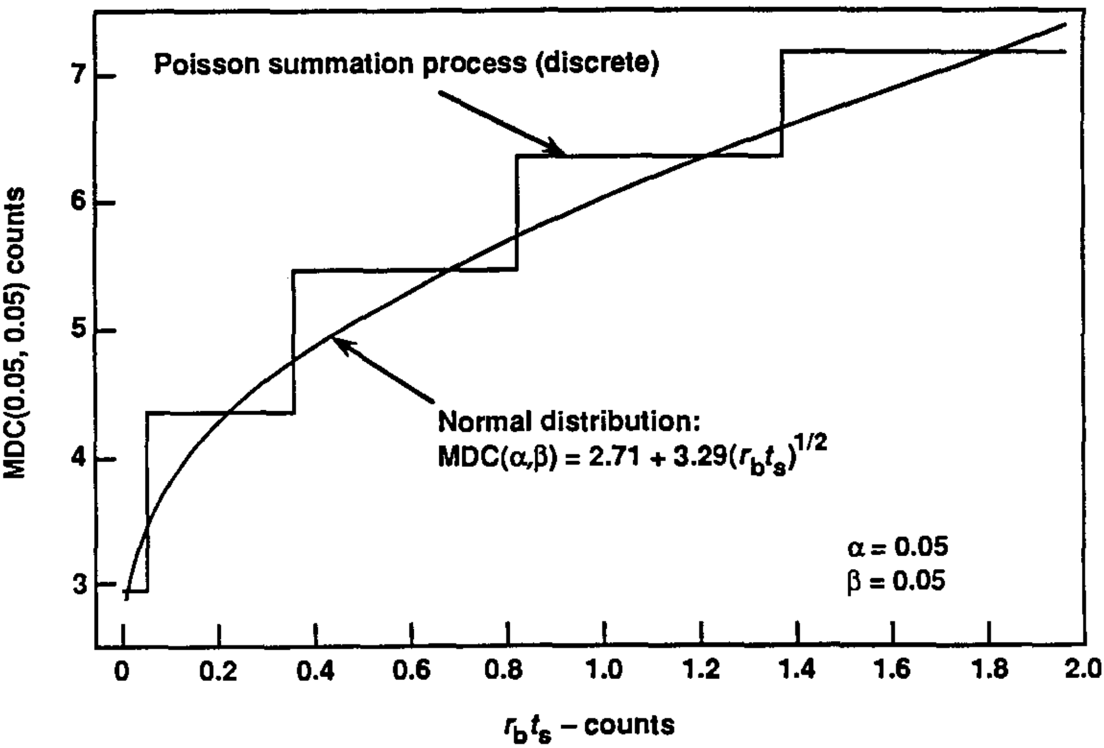

Figure 4. MDC(α,β) as a function of background, $r_b t_s$, for $\alpha = \beta = 0.05$ using Eq. (15).