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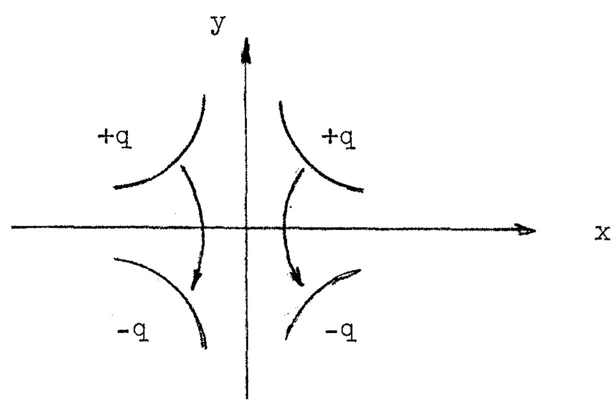
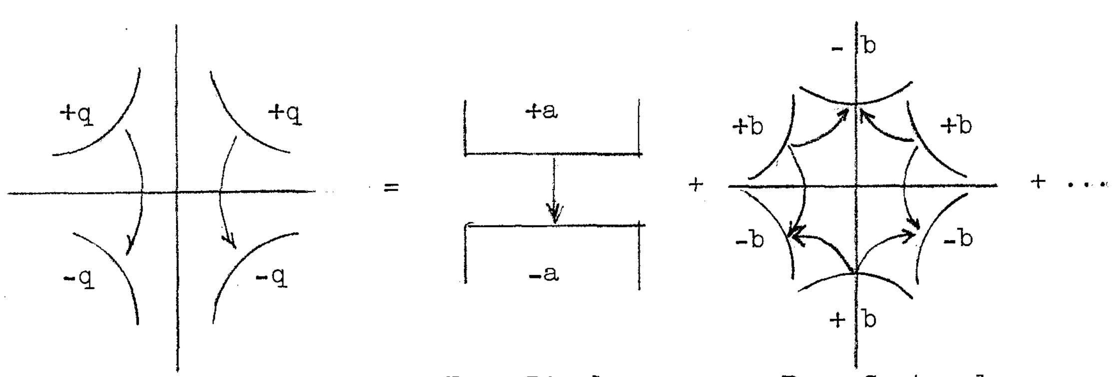
SEXTUPOLE COMPONENT IN THE MAGNETIC FIELD OF A SHUNTED QUAD

Consider a quadrupole with both of the upper poles the same sign; it produces a steering field somewhat weaker at the center than at the sides.



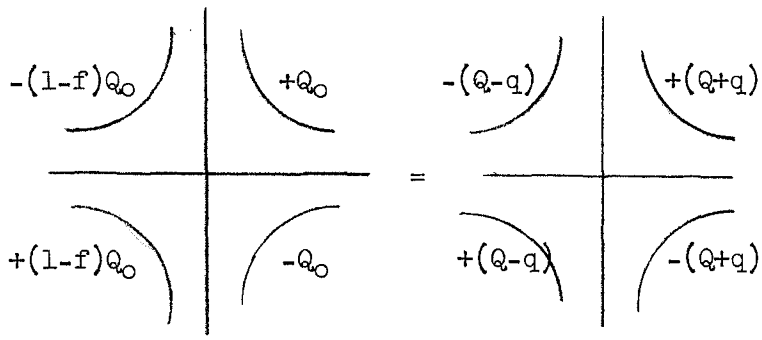
By symmetry an expansion of $B_y(x,y)$ in x about the origin will contain only even powers of x. This kind of an expansion is equivalent to the superposition of a sequence of pure multipole fields; the constant term is just a dipole, the x^2 term is a sextupole, etc., and one can make a sketch



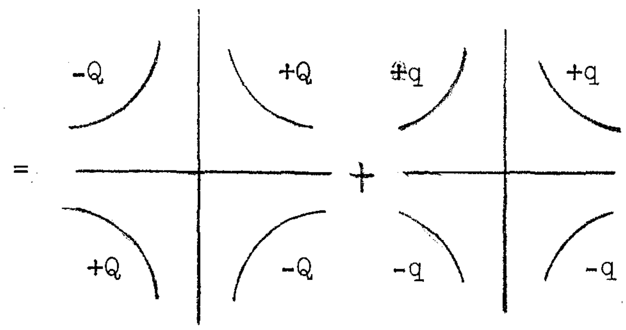
Pure Dipole

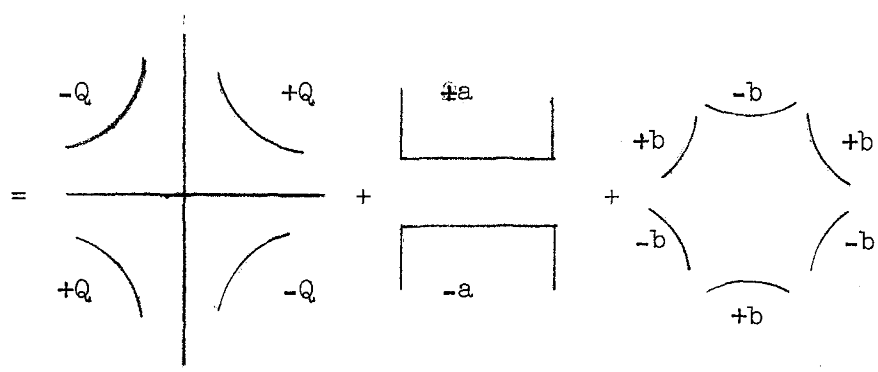
Pure Sextupole

Now consider a regular quadrupole with pole strength Q_o , two of whose poles are shunted by a factor f ; two poles have strength Q_o and two have strength $(1-f)Q$. If these strengths are written $Q \pm q$, then $q = Q_o f/2$ and $Q = Q_o\left(1 - \frac{f}{2}\right)$; and $f = 2(q/Q)/(1 + q/Q)$. Note that to get horizontal steering one shunts a vertical pair of poles.



Shunted Quads





Pure Quadrupole Pure Dipole Pure Sextupole

To get quantitative results a shunted quad was measured and also computed; the results agreed well. Joe Cobb made the measurements. The calculation used the program of Bill Herrmannsfeldt. The degree of shunting was $q/Q = 1/9$. The measured quad had a bore radius of 0.6 inches and a pole tip strength of about 3 kG. The following results are based on the computer run; a denotes the aperture radius, and B_o the average field at the pole tips of the quad.

Multipole Contribution	B_y
Quadrupole	$B_q = B_o(x/a)$
Dipole	$b_d = 0.564\ B_o(q/Q)$
Sextupole	$b_s = 0.456\ B_o(x/a)^2(q/Q)$

These results are based on the assumption that the quadrupole field is proportional to Q and that the dipole and sextupole fields are proportional to q. The total fields were calculated for the shunted and non-shunted cases; then the difference was fit to $c + dx^2$. The fit was quite good, which indicates that there is not much contribution from higher multipoles (see Fig. 1).

As a possible application consider the shunting required to displace the optic axis of a quad by a distance w. Then

$$\frac{q}{Q} = \frac{1}{0.564}\left(\frac{w}{a}\right) = 1.77\left(\frac{w}{a}\right)$$

A subject of interest is the consequences of the sextupole component. I don't see any simple way to evaluate this. Figure 2 shows the dipole plus sextupole field as a function of radius; the total field variation is fairly large, although if only the center part of the aperture is used, the variation is decreased substantially. If one takes a beam uniformly distributed over a radius b and centered on the axis of a sextupole, then the average field seen by the beam is

$$\langle b_s \rangle = \int_0^b b_{so} \left(\frac{x}{a}\right)^2 \sqrt{b^2 - x^2}\, dx/(\pi b^2/4)$$

$$\frac{\langle b_s \rangle}{b_{so}} = \frac{1}{4} \left(\frac{b}{a}\right)^2$$

a radius of aperture

b radius of beam

b_{so} sextupole field at $x = a$

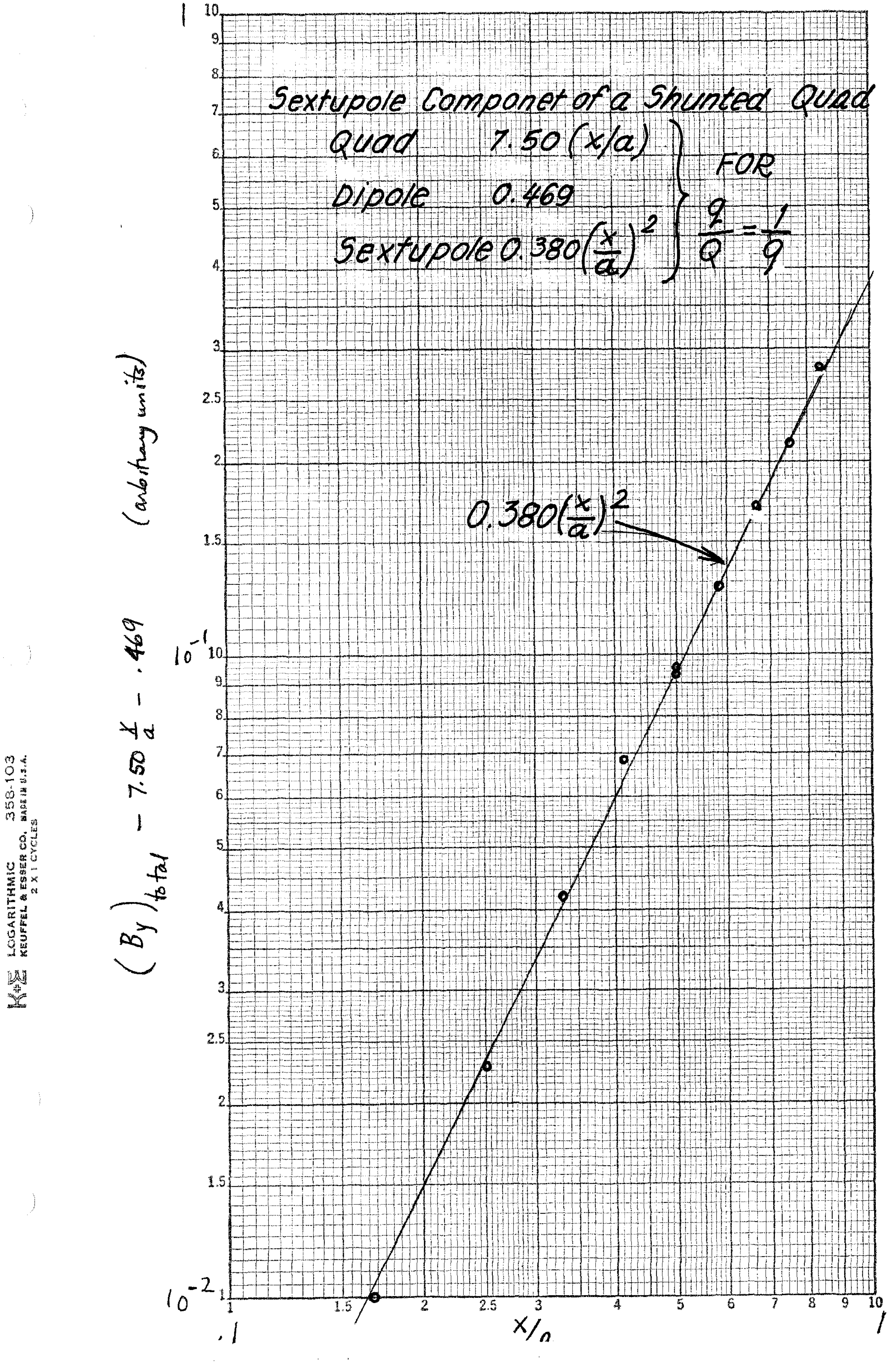

Sextupole Componet of a Shunted Quad
Quad 7.50 (x/a)
Dipole 0.469
Sextupole 0.380(x/a)^2
FOR q/Q = 1/9
0.380(x/a)^2
(B_y)_total - 7.50 x/a - .469 (arbitrary units)
x/a
1
10^-1
10^-2
.1
1
LOGARITHMIC KEUFFEL & ESSER CO. 358-103 MADE IN U.S.A.
2 X 1 CYCLES

Fig 1

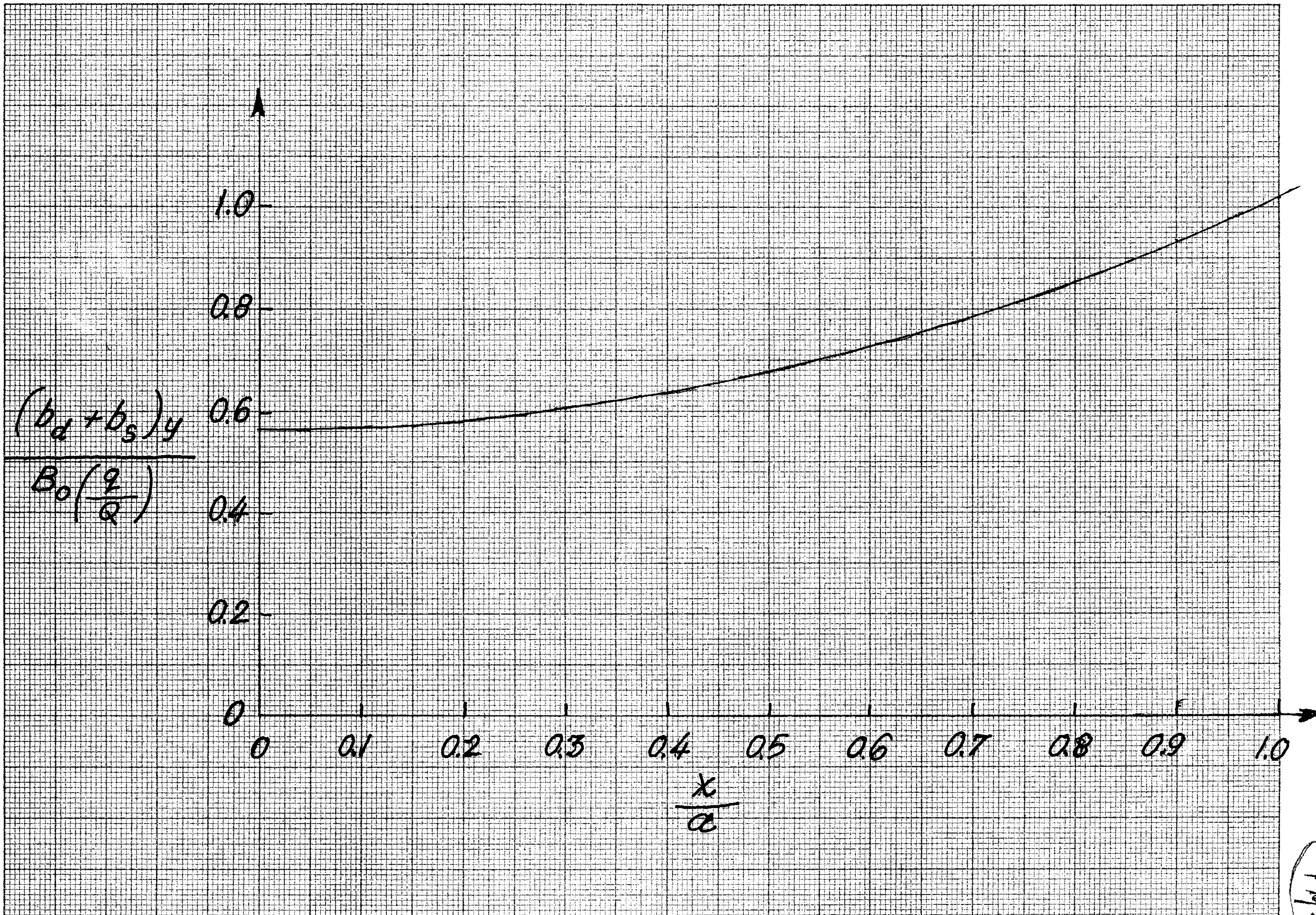


Fig. 2