

GEOMETRICAL DARK ENERGY AND LORENTZ SYMMETRY VIOLATION IN BRANE-WORLDS

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In this paper, we investigate the speed of gravitational waves in the context of brane-world theory without mirror symmetry or any form of junction conditions. Using the geometric dark energy, we show that the speed of the propagation of such waves is greater in the bulk than that on the brane. So, we expect the 4D Lorentz violation effects manifest themselves in the gravitational sectors. Finally, we study the effect of the geometric dark energy on the red shift of gravitational waves.

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1. Introduction

General relativity cannot describe gravity at high enough energies and must be replaced by quantum gravity theory. The physics responsible for making a sensible quantum theory of gravity is revealed only at the Planck scale. This cut-off scale marks the point where our old description of nature breaks down and it is not inconceivable that one of the victims of this break down is Lorentz invariance. During the last two decades theoretical studies and experimental observation of Lorentz invariance violation have received a lot of attention [1–3]. One possible consequence of Lorentz invariance violation is energy dependent photon propagation velocity. The energy dependence can be constrained by recording the arrival times of photons of different energies emitted by distant objects at approximately the same time [4–6]. One feature of Lorentz invariance violation to be considered is that the speed of light differs from that in special relativity. According to gravity theories with Lorentz violation, the speed of graviton or the speed of gravitational wave differ from that in general relativity (see *e.g.* [7]). Studying the speed of gravitational wave in a Lorentz violating gravity theory will provide different perspectives on quantum gravitational phenomena.

On the other hand, it is interesting to test the robustness of this symmetry at the highest energy scales [8–10]. As usual in high energy physics, if the scale characterizing new physics is too high then it cannot be reached directly in collider experiments. In this case, cosmology is the only place where the effects of new physics can be indirectly observed. Brane-world models offer a phenomenological way to test some of the novel predictions and corrections to general relativity that are implied by M-theory. Such models usually assume that c is a universal constant. For alternative approaches where the speed of gravity can be different from c in a brane-world context, see [11–13]. It should be emphasized that the assumption that the maximal velocity in the bulk coincides with the speed of light on the brane must not be taken for granted. In this regard, theories with two metric tensors have been suggested with the associated two sets of “null cones”, in the bulk and on the brane [14]. This is the manifestation of violation of the bulk Lorentz invariance by the brane solution. In some brane-world scenarios, the space-time globally violates 4D Lorentz invariance, leading to apparent violations of Lorentz invariance from the brane observer’s point of view due to bulk gravity effects. These effects are restricted to the gravity sector of the effective theory while the well measured Lorentz invariance of particle physics remains unaffected in these scenarios [15–17].

In a previous paper [18], we studied Lorentz violation in a brane-world model with Z_2 symmetry through the well known SMS procedure [19]. But in this work, we study local Lorentz violation in a brane-world scenario without using the Z_2 symmetry, or without postulating any junction condition. Here, the Friedman’s equation is modified by a geometrical term which is defined by the extrinsic curvature [20]. In order to evaluate the compatibility of the resulting cosmology with the observations, it is made an analogy with the phenomenological Λ CDM dark energy model. Based on the analysis of the deceleration parameter, it is found the universe expands in a bulk with signature $(4, 1)$, compatible with the de Sitter cosmology. Then, we address the effect of dark energy on the speed of propagation of gravitational waves in the bulk as well as on the brane. We find a relation between the maximum velocity in the bulk and the speed of light on the brane. Next, we compare the red shift experienced by gravitational waves travelling in the bulk with that of the electromagnetic waves on the brane and show that they are different. Therefore, if there is a possible detection mechanism in gravitational wave experiments, we can obtain some information about the state equation of geometric dark energy.

2. Field equation

In the usual brane-world scenarios, the space-time is identified with a singular hypersurface (or 3-brane) embedded in a five-dimensional bulk. Suppose now that the background manifold $\bar{v}_4$ is isometrically embedded in a pseudo-Riemannian manifold v_5 by the map $\mathcal{Y} : \bar{v}_4 \rightarrow v_5$ such that

$$\mathcal{Y}^A_{,\mu} \mathcal{Y}^B_{,\nu} g_{AB} = \bar{g}_{\mu\nu}, \quad \mathcal{Y}^A_{,\mu} N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon, \quad (1)$$

where g_{AB} ($\bar{g}_{\mu\nu}$) is the metric of the bulk (brane) space v_5 ($\bar{v}_4$) in arbitrary coordinate, $\mathcal{Y}^A(\mathcal{X}^\mu)$ is the basis of the bulk (brane) and N^A is normal unite vector, orthogonal to the brane. The perturbation of $\bar{v}_4$ with respect to a small positive parameter y along the normal unit vector N^A is given by

$$\mathcal{Z}^A(x^\alpha, y) = \mathcal{Y}^A + y N^A. \quad (2)$$

The integrability conditions for the perturbed geometry are the Gauss and Codazzi equations. The perturbation (2) induces a perturbation on the metric $g_{\mu\nu}$ which can be written as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \mathcal{X}_{\mu\nu}(x^\alpha, y). \quad (3)$$

In particular, the linear perturbation obtained from the expansion in y is

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + y \gamma_{\mu\nu}(x^\alpha). \quad (4)$$

To find the perturbed metric, we follow the same definitions as in the geometry of surfaces. Consider the embedding equations of the perturbed geometry written in the particular Gaussian frame defined by the embedded geometry and the normal unit vector

$$\mathcal{Z}^A_{,\mu} \mathcal{Z}^B_{,\nu} g_{AB} = g_{\mu\nu}, \quad \mathcal{Z}^A_{,\mu} N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon. \quad (5)$$

Substituting equation (2) in (5), we may express the perturbed metric in the Gaussian frame defined by the embedding as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2y \bar{K}_{\mu\nu} + y^2 \bar{g}^{\alpha\beta} \bar{K}_{\mu\alpha} \bar{K}_{\nu\beta}, \quad (6)$$

where $\bar{K}_{\mu\nu}$ is the extrinsic curvature of the original brane and the metric of our space-time is obtained at $y = 0$ ($g_{\mu\nu} = \bar{g}_{\mu\nu}$). It can also be obtained York's relation for the extrinsic curvature

$$K_{\mu\nu} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial y}, \quad (7)$$

when the brane-world gravitational field propagates in the bulk. Now, if the Friedman–Robertson–Walker (FRW) universe is seen as a brane-world,

a five-dimensional bulk with constant curvature is sufficient as it does not require any additional conditions [20]. The constant curvature bulk is characterized by the Riemann tensor

$${}^{(5)}R_{ABCD} = k_* (g_{AC}g_{BD} - g_{AD}g_{BC}) , \quad (8)$$

where k_* is either zero for the flat bulk, or proportional to a positive or negative bulk cosmological constant respectively, corresponding two possible signature (4, 1) for the dS₅ bulk and (3, 2) for the AdS₅ bulk. We take, in the embedding equation, $g^{55} = \varepsilon = \pm 1$. With this assumption the Gauss–Codazzi equations reduce to

$$R_{\alpha\beta\gamma\delta} = \frac{1}{\varepsilon} (K_{\alpha\gamma}K_{\beta\delta} - K_{\alpha\delta}K_{\beta\gamma}) + k_* (g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) , \quad (9)$$

$$K_{\alpha[\beta;\gamma]} = 0 . \quad (10)$$

Using the equations above, we can obtain Einstein's equations directly. These equations are modified by the presence of the extrinsic curvature

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \lambda g_{\mu\nu} = -8\pi GT_{\mu\nu} + Q_{\mu\nu} , \quad (11)$$

where λ is the effective cosmological constant in four dimensions, $T_{\mu\nu}$ is the confined matter energy-momentum tensor and the last term in (11) is completely geometrical quantity given by

$$Q_{\mu\nu} = \frac{1}{\varepsilon} \left(K_{\mu}^{\alpha}K_{\alpha\nu} - hK_{\mu\nu} - \frac{1}{2} (K^2 - h^2) g_{\mu\nu} \right) , \quad (12)$$

where $h = g^{\mu\nu}K_{\mu\nu}$ and $K^2 = K^{\mu\nu}K_{\mu\nu}$. Notice that the quantity Q is identically conserved in the sense that

$$Q_{;\nu}^{\mu\nu} = 0 , \quad (13)$$

then, there is no exchange of energy between this geometrical correction and the confined matter. In order to specify a cosmological model, it is usual to add a condition on the extrinsic curvature, such as the Israel–Lanczos junction conditions. If we solve the modified Einstein's equations on brane without any conditions, we can obtain more general solutions for these equations which they permit an adjustment with the observational results.

For the purpose of the embedding of FRW universe in a five-dimensional bulk with maximal symmetry, it is convenient to parameterize the FRW metric as

$$dS^2 = -dt^2 + a^2 [dr^2 + f(r) (d\theta^2 + \sin^2 \theta d\varphi^2)] , \quad (14)$$

where $f(r) = \sin r, r, \sinh r$ correspond to $k = 1, 0, -1$ respectively. Therefore, since $K_{\mu\nu}$ is diagonal for the FRW metric, (10) is reduced to

$$\begin{aligned} K_{ii,k} - K_{il}\Gamma_{ik}^l &= K_{ik,i} - K_{kl}\Gamma_{ii}^l, \\ K_{ii,0} - K_{ii}\frac{\dot{a}}{a} &= -a\dot{a}(\delta_i^1\delta_i^1 + f^2\delta_i^2\delta_i^2 + f^2\sin^2\theta\delta_i^3\delta_i^3)K_{00}, \end{aligned} \quad (15)$$

where the spatial indices in the brane are $i, j, k, l = 1, 2, 3$. The first equation for $k \neq 1$ gives $K_{11,k} = 0$ so that K_{11} is a time function $b(t)$. From the second equation we obtain $K_{00} = \frac{-1}{a}\frac{d}{dt}(\frac{b}{a})$. Repeating the same arguments for K_{22} and K_{33} , we obtain the general solutions for equation (10)

$$K_{ij} = \frac{b}{a^2}g_{ij}, \quad K_{00} = -\frac{1}{a}\frac{d}{dt}\left(\frac{b}{a}\right). \quad (16)$$

By using $B = \frac{\dot{b}}{b}$ and $H = \frac{\dot{a}}{a}$, we have

$$Q_{ij} = \frac{1}{\varepsilon}\frac{b^2}{a^4}\left(2\frac{B}{H} - 1\right)g_{ij}, \quad Q_{00} = -\frac{1}{\varepsilon}\frac{3b^2}{a^4}. \quad (17)$$

Substituting equation (17) in (11), we obtain Friedmann's equation modified by the presence of the extrinsic curvature

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} + \frac{1}{\varepsilon}\frac{b^2}{a^4}. \quad (18)$$

As can be seen, the correction term with respect to the standard Friedmann equation is given by components of the extrinsic curvature where $b(t)$ is an arbitrary function. In the next section, we try to find the dynamical role of this function.

3. The effects of the geometric dark energy

Using equation (13), we found that $Q_{\mu\nu}$ is independently conserved. So, we can suggest for $Q_{\mu\nu}$ a conserved energy-momentum tensor as

$$Q_{\mu\nu} = -\frac{1}{8\pi G}[(\rho_x + P_x)U_\mu U_\nu + P_x g_{\mu\nu}], \quad U_\mu = \delta_\mu^0, \quad (19)$$

where ρ_x and P_x are the geometric energy density and the geometric pressure, respectively. In fact, we consider a practical example in the Λ CDM model. Using equation (19) and equation (17), we can obtain

$$P_x = -\frac{1}{8\pi G}\frac{1}{\varepsilon}\frac{b^2}{a^4}\left(2\frac{B}{H} - 1\right), \quad \rho_x = \frac{3}{8\pi G}\frac{1}{\varepsilon}\frac{b^2}{a^4}. \quad (20)$$

Notice that sign $(-)$ in (19) was chosen in accordance with the weak energy condition such that $\rho_x > 0$ and $P_x < 0$ with $\varepsilon = 1$. As the Λ CDM model, we can consider a state-like equation for the geometric dark energy fluid

$$P_x = \omega_x \rho_x, \quad (21)$$

where ω_x may be a function of time. Substituting the expressions of B and H in equation above, we have the following equation for $b(t)$

$$\frac{\dot{b}}{b} = \frac{1}{2}(1 - 3\omega_x)\frac{\dot{a}}{a}. \quad (22)$$

Since ω_x is not known, we cannot solve the equation above. But if we assume that ω_x is constant with time, we obtain this solution

$$b(t) = b_0 \left(\frac{a}{a_0} \right)^{\frac{1}{2}(1-3\omega_x)}, \quad (23)$$

where a_0 is the present value of the expansion parameter and b_0 is a positive constant. As we know b_0 must not vanish, otherwise all extrinsic curvature components would vanish, then the brane-world would behave just as a trivial plane. By replacing equation (23) in (18), the modified Friedmann equation can be written

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} + \frac{1}{\varepsilon} \frac{b_0^2 a^{-3(1+\omega_x)}}{a_0^{(1-3\omega_x)}}. \quad (24)$$

As we can see, the modified Friedmann's equation depends on the signature of bulk. The different researches show that the present geometrical model with the de Sitter bulk is consistent with the latest experimental observation in limit of the weak energy condition. For example, acceleration red shift for such models are in good agreement with observational data [21]. Also, using the wealth of available data from the recent measurements, we can determine limits on the values of ω_x in our geometric model. For example, distance estimates of galaxy clusters from interferometric measurements of the Sunyaev-Zeldovich effect and X-ray observations along with SNe Ia and CMB data requires $\omega_x = -1.2^{+0.11}_{-0.18}$ [22, 23].

On the other hand, to obtain the equations of gravitational waves in the bulk, first we assume that the bulk space is empty. Using equation (6), the metric of the perturbed brane can be written as follows

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi K_{\mu\nu}, \quad (25)$$

where ξ is a small parameter. By using the Einstein gauge, the equations of gravitational wave become

$$\square K_{\mu\nu} = 0, \quad (26)$$

since $K_{\mu\nu}$ is related to the conserved energy-momentum tensor $Q_{\mu\nu}$, so we can conclude these wave are generated by the geometric dark energy.

3.1. The speed of gravitational wave

In this section, we want to obtain a relation between the maximal velocity of propagation in bulk and on the brane. Let us start by considering the metric for our 4D universe as

$$\bar{g}_{\mu\nu} = \text{diag}(-c_b^2, a(t)^2 \Upsilon_{ij}), \quad (27)$$

with coordinates (t, x^i) and the 3-metric Υ_{ij} on the spatial slices of constant time. Now, using equations (16) and (23), we have

$$\begin{aligned} \bar{K}_{00} &= \frac{b_0 \left(\frac{1+3\omega_x}{2} \right)}{a_0^{\frac{1}{2}(1-3\omega_x)}} a^{-\frac{3}{2}(1+\omega_x)}, \\ \bar{K}_{ii} &= \frac{b_0 \bar{g}_{ii}}{a_0^{\frac{1}{2}(1-3\omega_x)}} a^{-\frac{3}{2}(1+\omega_x)}. \end{aligned} \quad (28)$$

Substituting the above equations in equation (6), we find that the different 4D sections of the bulk in the vicinity of the original brane will have the metric

$$g_{\mu\nu} = \Omega^2 \text{diag}(-D^2 c_b^2, a(t)^2 \Upsilon_{ij}), \quad (29)$$

where

$$\begin{aligned} \Omega^2 &= \left[1 + \frac{b_0 y}{a_0^{\frac{1}{2}(1-3\omega_x)}} a^{-\frac{3}{2}(1+\omega_x)} \right]^2, \\ D &= \left[\frac{a_0^{\frac{1}{2}(1-3\omega_x)} - b_0 y \left(\frac{1+3\omega_x}{2} \right) a^{-\frac{3}{2}(1+\omega_x)}}{a_0^{\frac{1}{2}(1-3\omega_x)} + b_0 y a^{-\frac{3}{2}(1+\omega_x)}} \right]. \end{aligned} \quad (30)$$

From (27), we see that the constant c_b represents the speed of light on the original brane, whereas from (29) the speed of propagation of gravitational waves in this model is $D c_b$. Since the current observation data show that $\omega_x < -1$, so D is always greater than unity ($D > 1$). This leads to apparent violations of Lorentz invariance from the brane observer's point of view due to bulk gravity effects, because the maximal velocity in the bulk becomes more than the speed of light on the brane.

An interesting analogy exists between the behavior of gravitational waves propagating into the bulk from the brane and the electromagnetic waves crossing one medium into another with different indexes of refraction. This is a reflection of Fermat's principle where the greater speed achieved by gravitational waves in the bulk is taken advantage of when such waves bend slightly into the bulk with the result that they arrive earlier than electromagnetic waves, the latter being stuck to the brane. Therefore, gravitational waves traveling faster than light would be a possibility! These faster than light signals, however, do not violate causality since the apparent violation of causality from the brane observer's point of view is due to the fact that the region of causal contact is actually bigger than one would naively expect from the ordinary propagation of light in an expanding universe, with no closed timelike curves in the 5D spacetime that would make the theory inconsistent [24].

3.2. The red shift of gravitational wave

As the present day observations of distant objects involve red shifted spectra, knowing the behavior of the red shift of different waves is necessary for analyzing data. In this section, we compare the red shift of gravitational waves in the bulk with that of the electromagnetic waves on brane. In doing so we consider the usual FLRW line element on the brane with $a(t)$ as scale factor. Therefore, the red shift of electromagnetic waves on the brane is obtained by the usual formula

$$1 + z = \frac{\lambda}{\lambda_0} = \frac{a(t)}{a(t_0)}, \quad (31)$$

where λ and λ_0 are the detected and emitted wavelengths, respectively. On the other hand, using equations (29), (30), the red shift of gravitational waves on brane is given by

$$1 + z = \frac{\lambda}{\lambda_0} = \sqrt{\frac{D(t_0)}{D(t)}} \frac{a(t)}{a(t_0)}. \quad (32)$$

Now, the ratio of the red shift of the gravitational waves to that of the electromagnetic waves is $\sqrt{\frac{D(t_0)}{D(t)}}$. Therefore, it depends on the function $D(t)$. In an expansion universe (as our universe), we can consider $a(t_0) = na_0$, $a(t_2) = ma_0$ such that $m > n$ and a_0 is the scale factor of early universe. Using equation (30), we have

$$D(t_0) = \left[\frac{a_0^2 - b_0 y \left(\frac{(1+3\omega_x)}{2} \right) n^{-\frac{3}{2}(1+\omega_x)}}{a_0^2 - b_0 y n^{-\frac{3}{2}(1+\omega_x)}} \right],$$

$$D(t) = \left[\frac{a_0^2 - b_0 y \left(\frac{1+3\omega_x}{2} \right) m^{-\frac{3}{2}(1+\omega_x)}}{a_0^2 - b_0 y m^{-\frac{3}{2}(1+\omega_x)}} \right], \quad (33)$$

and

$$D(t) - D(t_0) = \frac{b_0 a_0^2 y \left[1 - \frac{(1+3\omega_x)}{2} \right] \left[m^{-\frac{3}{2}(1+\omega_x)} - n^{-\frac{3}{2}(1+\omega_x)} \right]}{\left[a_0^2 - b_0 y m^{-\frac{3}{2}(1+\omega_x)} \right] \left[a_0^2 - b_0 y n^{-\frac{3}{2}(1+\omega_x)} \right]}. \quad (34)$$

Since $m > n$, we can obtain $D(t) - D(t_0) > 0$. Then, for the de Sitter bulk dS_5 , $\sqrt{\frac{D(t_0)}{D(t)}}$ is less than unity so the red shift, due to gravitational waves, is smaller than that of the electromagnetic waves. Since the red shift of gravitational waves is different from that of the electromagnetic waves, this issue may have effects on the detection of gravitational waves. For instance, through the process of finding a correlation between the small scale CMB polarization fluctuations and the galaxy number density at a given red shift, one can determine the local quadrupole moments of the CMB at that red shift. Then, considering these quadrupoles at different patches on the sky and at different red shifts, one can obtain a map of the quadrupole moments during the reionization era [25]. A small part of this quadrupole pattern can be produced by the tensor modes of fluctuations, *i.e.*, gravitational waves. Using the correlation between galaxy distribution and the CMB polarization anisotropies, we can constrain the strength of the primordial gravitational waves which are really important to physicists, see also [26–29]. Since the red shift of gravitational waves is different from that of electromagnetic waves, one expects the strength of primordial gravitational waves to be modified.

Another interesting outcome is that if there is a possible detection mechanism in gravitational wave experiments, by studying the speed and red shift of these waves, we can find some information about the state equation of geometric dark energy.

3.3. A comparison with the SMS formalism

In the spirit of the usual SMS formalism [19], we assume Z_2 symmetry about the brane which considered to be a hypersurface Σ at $y = 0$. Using Z_2 symmetry, the Israel's junction conditions are written as

$$K_{\mu\nu}|_{\Sigma^+} = -K_{\mu\nu}|_{\Sigma^-} = -\frac{\epsilon k_5^2}{2} \left[\tau_{\mu\nu} - \frac{1}{3} g_{\mu\nu} \tau \right]. \quad (35)$$

Then, from the embedding equations, it follows that

$$(\tau_\nu^\mu)_{;\mu} = 0, \quad (36)$$

showing that the energy-momentum tensor $\tau_{\mu\nu}$ is conserved on the brane and represents the total vacuum plus matter energy-momentum. It is usually separated in two parts,

$$\tau_{\mu\nu} = \sigma g_{\mu\nu} + T_{\mu\nu}, \quad (37)$$

where σ is the tension of the brane in 5D, which is interpreted as the vacuum energy of the brane world and $T_{\mu\nu}$ represents the energy-momentum tensor of ordinary matter in 4D. In our previous paper [18], we obtain a relation between the maximal velocity of propagation in bulk and on the brane through the SMS formalism [19]. We start by assuming a perfect fluid configuration on the brane as

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, \quad (38)$$

where $\mathbf{u}$, ρ and p are the unit velocity, energy density and pressure of the matter fluid respectively. We also assume a linear isothermal equation of state for the fluid

$$p = \omega \rho, \quad 0 \leq \omega \leq 1. \quad (39)$$

The weak energy condition imposes the restriction $\rho \geq 0$ [30]. In that paper, we consider non-tilted homogeneous cosmological models on the brane, *i.e.* we assume that the fluid velocity is orthogonal to the hypersurfaces of homogeneity [31]. Then, using the Israel's junction condition, we obtain

$$\begin{aligned} \bar{K}_{00} &= \frac{\epsilon k_5^2 \bar{g}_{00}}{6} [\sigma - (2 + 3\omega) \rho], \\ \bar{K}_{ii} &= \frac{\epsilon k_5^2 \bar{g}_{ii}}{3} [\sigma + \rho]. \end{aligned} \quad (40)$$

Substituting the above equations in equation (6), we find that the different 4D sections of the bulk in the vicinity of the original brane will have the metric

$$g_{\mu\nu} = \Omega^2 \text{diag}(-D^2 c_b^2, a(t)^2 \Upsilon_{ij}), \quad (41)$$

where

$$\begin{aligned} \Omega^2 &= \left[1 - \frac{\epsilon k_5^2}{6} y (\sigma + \rho) \right]^2, \\ D &= \left[\frac{6 - \epsilon k_5^2 y (\sigma - (2 + 3\omega) \rho)}{6 - \epsilon k_5^2 y (\sigma + \rho)} \right]. \end{aligned} \quad (42)$$

From (27), we see that the constant c_b represents the speed of light on the original brane, whereas from (41) the speed of the propagation of gravitational waves on the 4D section of the bulk is $D c_b$. Now, if the extra

dimension is space-like ($\epsilon = 1$), D is always greater than unity ($D > 1$). Then, the maximal velocity in the bulk becomes more than the speed of light on the brane. This leads to apparent violations of Lorentz invariance from the brane observer's point of view due to bulk gravity effects. On the other hand, the ratio of the red shift of the gravitational waves to that of the electromagnetic waves depends on $\sqrt{\frac{D(t_0)}{D(t)}}$. In [18], it is shown that if the extra dimension is space-like, $\sqrt{\frac{D(t_0)}{D(t)}}$ is less than unity so the red shift, due to gravitational waves, is smaller than that of the electromagnetic waves. As we see, the results are quite matching with the results in the previous sub-sections.

4. Conclusions

In this paper, we have considered a brane-world scenario where the modified field equations on the brane were obtained without using the Z_2 symmetry or any junction condition. We also showed that if bulk is the de Sitter spacetime, the 4D Lorentz invariance in the gravitational sector is broken in the sense of having a propagation speed greater than that of the light. Gauge fields will not feel these effects, but gravitational waves are free to propagate into the bulk and they will necessarily feel the effects of the variation of the speed of light along the extra dimension. Based on this scenario we then showed that the red shift associated with gravitational waves moving through the bulk is not equal to the red shift of electromagnetic waves propagating on the brane. Such a difference could be used, in the event of the detection of gravitational waves, to find some information about the state equation of geometric dark energy.

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