SLAC-PUB-2572
July 1980
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A MONTE CARLO STUDY OF SU(2) YANG-MILLS THEORY AT FINITE TEMPERATURE*

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ABSTRACT

We employ Monte Carlo methods to study SU(2) Yang-Mills theory in the absence of fermions, on a lattice, at finite temperature. We determine the temperature at which a second-order transition takes place between confined and unconfined phases, in terms of the string tension.

Submitted to Physics Letters

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Systems of quarks and gluons at finite temperature and density have been the subject of much recent interest [1-3]. Assuming the existence of a quark-freeing transition, perturbative calculations, and non-perturbative calculations involving dilute instanton gases, can be and have been performed [1-4]. At high densities and temperatures, the QCD coupling strength is small and such calculations are self-consistent.

At lower temperatures and densities, non-perturbative confinement effects are important and perturbative calculations may lose their validity. A non-perturbative estimate of the transition temperature and the order of the transition would be useful, as would calculations of the entropy density, the energy density, and the quark-quark potential both in the confined phase and in the quark-gluon plasma. These results could be employed in the study of such plasmas as they may occur in neutron stars, heavy ion collisions, or the early universe [2-3],[5-6].

In this letter, we present some results of a Monte Carlo lattice calculation [7] for a gluon plasma at finite temperature. We have treated an SU(2) gauge theory in the absence of fermions. The existence of a quark-freeing transition in this system has been demonstrated, in the strong coupling limit of the Hamiltonian theory, by Susskind and by Polyakov [8]. An extrapolation to the continuum limit is however not possible without information about the intermediate coupling regime, which the Monte Carlo calculation provides.

In this note we shall display only our results for the critical transition temperature and the order of the transition. The energy density, entropy density, and quark-quark potential will be discussed at length elsewhere.

The partition function of the SU(2) gauge theory at temperature $T = 1/\beta$ may be written as a Euclidean path integral

$$Z = \int \mathscr{D}A^{\mu} \exp\left[-\int_{0}^{\beta} dt \int d^3x \; \mathscr{S}_E\right] \quad , \tag{1}$$

where we have indicated in the exponent the integral of the Euclidean action density

$$\mathscr{S}_E = \frac{1}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} \tag{2}$$

over a finite interval of imaginary time $0 \leq \tau \leq \beta$; we impose periodic boundary conditions

$$A_{\mu}(\vec{x}, \tau) = A_{\mu}(\vec{x}, \tau + \beta) \quad . \tag{3}$$

An ultraviolet cutoff may be introduced into (1) by formulating the theory on a space-time lattice of points $\{\vec{x},\tau\}$ with the degrees of freedom $U_{\mu}(\vec{x},\tau) \equiv \exp ia\sigma^a A^a_{\mu}(\vec{x},\tau) \in SU(2)$ defined on the links $\{\mu\}$ at each site (a is the lattice spacing). Then (1) is replaced by

$$Z = \int \left[\prod_{\text{links}} dU\right] \exp\left[-\frac{1}{g^2} \sum_{p} \left(1 - \frac{1}{2} \mathrm{Tr}(UUUU)_p\right)\right] , \tag{4}$$

where we have written the lattice action as a sum over plaquettes $\{p\}$ of the trace of the product of U's around each plaquette. dU represents the invariant integration measure on SU(2) at each link. Our lattice is comprised of $N_t \times N_x^3$ sites: N_x, the number of sites along any spatial axis, is finite only as a matter of convenience, while the finiteness of N_t, the number of sites along the time axis, is supposed to introduce effects of nonzero temperature when $N_t \ll N_x$.

A convenient order parameter with which to study the phase structure of (4) is provided by an adaptation of the Wilson loop integral. Consider

$$L(\vec{x}) = \mathrm{Tr} \prod_{\tau=1}^{N_t} U_0(\vec{x},\tau) \qquad , \qquad (5)$$

which is the trace of the product of U's along a time-oriented string running the temporal length of the lattice [8]. By virtue of the periodic boundary condition, the string is a closed loop, and $L(\vec{x})$ is therefore gauge invariant.

The expectation value of $L(\vec{x})$ in the ensemble defined by (4) yields the free energy F_q of an isolated quark (relative to the vacuum) via

$$e^{-\beta F_q} = \langle L(\vec{x}) \rangle \qquad ; \qquad (6)$$

a review of the elementary derivation of the path integral (1) makes this apparent. Further, the two-point function of $L(\vec{x})$ is related to the free energy $V(\vec{R})$ of a $q\bar{q}$ pair according to

$$e^{-\beta V(\vec{R})} = \langle L(\vec{x})\ L(\vec{x}+\vec{R}) \rangle \qquad . \qquad (7)$$

We will refer to $V(\vec{R})$ as the $q\bar{q}$ potential.

There is a global symmetry operation on the system (4) which reverses the sign of L. To display it, it is convenient first to do a partial gauge-fixing. The gauge invariance of (5) shows that it is impossible in general to fix $A_0 = 0$ (i.e., $U_0 = 1$) everywhere; the best we can do is $U_0(\vec{x},\tau) = 1$ for $\tau \neq 1$. Then $L(\vec{x}) = \mathrm{Tr}\, U_0(\vec{x},1)$. Now it is apparent that the action is invariant under the global transformation $U_0 \to -U_0$, which transforms $L \to -L$.

Thus $\langle L \rangle$ is reminiscent of the magnetization in a three-dimensional Ising system. If $\langle L \rangle = 0$, meaning $F_q = \infty$, we expect $\langle L(\vec{x})\, L(\vec{x}+\vec{R}) \rangle \xrightarrow[|\vec{R}| \to \infty]{} e^{-|\vec{R}|/\xi}$, showing that an isolated quark has infinite free energy and the $q\bar{q}$ potential is linear. On the other hand, a spontaneous symmetry-breaking magnetization $\langle L \rangle = M$ will lead to $\langle L(\vec{x})\, L(\vec{x}+\vec{R}) \rangle \xrightarrow[|\vec{R}| \to \infty]{} M^2$, meaning that $V(\vec{R}) \to$ constant and free quarks exist.

We have studied (4) on a small lattice, with Monte Carlo methods. These techniques have been described adequately elsewhere [7]. Choosing N_t, N_x, and g^2, we would either start with a magnetized lattice or an unmagnetized lattice and wait until $\langle L \rangle$ stabilized. Sometimes an otherwise stable nonzero magnetization would flip sign in a small number of iterations, showing nucleation and growth of a bubble (see fig. 1). This means, of course, that one must check for other finite volume effects.

We have found that, for various values of N_x and $N_t < N_x$, there is indeed a transition between an unmagnetized (confined) strong coupling phase and a magnetized (liberated) weak coupling phase as we vary g^2. Figure 2 shows this behavior for $N_t = 3$. The curves for $N_x = 6$ and $N_x = 7$ coincide, showing that effects of finite spatial volume have disappeared and that, on a scale set by $\beta \propto N_t$, we have reached the thermodynamic limit. $N_x = 5$ shows distortion of the critical behavior.

In order to understand the implications of our results for the continuum theory, we must adopt a renormalization scheme. As the lattice space goes to zero, a sensible physical parameter to keep fixed is the string tension at zero temperature. For all intents and purposes, zero temperature is reached when $N_t = N_x$, and we may look to Creutz's work [7] for the desired variation of bare coupling with lattice spacing. We write the continuum inverse temperature as a limit as $a, g \to 0$ and $N_t, N_x \to \infty$:

$$\beta = \lim a(g^2) N_t \quad . \tag{8}$$

In particular, the critical β depends on the critical g^2, which in turn depends on N_t:

$$\beta_{cr} = \lim_{N_t \to \infty} a\left(g^2_{cr}(N_t)\right) N_t \quad . \tag{9}$$

Creutz's work shows that lowest-order strong and weak coupling perturbation theory account well for the behavior of $a(g^2)$ except in a small interval around $1/g^2 = 2$:

$$a^2 = \frac{1}{K}\left(-\log \frac{1}{4g^2}\right) \qquad \frac{1}{g^2} \lesssim 2 \tag{10}$$

$$a^2 = \frac{1}{K} \exp\left(-\frac{6\pi^2}{11}\left(\frac{1}{g^2} - 2\right)\right) \qquad \frac{1}{g^2} \gtrsim 2 \tag{11}$$

K is the string tension, related to the Regge slope $\alpha' \sim 1$ GeV^{-2} by $K = 1/2\pi\alpha'$ (for definiteness we use this number from the real world to discuss the SU(2) Yang-Mills theory). In table I we display values of $1/g^2_{cr}$, $a(g^2_{cr})$, and aN_t for several (finite) values of N_t. Note that the approximants to β_{cr} for $N_t = 2$ and for $N_t = 3$ are close together, showing that the continuum limit is well represented by $N_t = 3$.

Finally, we note that the continuity of the order parameter across the transition, as shown in fig. 2, is characteristic of a second-order transition.

ACKNOWLEDGEMENTS

We thank M. Creutz for providing us with a copy of his Monte Carlo computer program. We are grateful for useful conversations with him, with T. DeGrand, R. Giles, G. Lasher, S. Shenker, and N. Weiss, and with our colleagues at SLAC. After the work reported herein was completed, similar work by J. Kuti, J. Polonyi, and K. Szlachanyi came to our attention. This work was supported by the Department of Energy under contract DE-AC03-76SF00515.

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TABLE I

Approximants to β_{cr}. Typical error in $1/g^2_{cr}$ may be estimated from fig. 2; most of the error in T_{cr} will come from Creutz's determination of the renormalization curve.

Lattice Size ($N_t \times N_x^3$)	$1/g^2_{cr}$	$a(g^2_{cr})$ (GeV^{-1})	aN_t (GeV^{-1})	T_{cr} (MeV)
1×5^3	.75	3.2	3.2	310
2×5^3	1.8	2.2	4.4	230
3×6^3	2.2	1.46	4.4	230

FIGURE CAPTIONS

Fig. 1. Fluctuations in magnetization for $N_t = 1$, $1/g^2 = .85$. In this case $1/g^2_{cr} \sim .75$.

Fig. 2. Magnetization curves for $N_t = 3$. We display $\langle |L| \rangle$ rather than $\langle L \rangle$ to remove effects of domain nucleation as shown in fig. 1. Points for $N_x = 5$ and for $N_x = 7$ are joined to guide the eye.

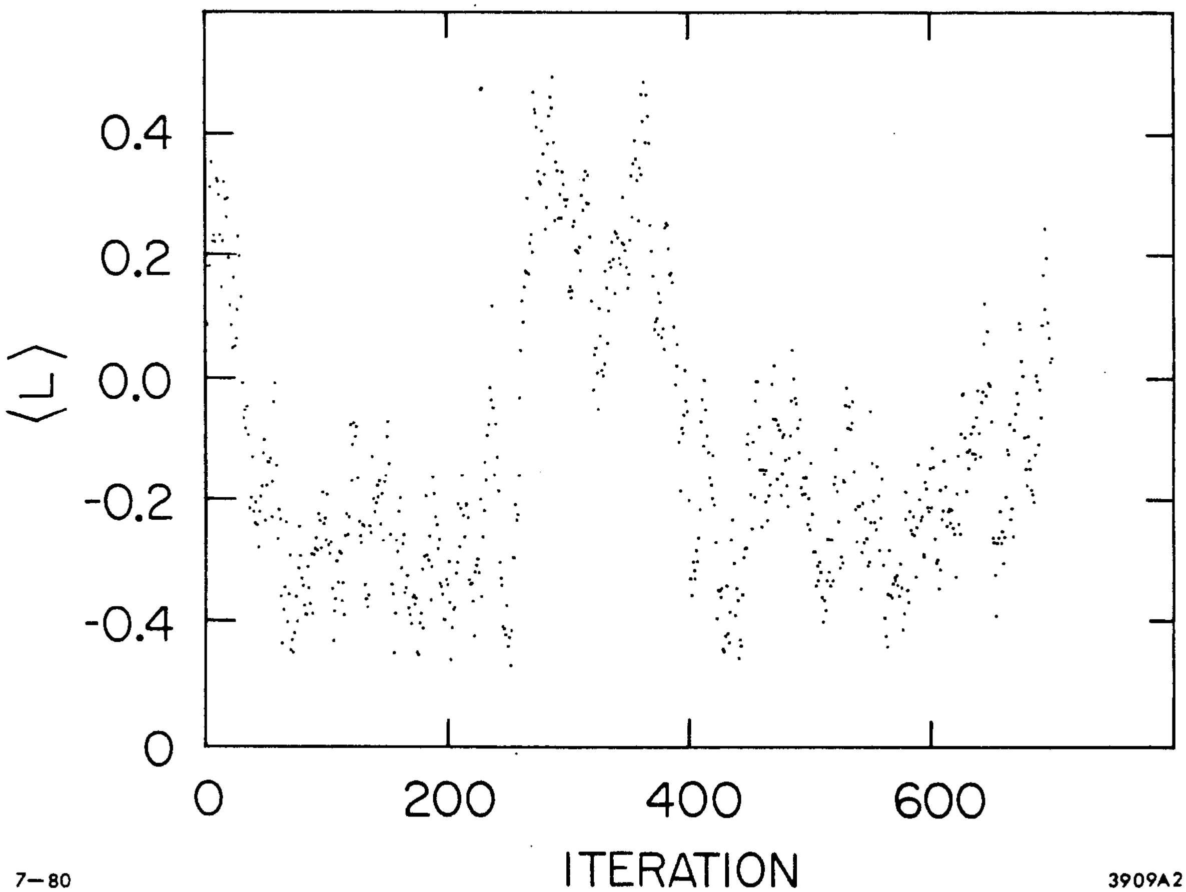

Fig. 1

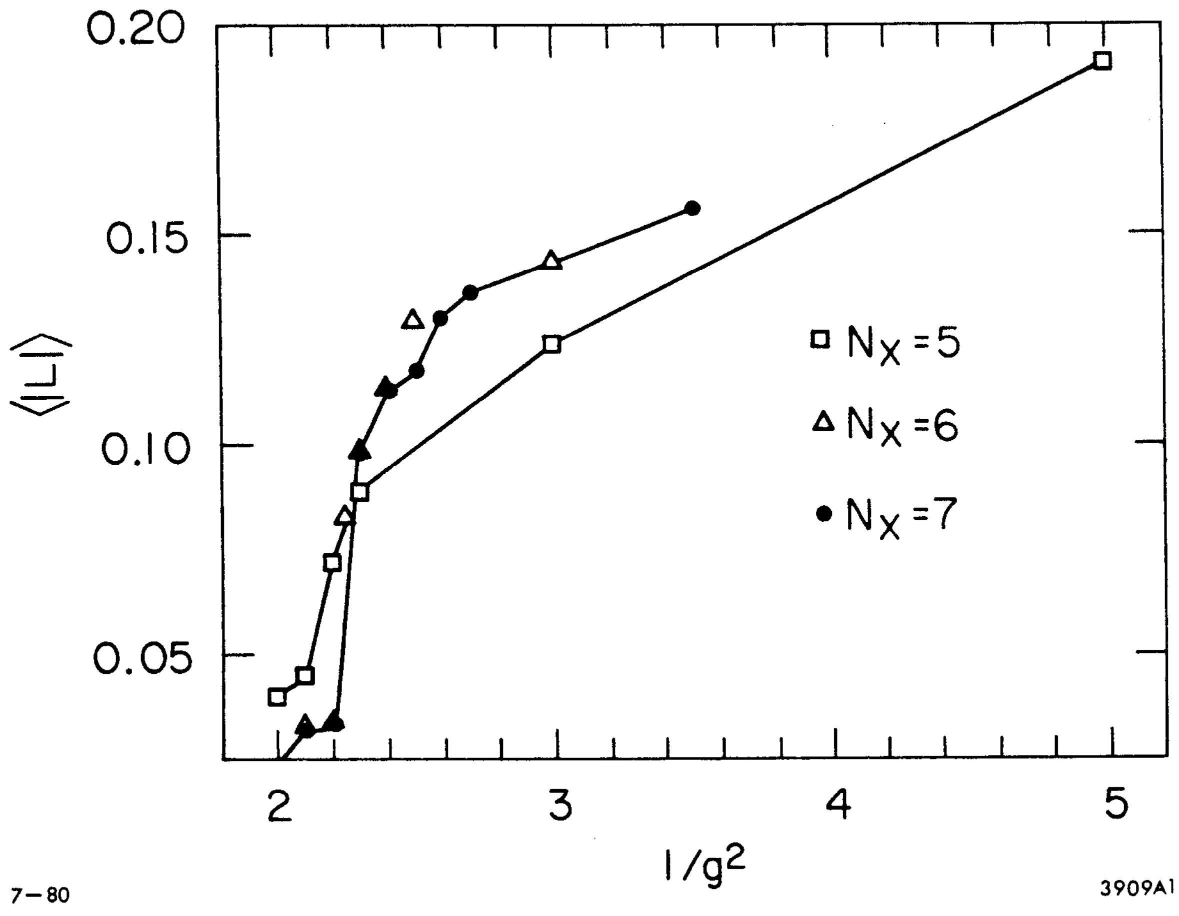

Fig. 2