PHASE-SHIFT ANALYSIS OF p-p-SCATTERING AT 95, 150 AND 310 MeV

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Until now, phase analysis has been carried out by means of the method of gradient descent from arbitrarily selected points. This method, however, has many disadvantages. Thus, with a limited number of selected initial points, we have no certainty of finding all the minima of the function χ^2. The gradient descent method does not provide us with a picture of the shape of the functions, and in this connection it does not make it possible to determine accurately the permissible boundaries of the regions of the varying parameters.

Gelfand, Grashin, Ivanova, Pomeranchuk and Smorodinsky have recently carried out in Moscow a phase analysis of *p-p* scattering data of energies of 95, 150 and 310 MeV by means of a new calculational method which had been suggested by Gelfand [1]. This new method is specially adapted to the investigation of the shape near the minimum of the function χ^2. The computer follows the bottom of the minimum, and does not stop at small dips.

In the course of the analysis, there were variations of a certain number of parameters, while the others were determined as a one-meson contribution from the pole. The constant g^2 was taken equal to 14.5.

At energies of 95 MeV there were variations of 5 parameters. Two regions of solutions were discovered, separated by a small ridge.

At energies of 150 and 310 MeV, there were variations of 9 parameters.

At energies of 150 MeV there were two regions of solutions, corresponding to those at 95 MeV but more clearly separated.

At an energy of 310 MeV, there were three separate regions of solutions. One of them corresponds to the first region obtained at 150 MeV, and includes the first and third solutions of Stapp [2]. The second corresponds to the second region at 150 MeV, and includes the second and fourth solutions of Stapp [2]. The third region has no equivalent at lower energies and includes the fifth and the eighth solution of Stapp [2].

The region of permissible values of the phases was considerably greater than the errors of the phases stated by Stapp et al [2].

Table I. Boundaries of the region of solutions for 95 MeV and some solutions

Phase-shifts	Common boundaries of region N1 and N2		Two solutions from region N1 $\chi^2 = 20$ $\chi^2 = 24$		Solution from region N2 $\chi^2 = 34$
$^1S_0$	−0.45	0.45	0.17	0.35	−0.25
$^1D_2$	−0.07	0.16	−0.06	0.105	0.13
$^3P_0$	−0.35	0.6	0.45	0.25	−0.23
$^3P_1$	−0.23	0.16	−0.046	−0.20	0.13
$^3P_2$	0.15	0.32	0.27	0.28	0.28

Table II. Boundaries of regions of solutions for 150 MeV and some solutions

Phase shifts	Boundaries of region N1 with $\chi^2 \leqslant 2\overline{\chi^2}$		Boundaries of region N1 with $\chi^2 \leqslant 3\overline{\chi^2}$		Solution from region N1 with $\chi^2 = 37$	Boundaries of region N2 with $\chi^2 \leqslant 3\overline{\chi^2}$		Solution from region N2 with $\chi^2 = 58$
$^1S_0$	0.2	0.4	0.2	0.45	0.296	−0.6	0.1	−0.155
$^1D_2$	0.09	0.16	0.08	0.16	0.134	0.05	0.17	0.139
$^3P_0$	0.02	0.2	0.0	0.25	0.132	−0.5	−0.35	−0.415
$^3P_1$	−0.35	−0.27	−0.35	−0.27	−0.295	0.04	0.13	0.090
$^3P_2$	0.25	0.3	0.24	0.3	0.280	0.3	0.36	0.321
ξ_2	−0.06	0.02	−0.06	−0.02	−0.034	−0.1	0.06	−0.079
$^3F_2$	−0.04	0.02			−0.024			0.007
$^3F_3$	−0.01	0.03			−0.001			−0.007
$^3F_4$	−0.02	0.07			0.016			−0.034

Table III. Boundaries of regions of solutions for 310 MeV and some solutions

Phase shifts	Boundaries of region N1 with $\chi^2 \leqslant 2\overline{\chi^2}$		Boundaries of region N1 with $\chi^2 \leqslant 3\overline{\chi^2}$		Five solutions from region 1 $\chi^2 = 30$	43	45	49	50
$^1S_0$	−0.4	0	0.45	0.05	0.204	−0.267	−0.387	−0.14	−0.158
$^1D_2$	0.18	0.27	0.16	0.28	0.240	0.228	0.248	0.25	0.200
$^3P_0$	−0.45	0	−0.5	0.1	−0.217	−0.324	−0.245	−0.013	−0.223
$^3P_1$	−0.55	−0.35	−0.55	−0.3	−0.427	−0.431	−0.365	−0.384	−0.517
$^3P_2$	0.26	0.4	0.25	0.45	0.309	0.271	0.304	0.374	0.273
ξ_2	−0.07	0.02	−0.07	0.02	−0.018	−0.009	0.001	−0.014	−0.027
$^3F_2$	−0.05	0.07			0.029	0.030	0.030	−0.040	0.017
$^3F_3$	−0.11	−0.00			−0.045	−0.077	−0.076	−0.027	−0.036
$^3F_4$	0.01	0.08			0.067	0.064	0.077	0.016	0.055
Phase shifts	Boundaries of region N2 with $\chi^2 \leqslant 2\overline{\chi^2}$		Boundaries of region N2 with $\chi^2 \leqslant 3\overline{\chi^2}$		Five solutions from region 2 $\chi^2 = 36$	43	45	48	49
$^1S_0$	−0.65	−0.2	−0.7	−0.1	−0.515	−0.515	−0.417	−0.413	−0.250
$^1D_2$	0.05	0.15	0.03	0.16	0.102	0.142	0.102	0.096	0.034
$^3P_0$	−0.85	−0.25	−1.00	−0.15	−0.461	−0.481	−0.697	−0.33	−0.805
$^3P_1$	−0.23	−0.05	−0.25	−0.04	−0.122	−0.119	−0.187	−0.193	−0.175
$^3P_2$	0.35	0.45	0.34	0.46	0.430	0.406	0.364	0.45	0.371
ξ_2	−0.15	−0.1	−0.15	−0.1	−0.108	−0.117	−0.133	−0.132	−0.134
$^3F_2$	−0.03	0.07			0.004	−0.008	0.009	0.008	0.036
$^3F_3$	0	0.04			0.023	0.015	0.015	0.028	0.011
$^3F_4$	0.03	0.1			0.063	0.007	0.069	0.076	0.068

LIST OF REFERENCES AND NOTES

1. A detailed description of this method will be published separately.
2. Stapp, H. P., Ypsilantis, T. J. and Metropolis, N. Phys. Rev. **105**, p. 302 (1957).

DISCUSSION

SEGRE : Did the calculation start from a set of random numbers?

TYAPKIN : The calculations were started from an arbitrarily chosen point. Then the calculation was carried out again using another arbitrarily chosen starting point at which time the same minima were obtained again.

SEGRE : Were any new solutions found beside the ones found by Stapp, Ypsilantis, and Metropolis?

TYAPKIN : No new solutions were found, but it was shown that some solutions were nearly the same.

SEGRE : Yes, we know that. Thank you.

TYAPKIN : It was found that for instance solution 1 and 3 are found within one large permissible region in small dips. The investigation of the construction showed that the error ellipse had hitherto led to inaccurate conclusions concerning the width of these regions. The investigation of the bottom of the region showed that there was a wide region. This is an interesting result but a sad one for physicists. The dips in the functions are due to the small number of points that have been calculated and to the error in the measurements themselves.

LOMON : Did the analysis at 150 MeV include the somewhat newer small angle data from Harvard which came out about a year ago, at about 8° or so?

TYAPKIN : The depolarization has been taken from the work of Harvard. Use has also been made of the data from Harwell but this led only to an insignificant shift in the position of the regions of solution. Again you have two regions of solutions but they were somewhat shifted.

LOMON : In the analysis published in the January Nuovo Cimento by Staebler and myself of the 150 MeV region we also tested the shape somewhat and tracked along the trough by taking the second derivative as well as the gradient. We indeed had found two solutions at an early stage but we found that on including the smallest angle data which was in the Coulomb interference region one of the solution regions disappeared. This is why I was particularly interested in the small angle data.

BREIT : In the first place, I have to state that to my knowledge at this conference our errors were actually not given. Now of course, Tyapkin has seen our errors last night privately.

TYAPKIN : I was merely referring to the method. I know what the errors inherent in the gradient method are. So even if you keep your errors in your briefcase I still know what they are.

BREIT : I understand this reasoning, of course. I would like to add the following. Our limits of errors are not claimed to be correct. We do include the sums of the squares of the deviations as a factor in determining the mean square error, which makes some allowance for the presence of unknown systematic deviations. Furthermore, the errors are usually determined by employing a collection of data in a rather wide energy range, and making a parallel shift of the phase parameter curve. In that way, presumably more data are employed, and therefore the chance of a local minimum is, I believe, appreciably diminished. We also had other tests on the determination of errors, which were made by the method of the linear variation and in that we obtained approximately the same estimates. I should also mention a third piece of evidence and that is that employing the different starting points we obtained a family of phase parameter curves which falls approximately within the error bounds determined by the matrix method. But, in spite of all this, no accurate claim for the accuracy of the error limits can be made at this time.

TYAPKIN : The good thing of course, would be to combine the methods of the joint energetic analysis with the method suggested by Gelfand.

CASSELS : This sounds like a good idea.