MULTIGRAVITY EFFECTS IN HIGH ENERGY COLLISIONS

A.V. Kisselev*and **V.A. Petrov**†
Institute for High Energy Physics, Protvino, Russia

The gravity effects in a four-dimensional eikonal amplitude related to Kaluza-Klein gravireggeons are calculated. It is demonstrated that the real part of the eikonal (with a massless mode subtracted) dominates its imaginary part at zero and large impact parameters. Both the real and imaginary part of the scattering amplitude exhibit an exponential falloff at large momentum transfer.

At present, models with compact extra dimensions are of particular interest. Depending on a number of extra spatial dimensions, $d = D - 4$, their compactification radius, R_c, can vary from 1 fm to 1 mm [1]. The main ingredient of the models in extra dimensions is massive Kaluza-Klein (KK) excitations of the graviton. In a D-dimensional space-time, the coupling of all KK gravitons (both massless and massive) to the SM fields is given by the Newton constant, $G_N = 1/M_{Pl}^2$, where M_{Pl} is the Planck mass. Since $M_{Pl} = 1.2 \cdot 10^{19}$ GeV, this coupling is extremely weak. Nevertheless, summing up real KK gravitons results in a D-dimensional gravitational coupling $G_D \sim 1/M_D^{2+d}$, with a fundamental Planck scale M_D of order of 1 TeV [1]. We will demonstrate that the same effect takes place for virtual KK excitations as well.

In the present paper we study the scattering of two particles confined on a 4-dimensional brane in a transplanckian kinematical region:

$$\sqrt{s} \gg M_D, \ -t, \tag{1}$$

$t = -q_\perp^2$ being four-dimensional momentum transfer. For the sake of simplicity, we consider first the case with one compact extra dimensions ($d = 1$). Then the generalization for the case $d > 2$ will be given. Thus, we start from the consideration of the scattering of bulk particles in four spatial dimensions, one of which is compactified with the large radius R_c.

In the eikonal approximation, the scattering amplitude is presented in the form

$$A(s,t) = 2is \int d^2b \, e^{iq_\perp b} \left[1 - e^{i\chi(s,b)}\right], \tag{2}$$

with the eikonal given by

$$\chi(s,b) = \frac{1}{4\pi s} \int\limits_0^\infty q_\perp dq_\perp \, J_o(q_\perp b) \sum_{n=-\infty}^{\infty} A^B(s, -q_\perp^2, n). \tag{3}$$

One can see that at $s < 4/R_c^2$ only mode with $n = 0$ contributes in the sum in Eq. (3). So, at low energy the scattering amplitude does not feel extra dimensions.

*E-mail: alexandre.kisselev@mail.ihep.ru
†E-mail: vladimir.petrov@mail.ihep.ru

In the transplanckian kinematical region (1), the eikonal is given by a sum of reggeized gravitons in the t-channel. We assume that both massless graviton and its KK massive excitations lie on linear Regge trajectories:

$$\alpha(t_D) = \alpha(0) + {\alpha'}_g \, t_D, \tag{4}$$

where t_D denotes D-dimensional momentum transfer. For one extra compact dimension with the radius R_c, we come to splitting of the Regge trajectory (8) into a leading vacuum trajectory

$$\alpha_0(t) \equiv \alpha_{grav}(t) = 2 + \alpha'_g t \tag{5}$$

and infinite sequence of secondary, called *gravireggeons* [2]:

$$\alpha_n(t) = 2 - \frac{\alpha'_g}{R_c^2} n^2 + \alpha'_g t, \quad n \geqslant 1. \tag{6}$$

In the string theory the slope of the gravireggeon trajectory is universal for all s, and $\alpha'_g = 1/M_s^2$, where M_s is the string scale.

The Born amplitude is, therefore, of the form [3]

$$A^B(s,t,n) = G_N s^2 \, \frac{\alpha'_g \pi}{2} \left[i - \cot \frac{\pi}{2} \Big(\alpha'_g t - \frac{\alpha'_g n^2}{R_c^2} \Big) \right] \left(\frac{s}{s_0} \right)^{\alpha'_g t - \alpha'_g n^2 / R_c^2} . \tag{7}$$

We start from the calculation of the imaginary part of the eikonal. Let us define the ratio:

$$a = \frac{R_c}{R_g(s)}. \tag{8}$$

From equations (3) and (7) we obtain:

$$\mathrm{Im}\,\chi(s,b) = G_N s \, \frac{\alpha'_g}{8R_g^2(s)} \, \exp \Big[- b^2 / 4R_g^2(s) \Big] \, \theta_3(0,q), \tag{9}$$

where

$$R_g(s) = \sqrt{\alpha'_g \, \ln(s/s_0)} \tag{10}$$

is a "Regge gravitational radius", and $q = \exp(-1/a^2)$.

The quantity θ_3 in (9) is one of Jacobi θ-functions [4]:

$$\theta_3(0,q) = 1 + 2 \sum_{n=1}^{\infty} q^{n^2}. \tag{11}$$

Since $R_c \gg R_g(s)$ even at ultra-high energies, we have $a \gg 1$. Thus, we need to know the behavior of $\theta_3(0,q)$ at $q \to 1$. It can be derived by using unimodular transformation of θ_3-function [4]:

$$\theta_3(0,q) = \left(- \frac{\pi}{\ln q} \right)^{1/2} \sum_{n=-\infty}^{\infty} e^{(\pi n)^2 / \ln q}. \tag{12}$$

The series in the RHS of (12) converges very quickly at $q \to 1$ (provided that $q < 1$), contrary to original series (11). In variable a, it looks like

$$\theta_3(0,q) = a\sqrt{\pi} \Big[1 + 2 \sum_{n=1}^{\infty} e^{-(\pi n a)^2} \Big]. \tag{13}$$

As a result, we get

$$\mathrm{Im}\,\chi(s,b) = (G_N R_c) \, s \, \frac{\alpha'_g \pi^{1/2}}{8R_g^3(s)} \, \exp \Big[- b^2 / 4R_g^2(s) \Big]. \tag{14}$$

The expression (9) is directly generalized for arbitrary $d \geqslant 1$:

$$\mathrm{Im}\,\chi(s,b) = G_N s \,\frac{\alpha'_g}{8R_g^2(s)}\, \exp\big[-b^2/4R_g^2(s)\big]\,[\theta_3(0,q)]^d, \tag{15}$$

that results in [3]

$$\begin{aligned}\mathrm{Im}\,\chi(s,b) &= \frac{1}{\pi^{d/2-1}}\,\frac{s}{M_D^2}\left(\frac{M_s}{2M_D}\right)^d \left[\ln\left(\frac{s}{s_0}\right)\right]^{-(1+d/2)} \\ &\times \exp\big[-b^2/4R_g^2(s)\big]. \end{aligned}\tag{16}$$

Here the relation $M_{Pl}^2 = (2\pi R_c)^d\, M_D^{2+d}$ was used [1]. As one can see, $\mathrm{Im}\,\chi(s,0)$ can be of order of unity at $\sqrt{s} \sim M_D$ ($\approx M_s$), while in the 4-dimensional space-time, a gravitational contribution to the eikonal is suppressed by the extremely small factor s/M_{Pl}^2. It is a contribution of infinite number of virtual KK graviton modes that compensates this smallness.

Now we consider the real part of the eikonal in one extra dimension. The general case ($d > 2$) will be analyzed below. From Eqs. (3), (7), we obtain ($q_\perp^2 = -t$):

$$\begin{aligned}\mathrm{Re}\,\chi(s,b) &= G_N s \frac{\alpha'_g}{8}\int\limits_0^\infty q_\perp dq_\perp\, J_0(q_\perp b)\, e^{-q_\perp^2 R_g^2(s)} \\ &\times \sum_{n=-\infty}^{\infty} \cot\big[\frac{\pi\alpha'_g}{2}\big(-t + \frac{n^2}{R_c^2}\big)\big] e^{-n^2 R_g^2(s)/R_c^2}, \end{aligned}\tag{17}$$

Formally, there exist poles in the sum in Eq. (17) corresponding to negative angular momenta $\alpha_n(t) = -2k$, where $k = 0, 1\ldots$ These unphysical (tachion) singularities should be suppressed by appropriate vertices and give no contributions to $\mathrm{Re}\,\chi(s,b)$.

In what follows, we will assume that

$$\alpha'_g\,|t| \ll 1. \tag{18}$$

The inequality (18) is equivalent to $|t| \ll M_s^2$. The sum in (17) is effectively cut from above, $n \lesssim n_{\max} = R_c/R_g(s)$. It means that one can put $\alpha'_g n^2/R_c^2 \lesssim [\ln(s/s_0)]^{-1} \ll 1$.

Let us define $\mathrm{Re}\,\check{\chi}(s,b)$ to be the real part of the eikonal with a pole term (corresponding to $n = 0$ in (17)) subtracted. With all mentioned above, it can be written as follows:

$$\begin{aligned}\mathrm{Re}\,\check{\chi}(s,b) &= G_N R_c^2 s \frac{1}{2\pi}\int\limits_0^\infty q_\perp dq_\perp\, J_0(q_\perp b)\, e^{-q_\perp^2 R_g^2(s)} \\ &\times \sum_{n=1}^{\infty} \frac{1}{n^2 + R_c^2|t|}\, e^{-n^2 R_g^2(s)/R_c^2}. \end{aligned}\tag{19}$$

As was already mentioned above, $R_c \gg R_g(s)$ even at ultra-high (cosmic) energies s. So, the parameter a (8) is taken to be large everywhere in our calculations. Under approximation (18), one obtains

$$\mathrm{Re}\check{\chi}(s,b)\Big|_{b\gg R_c} \simeq -G_N s \frac{\pi}{12}\left(\frac{R_c}{b}\right)^2 J_2\left(\frac{a_1 b}{R_c}\right), \tag{20}$$

where $a_1 = 3/\pi$.

At zero impact parameters, the eikonal looks like

$$\mathrm{Re}\,\check{\chi}(s,b=0) \sim (G_N R_c) s\,\frac{1}{R_g(s)}. \tag{21}$$

So, the real part dominates the imaginary part at $b = 0$,[1] $\mathrm{Re}\,\check{\chi}(s,0)/\mathrm{Im}\,\check{\chi}(s,0) \sim \ln s$, and it has a power-like behavior (with oscillations) at $b \to \infty$ (20), while the imaginary part decreases exponentially at large b.

The expression for the real part of the eikonal (19) is easily generalized for $d > 2$:

$$\begin{aligned} \mathrm{Re}\,\check{\chi}(s,b) &= G_N R_c^2 s \,\frac{1}{2\pi} \int\limits_0^\infty q_\perp dq_\perp\, J_0(q_\perp b)\, e^{-q_\perp^2 R_g^2(s)} \\ &\times \sum_{n=1}^{\infty} \frac{1}{n^2 + R_c^2|t|}\, e^{-n^2/a^2} \sum_{n_1^2+n_2^2+\cdots n_{d-1}^2 \leqslant n^2} , \end{aligned} \tag{22}$$

where the notation $n^2 = \sum_i^d n_i^2$ is introduced. The main contribution to the sum in the RHS of Eq. (22) comes from the region $n^2 \sim (d-2)\,a^2 \gg 1$. Thus, to estimate the sum in $(n_1, n_2, \ldots n_{d-1})$ analytically, we can replace the sum by the integral:

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n^2 + R_c^2|t|}\, e^{-n^2/a^2} \sum_{n_1^2+n_2^2+\cdots n_{d-1}^2 \leqslant n^2} &\to \sum_{n=1}^{\infty} \frac{1}{n^2 + R_c^2|t|}\, e^{-n^2/a^2} \int\limits_{\vec{x}^2 \leqslant n^2} \!\!\cdots\! \int d\vec{x} \\ &= \frac{\pi^{(d-1)/2}}{\Gamma\left(\frac{d+1}{2}\right)} \sum_{n=1}^{\infty} \frac{n^{d-1}}{n^2 + R_c^2|t|}\, e^{-n^2/a^2} . \end{aligned} \tag{23}$$

The calculations result in the expression

$$\mathrm{Re}\,\check{\chi}(s,b)\Big|_{b \gg R_g(s)} \simeq G_D s\, {\alpha_g'}^{-d/2}\, C(d) \left[\ln\left(\frac{s}{s_0}\right)\right]^{-d/2} \left(\frac{R_g(s)}{b}\right)^2 J_2\left(\frac{a_d b}{R_g(s)}\right) . \tag{24}$$

Here $a_d^2 = (d-2)/2$ and $C(d)$ is a constant depending on the number of the extra dimensions.

At zero impact parameter, we get:

$$\mathrm{Re}\,\check{\chi}(s,0) \sim G_D s\, {\alpha_g'}^{-d/2} \left[\ln\left(\frac{s}{s_0}\right)\right]^{-d/2} . \tag{25}$$

The expression for the imaginary part of the eikonal for $d \geqslant 1$ was calculated in Ref. [3]:

$$\mathrm{Im}\,\chi(s,b) = \frac{G_D s\, {\alpha_g'}^{-d/2}}{\pi^{d/2-1}} \left[\ln\left(\frac{s}{s_0}\right)\right]^{-(1+d/2)} \exp[-b^2/4R_g^2(s)]. \tag{26}$$

As one can see from (25) and (26),

$$\frac{\mathrm{Re}\,\check{\chi}(s,0)}{\mathrm{Im}\,\chi(s,0)} \sim \ln s. \tag{27}$$

Let us remind the asymptotic behavior of the eikonal derived in the framework of the string theory for the scattering of *D-dimensional fields in a flat space-time* [5]:

$$\chi_D(s,b)\Big|_{b^2 \gg \alpha' \ln s} \simeq \left(\frac{b_c}{b}\right)^d + i\pi^2 \frac{G_N^D s\, \alpha'^{-d/2}}{(\pi \ln s)^{1+d/2}} \exp\Big(-\frac{b^2}{4\alpha' \ln s}\Big), \tag{28}$$

where $b_c = [2\pi^{-d/2}\Gamma(d/2) G_N^D s]^{1/d}$, G_N^D being the Newton constant in D flat dimensions. Note, the real part of $\chi_D(s,b)$ exhibits power-law falloff which does not depend on the string tension $\alpha' \equiv \alpha_g'$.

The *imaginary parts* of $\chi(s,b)$ and $\chi_D(s,b)$ *coincide* at $b \gg \alpha_g' \ln s$, taking into account the definition of the gravitational radius $R_g(s)$ (10). As for the real part of the eikonal, $\mathrm{Re}\,\check{\chi}(s,b)$ decreases as a *fixed (d-independent) power of* b at $b \to \infty$, contrary to (28). The scales in the real

[1] The singular term was subtracted in $\mathrm{Re}\,\check{\chi}(s,b)$.

parts (associated with the impact parameter b) are also different: dynamical radius $R_g(s)$ in our case, and $b_c \sim (G_N^D s)^{1/d}$ in $\chi_D(s,b)$ (28).

Because of the inequality $R_g(s) \ll R_c$, our formulae contain the compactification radius R_c only via D-dimensional coupling $G_D = G_N(2\pi R_c)^d$. However, at extremely high energies, when the dynamical radius $R_g(s)$ becomes comparable with (or larger than) R_c, the eikonal profile in impact parameter space should " feel" the size of the compact dimensions R_c [3].

In this connection, let us mention the SM in a *D-dimensional space-time* with compact extra dimensions, but *without gravity* [6]. In such a case, the dynamical radius, $R(s)$, is proportional to $\ln(s/s_0)/\sqrt{t_0}$, where t_0 denotes the nearest (non-zero) singularity in the t-channel (for instance, $t_0 = m_\pi^2$, if only strong interactions are taken into account).

The expression for four-dimensional eikonal amplitude (in the presence of d compact extra dimensions) is of the form:

$$A(s,t) = 2is \int d^2b\, e^{iq_\perp b} \left[1 - e^{i\chi(s,b)}\right]. \tag{29}$$

At not extreme energies, namely, for $\sqrt{s} \lesssim M_D \sim M_s$, we have inequalities $\mathrm{Re}\,\check{\chi}(s,b)$, $\mathrm{Im}\,\chi(s,b) \ll 1$, and Eq. (29) is given by

$$\begin{aligned} \check{A}(s,t) &\simeq 4\pi s \int_0^\infty db\, b\, J_0(q_\perp b)\, [\mathrm{Re}\,\check{\chi}(s,b) + i\,\mathrm{Im}\,\chi(s,b)] \\ &= \mathrm{Re}\check{A}(s,t) + i\,\mathrm{Im}\,A(s,t). \end{aligned} \tag{30}$$

The imaginary part of the scattering amplitude exhibits exponential falloff at large $|t|$:

$$\mathrm{Im}\,A(s,t)\Big|_{\alpha'_g |t| \ln(s/s_0 \gg 1} \simeq G_D s^2\, \frac{8\, {\alpha'_g}^{1-d/2}}{\pi^{d/2-2}} \left[\ln\left(\frac{s}{s_0}\right)\right]^{-d/2} \exp\left(t\alpha'_g \ln(s/s_0)\right). \tag{31}$$

As for the real part of the amplitude, we obtain the following expression:

$$\mathrm{Re}\,\check{A}(s,t)\Big|_{\alpha'_g |t| \ln(s/s_0 \gg 1} \simeq \frac{G_D s^2}{-t}\, \frac{{\alpha'_g}^{-d/2}\, \Gamma\left(\frac{d}{2}\right)}{2^d \pi^{(d+1)/2} \Gamma\left(\frac{d+1}{2}\right)} \left[\ln\left(\frac{s}{s_0}\right)\right]^{-d/2} \exp\left(t\alpha'_g \ln(s/s_0)\right). \tag{32}$$

Note, that $\mathrm{Im}\,A(s,t) \ll \mathrm{Re}\,\check{A}(s,t)$ in kinematical region (18).

These asymptotics are quite different from the behavior of the eikonal amplitude in both the string theory [5] and in a model with Regge exchanges in *D flat dimensions* [7]:

$$A(s,t)\Big|_{|t| \gg b_c^{-2}} \sim G_N^D s^2\, {\alpha'_g}^{(1-d)/2} |t|^{-(d+2)^2/4(d+1)}\, e^{i\phi_D(t)}, \tag{33}$$

where $\phi_D(t) \sim |t|^{d/2(d+1)}$, and b_c is defined after formula (28). They are also different from the asymptotic behavior of $A(s,t)$ in the model with *compact extra dimensions*, when non-reggeized KK graviton exchanges are summed up [8].

Thus, in the framework of the model with d extra compact dimensions, we have studied the quantum gravity effects related to Kaluza-Klein gravireggeons. For the scattering of the SM fields living on the 4-dimensional brane, the eikonal is calculated. It is shown that the real part of the eikonal (with the massless mode subtracted) decreases as the power of b (with oscillations) at large values of the impact parameter b. This power depends on the gravitational slope, not on d, contrary to the case when the colliding fields are allowed to propagate in the bulk. The imaginary part of the eikonal decreases exponentially at large b. At zero momentum transfer, the real part also dominates the imaginary part. It is also shown that both the real part of the amplitude and its imaginary part decrease exponentially at large momentum transfer.

The gravitational effects considered here can be seen in leptonic or semileptonic collisions at transplanckian energies ($\sqrt{s} \gtrsim M_D$), for instance, in interactions of cosmic neutrinos with the atmospheric nucleons [3].

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. **D59**, 086004 (1999).

[2] V.A. Petrov, *Proceedings TH 2002 Supplement* (2003) 253-255.

[3] A.V. Kisselev and V.A. Petrov, arXiv: hep-ph/0311356.

[4] A. Erdelyi, W. Magnus, F. Oberhettinger and F.C. Tricomi (eds.), *Higher Transcendental functions*, v.3 (Mc Graw-Hill Book Company, 1955).

[5] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. **B197**, 81 (1987); Int. J. Mod. Phys. **A3**, 1615 (1988); Nucl. Phys, **B347**, 550 (1990); Nucl. Phys, **B403**, 707 (1993).

[6] V.A. Petrov, Mod. Phys. Lett. **A16**, 151 (2001).

[7] I.J. Muzinich and M. Soldate, Phys. Rev. **D37**, 359 (1988).

[8] G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. Phys. **B630**, 293 (2002); A.V. Kisselev, arXiv: hep-ph/0306136, to appear in Eur. Phys. J. C (2004).