HELIUM GAS COOLED CURRENT LEADS IN A REFRIGERATOR COOLED SUPERCONDUCTIVE MAGNET SYSTEM

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Abstract

Operating conditions for large superconductive magnets are maintained by helium refrigerators in closed refrigeration loops. Their current leads are cooled by helium being withdrawn from the gas returning to the cold end of the last heat exchanger of the refrigerator. The cooling gas flowing at a rate m_w is warmed up by heat exchange with the current leads and fed back to the main compressor of the refrigerator. The rate of cold gas returning to the refrigerator is reduced by an amount \dot{m}_w and thus the refrigeration power is reduced as well. The influence of \dot{m}_W on the refrigeration performance of the magnet-refrigerator system is investigated both theoretically and experimentally. Using the excess refrigeration power \dot{Q}_{e} as a measure for the performance one can show that Q_e becomes rather small at $\dot{m}_W = 0$ due to the high heat conduction rate of uncooled copper leads. If m_w increases Q_p decreases again because no cold gas returns to the refrigerator.

In the DESY 1.4 m/2.2 Tesla superconductive magnet system Pluto, \dot{Q}_e was measured as a function of \dot{m}_w for different currents between 0 and 1200 A. At the maximum $\dot{Q}_e = (\dot{Q}_e)_{max}$ the optimal value for the cooling flow rate $\dot{m}_w = \dot{m}_{w0}$ can be found. A new method for separating the heat losses of the current leads from all other static heat losses of the system is derived from the asymptotic behavior of the $\dot{Q}_e = f(\dot{m}_w)$ relation.

I. Introduction

Superconductive magnets operating at liquid helium temperature are connected to their power supplies at room temperature by means of electrical leads of copper or other materials. A certain amount \dot{Q}_L of parasitic heat is caused to enter the cold part of the system due to heat conduction and power dissipation in these current leads. In general this heat load can be reduced by introducing a heat exchange between the cold evaporating gas and the current leads (Fig. 1). Provided that a good quality of heat exchange is established a certain amount of heat can be transferred already at temperature levels $T_{He} < T < T_{room}$ from the conductor to the gas. The enthalpy of the gas is increased and the heat entering

the magnet at $T = T_{He}$ ($T_{He} \approx 4.2$ K = temperature of liquid helium boiling at atmospheric pressure) is reduced.

II. Theory

The heat losses \dot{Q}_1 of a magnet suspended in a helium filled cryostat are the sum of the heat flux \dot{Q}_L from the electrical current leads into the liquid and the basic load of the cryostat \dot{Q}_0 due to heat conduction, radiation etc. We have

$$\dot{Q}_1 = \dot{Q}_L + \dot{Q}_0 \tag{1}$$

At a certain current I and for a given geometry of a current lead \dot{Q}_1 is a function of the cooling gas flow rate \dot{m}_w . Consequently the heat flux \dot{Q}_1 (\dot{m}_w) decreases with increasing \dot{m}_w (Fig. 2).

In the same diagram is indicated the heat flux $\dot{Q}_{v} = \lambda m_{v}$ which is necessary to evaporate the liquid at a rate of \dot{m}_{v} (λ = heat of evaporation). If all vapor being produced in the cryostat is used for the lead cooling, the point of intersection given by

 $\dot{Q}_1 = \dot{Q}_{w}$

or

$$\dot{\dot{Q}}_{L} + \dot{\dot{Q}}_{o} = \dot{h} \dot{m}_{w}$$
 (2)

is a point of stable equilibrium between the vapor produced in the cryostat and the total heat entering the liquid. An increase of m_w would reduce the heat flux through the conductor and the rate of evaporation decreases, or vice versa.

The stable cooling gas flow rate is

$$\dot{m}_{ws} = \frac{\dot{Q}_L + \dot{Q}_O}{4}$$
(3)

Thus in the case of a cryostat filled with liquid helium from a dewar the most economical way of operation will be to use all cold helium gas available in the cryostat for the cooling of the leads.

The situation changes completely in the case of large magnets with long operation periods. Those units have to be cooled by refrigerators in a closed cycle. A simplified scheme giving only the details necessary for our considera-



Fig. 1. Dewar supplied magnet cryostat with vapor cooled current leads Legend: D = helium storage dewar; T = He transfer line; M = superconductive magnet coil; C = cryostat; L = current leads; R = He recovery line; PS = power supply; m_w = gas rate evaporated in the dewar.



Fig. 2. Heat load of a magnet cryostat with vapor cooled current leads Legend: 1: $\dot{Q}_1 = \dot{Q}_0 + \dot{Q}_L (\dot{m}_w);$ 2: $\dot{Q}_1 = \dot{Q}_0;$ 3: $\dot{Q}_1 = \dot{Q}_v = \lambda \cdot \dot{m}_w;$ $\dot{m}_{ws} =$ stable cooling gas rate

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Fig. 3. Refrigeration system of a superconductive magnet (schematic) Legend: K = compressor P = precooling; H = heat exchanger; J = Joule Thomson valve; Tr₁, Tr₂ = vacuum insulated transfer lines; C = cryostat; M = superconductive magnet; L = vapor cooled current leads; V = control valve for cooling gas rate; PS = power supply

tions is shown in Fig. 3. A helium compressor K forces high pressure helium gas at room temperature into the circuit. The rate of gas flow is $\dot{m}_0 + \dot{m}_p$ grams per second. In a precooling device P a fraction \dot{m}_0 of the gas is cooled successively to about 10 K. The precooling in P is performed by means of liquid nitrogen, by expansion engines and by counterflow of cold gas returning from the heat exchanger H. The remaining amount of gas \dot{m}_p being supplied by the compressor is used for the operation of the expansion engines.

In the last heat exchanger H the gas is further cooled by the cold gas returning from the system. An isenthalpic pressure reduction to about 1 bar, through the Joule Thomson valve J into the cryostat C reduces the temperature of the gas to its final value of about 4.2 K and causes a fraction of the gas to be liquefied. An amount of

$$\dot{\mathbf{m}}_{\mathbf{f}} = \boldsymbol{\varepsilon} \, \dot{\mathbf{m}}_{\mathbf{0}} \tag{4}$$

is collected as liquid in the cryostat and the non liquefied fraction of the gas

$$\dot{m}_{co} = (1 - \epsilon) \dot{m}_{o}$$
 (5)

returns to the heat exchanger H.

Due to the parasitic heat \dot{Q}_1 (Eq. 1) parts of the liquid begin immediately to evaporate.

If now \dot{Q}_r is the cooling capacity of the refrigerator, one will have to choose $\dot{Q}_r \ge \dot{Q}_1$ in order to maintain the operation of the system. Without any other heat load the condition $\dot{Q}_r > \dot{Q}_1$ would result in an



Fig. 4. Excess refrigeration power and heat load of a magnet refrigerator system Legend: 1: $\hat{Q}_e = (\hat{Q}_r)_{max} - \boldsymbol{\omega} \cdot \hat{\mathbf{m}}_w$; 2: $\hat{Q}_e = (\hat{Q}_r)_{max} - \boldsymbol{\omega} \cdot \hat{\mathbf{m}}_w$; 3: $\hat{Q}_e = (\hat{Q}_r)_{max} - \boldsymbol{\omega} \cdot \hat{\mathbf{m}}_w - \hat{Q}_o$; 4: $\hat{Q}_1 = \hat{Q}_o$; 5: $\hat{Q}_1 = \hat{Q}_o + \hat{Q}_L (\hat{\mathbf{m}}_w)$

increase of the liquid level in the cryostat. Thermodynamically this is not a stationary working condition of the system because one has to supply fresh gas at the rate of the liquefaction. In order to establish stationary conditions one can introduce an additional amount of heat \hat{Q}_e so that

$$\dot{Q}_r = Q_1 + \dot{Q}_e \tag{6}$$

 \dot{Q}_e can be provided by an electrical heater immersed in the liquid. On the other hand - and this is more interesting - \dot{Q}_e can be the heat extracted from another cryogenic system to be cooled. In other words, \dot{Q}_e is the excess cooling power which is available in addition to the consumption \dot{Q}_1 of the magnet. Of course we are interested to have \dot{Q}_e at its maximum.

As we have seen before \hat{Q}_L can be reduced by cooling the current leads at a certain cooling gas rate \hat{m}_w . This vapor is warmed up and returns directly to the low pressure side of the compressor. The maximum cold gas rate being available in the system is the sum of Eq. 5 and of the vapor produced by the heat inputs of \dot{Q}_0 , \dot{Q}_L and \dot{Q}_e , i.e.

$$(\dot{\mathbf{m}}_{c})_{max} = (1-\epsilon) \dot{\mathbf{m}}_{o} + \frac{1}{\lambda} (\dot{\mathbf{Q}}_{o} + \dot{\mathbf{Q}}_{L} + \dot{\mathbf{Q}}_{e})$$
 (7)

If one would use the total of $(\dot{\mathbf{m}}_{c})_{max}$ for cooling the leads the refrigerator would not work any further. In general one can only withdraw a certain amount $\dot{\mathbf{m}}_{w} < (\dot{\mathbf{m}}_{c})_{max}$ from the cold return gas which then reduces to

$$\dot{\mathbf{m}}_{\mathbf{c}} = (1 - \boldsymbol{\ell}) \, \dot{\mathbf{m}}_{\mathbf{o}} + \frac{1}{\lambda} \left(\dot{\mathbf{Q}}_{\mathbf{o}} + \dot{\mathbf{Q}}_{\mathbf{L}} + \dot{\mathbf{Q}}_{\mathbf{e}} \right) - \dot{\mathbf{m}}_{\mathbf{w}} \quad (8)$$

Since the lead cooling gas rate \dot{m}_W does not return to the heat exchanger H the down stream high pressure gas is cooled to a lesser degree. Consequently the performance of the liquefier is reduced and \dot{Q}_T decreases. To maintain stable conditions we also have to reduce \dot{Q}_e . One can show (see Appendix) that \dot{Q}_e decreases with increasing \dot{m}_W as

$$\dot{\hat{Q}}_{e} = (\dot{\hat{Q}}_{r})_{max} - \dot{\hat{Q}}_{o} - \dot{\hat{Q}}_{L}(\dot{\tilde{m}}_{w}) - \alpha \dot{\tilde{m}}_{w}$$
(9)

 $(\bar{Q}_r)_{max}$ is the maximum refrigeration power of the unloaded refrigerator and d is the rate of decrease of refrigeration power with increasing gas rate warmed up in a bypass to the heat exchangers of the refrigerator.

In the case of an unloaded refrigerator we have $\dot{Q}_1 = \dot{Q}_0 + \dot{Q}_L = 0$ and Eq. 9 gives

$$\dot{Q}_{e} = \dot{Q}_{r} = (\dot{Q}_{r})_{max} - d\dot{m}_{w}$$
(10)

The plot of Eq. 10 is a straight line as indicated in Fig. 4, curve 1. A constant load $\dot{Q}_1 = \dot{Q}_0$ changes Eq. 10 to

$$\dot{\hat{Q}}_{e} = \dot{\hat{Q}}_{eo} = (\dot{\hat{Q}}_{r})_{max} - \dot{\hat{Q}}_{o} - \varkappa \dot{m}_{w}$$
 (11)

which results in a parallel shift of curve 1 in Fig. 4 to curve 2.

The total load \dot{Q}_1 of a magnet with cryostat and current leads according to Eq. 1 is plotted in Fig. 4, curve 5. The excess refrigeration power \dot{Q}_e for this case (Eq. 9) is depicted in curve 3, Fig. 4. It is the difference between curve 1 and curve 5. \dot{Q}_e is rather low for small values of \dot{m}_w because of the high \dot{Q}_L of a badly cooled conductor. With increasing \dot{m}_w we reach a maximum of \dot{Q}_e at

$$\frac{d\hat{q}_{L}(\hat{m}_{w})}{d\hat{m}_{w}} = - \alpha$$
 (9a)

After having passed this optimal value of $\dot{m}_{W} = \dot{m}_{WO}$ a further increase of the cooling rate will result in a decrease of \dot{Q}_{e} because the refrigeration power disappears faster than \dot{Q}_{I} .

For large values of \mathring{m}_w the heat load \mathring{Q}_L becomes small compared with \mathring{Q}_0 , i.e. instead of Eqs. 1 and 9 we have



Fig. 5. Principle of the refrigeration system of Pluto. Legend: K = compressor; P = precooling; H = heat exchanger; VB = valve box; D = storage dewar; C = magnet cryostat; M = superconductive magnet coil; H_b = bypass heat exchanger; V₁, V₂ = control valves; Fl 1, Fl 2 = gas flow meters; PS = power supply; Im = level indicator.

and

for

$$\dot{q}_{e} \rightarrow (\dot{q}_{r})_{max} - \dot{q}_{o} - d\dot{m}_{w} = \dot{q}_{eo}$$
 (13)
for $\dot{m}_{e} \rightarrow \infty$

which is identical with Eq. 11. For large values of \dot{m}_{w} the excess refrigeration power curve \dot{Q}_{μ} as a function of \dot{m}_{w} approaches a straight line which is parallel to the un-loaded refrigerator performance curve \dot{Q}_{μ} (\dot{m}_{w}).

From Eq. 10 and 13 we find the difference between the two curves to be equal to the static losses \dot{Q}_0 of the cryostat:

$$\dot{Q}_{r}(\dot{m}_{w}) - \dot{Q}_{e}(\dot{m}_{w}) \rightarrow \dot{Q}_{o}$$
 (14)
for $\dot{m}_{v} \rightarrow \infty$

To the authors this seems to be the only way to measure separately the static losses of a system of a cryostat and a magnet without mechanically removing the current leads.

Once having identified Eq. 11 as the equation of the asymptotic of Eq. 9 we find the heat load due to the current leads to be the



Fig. 6. Excess refrigeration power of the Pluto magnet refrigeration system as a function of the lead cooling gas rate at different currents. Legend: 1: Q_e (m_w) of the unloaded refrigerator; 2; 3; 4: Q_e (m_w) at the magnet currents I = 0, 600, 1200 A respectively; 5: asymptotic of curves 2, 3, and 4.

difference between Eqs. 11 and 9:

$$\dot{q}_{J_{\perp}}(\dot{m}_{w}) = \dot{q}_{eo}(\dot{m}_{w}) - \dot{q}_{e}(\dot{m}_{w})$$
 (15)

III. Measurements and Results

The ideas of the preceding chapter have been checked in the DESY superconductive magnet Pluto¹. The main properties of this magnet are: flux density in the center 2.2 T, cylindrical field volume 1.4 m diameter and 1.15 m length, maximum current 1300 A, cooling capacity of the refrigerator 97 W at 4.4 K. A simplified scheme of the cryogenic circuit is shown in Fig. 5.

The circuit differs from that of Fig. 3 by a separate storage dewar D in addition to the cryostat C of the magnet M. A valve box VB allows the operation of the refrigerator with or without magnet. For measuring the zero load performance \dot{Q}_r (\dot{m}_w) (Eq. 10) the valve box VB is set into position a. The refrigerator works into the dewar alone, the cold gas goes back

to the refrigerator, a fraction \dot{m}_{W} can be warmed up in a bypass heat exchanger H_b. The flow rate through H_b is adjusted by means of V₁ and can be measured at Fl 1. \tilde{Q}_{e} is established and controlled by means of an electrical power supply PS and a resistor R in the dewar. For a given flow rate \dot{m}_w the heater was set to a value \dot{Q}_{e} , at which the liquid level in the dewar remained constant over about one hour. The result of these measurements is displayed in Fig. 6, curve 1. It might be mentioned that the static load of the dewar is considered to be a property of the liquefier. \dot{Q}_r is the cooling capacity available at the outgoing transferplugs of the valve box VB.

Setting the valve box into position b the refrigerator expands into the magnet cryostat C. Here the liquid helium is collected until the level reaches the end of the cold back gas transfer line which goes into the dewar. Thus the helium level in the cryostat is held constant and the excess helium is transferred into the dewar. The cold gas is returning to the heat exchanger H. The lead cooling gas rate can be controlled by means of V 2 and measured with Fl 2. \dot{Q}_{e} (\dot{m}_{w}) (Eq. 9) is measured for magnet currents I = 0, I = 600 A and I = 1200 A. The two conductors of the magnet and a third zero current conductor of less than 10% of the area of the current leads are laminated in order to increase the thermal contact with the gas. They are surrounded by a common insulated tube. The gas flow rates as well as the heat loads \dot{Q}_{L} indicated in the diagrams and texts always represent the sum of the values of all three conductors.

According to Eqs. 11 and 13 the common asymptotic \dot{Q}_{eo} (\dot{m}_w) of the three plots parallel to \dot{Q}_r (\dot{m}_w) is indicated (curve 5, Fig. 6). Using now Eq. 14 (difference between curves 1 and 5) we find the static losses of the cryostat (including the transfer line) to be

$$\dot{Q}_{o} = 18.0 \text{ W}$$
 (16)

Applying Eq. 15 (difference between curves 5 and 2, 3 or 4 respectively) we can calculate the heat load due to the current leads for the different currents. In Fig. 7 the total heat leak $\dot{Q}_1 = \dot{Q}_0 + \dot{Q}_L(\dot{m}_W)$ is plotted against \dot{m}_W . According to Fig. 2 the straight line of $\dot{Q}_V = \lambda \dot{m}_W$ is also indicated. In the case of operation without refrigerator the intersection point \dot{m}_{WS} would be the stable evaporation rate provided all vapor produced in the cryostat is used for cooling the leads. In the diagram the values $\mathbf{\hat{m}}_{wo}$ at which we get the best performance of the refrigerator are also indicated. As one can see, they are much lower than the stable evaporation rates $\mathbf{\hat{m}}_{ws}$ in the case of dewar operation.

IV. Summary and Conclusions

The most important results are summarized in Table 1. A cryogenic system consisting of a refrigerator and a superconductive magnet with gas cooled current leads can be characterized by the excess refrigeration power \dot{Q}_e as a function of the lead cooling gas flow rate \dot{m}_w . For each value of magnet current passing the leads there exists an optimal cooling gas rate which releases a maximum of residual refrigeration power \dot{Q}_e . The losses of the cryostat and the leads can be calculated separately from the excess refrigeration power \dot{Q}_e and the performance curve of the unloaded refrigerator.

According to Eq. 9a the optimal gas cooling rate for current leads is not only defined by the lead itself but it depends also on the properties of the refrigerator.

			<u>Table_1</u>		
I	=	0	600	1200	Α
$(\dot{q}_r)_{max}$	=	96.5	96.5	96.5	W
Q .	=	18.0	18.0	18.0	W
(Qe)max	=	47.5	39.0	24.0	W
m wo	=	0.25	0.30	0.375	g/sec
$\dot{Q}_{L} (\dot{m}_{wo})$	=	7.5	11.5	20	W
m ws	=	0.91	0.905	0.90	g/sec
0 _L (m _{ws})	=	5 1	\$ 1	£ 1	W

Appendix

<u>Calculation of \dot{Q}_e as a Function of a Gas Rate</u> \dot{m}_w warmed up in the Current Leads

We consider a refrigerator circuit as shown in Fig. 3. A high pressure gas current \dot{m}_0 is expanded isenthalpically through the Joule-Thomson valve J and the insulated transfer line Tr₁ into the cryostat C. The condition for isenthalpic expansion from point 2 to point 3 in the diagram is

$$\dot{\mathbf{m}}_{0}\mathbf{h}_{2} = \dot{\mathbf{m}}_{0}\mathbf{h}_{3} \tag{A1}$$

 h_2 and h_3 are the specific enthalpies of the helium at points 2 and 3 respectively. Since

after the expansion a fraction of the gas is liquefied we have, with the specific enthalpy of the liquid h_f and the heat of evaporation h

$$h_{3} = \boldsymbol{\varepsilon} h_{f} + (1 - \boldsymbol{\varepsilon}) (h_{f} + \boldsymbol{\lambda})$$
(A2)

and from Eqs. A1 and A2

$$\dot{\mathbf{m}}_{0}\mathbf{h}_{2} = \dot{\mathbf{m}}_{0}\boldsymbol{\varepsilon}\mathbf{h}_{f} + \dot{\mathbf{m}}_{0} (1-\boldsymbol{\varepsilon}) (\mathbf{h}_{f} + \boldsymbol{\lambda})$$
(A3)

For the heat exchanger H (which is considered to have 100 per cent heat exchange performance) we find

$$\dot{m}_{o} (h_{1}-h_{2}) = \dot{m}_{c} (h_{5}-h_{4})$$
 (A4)

with the index numbers of the enthalpies

corresponding to the pressures and temperatures at the points with the same numbers and with m_c to be the flow rate of cold gas coming back from the cryostat through the transfer line Trg.

In the normal operating region of the liquefier the cold gas has practically the same temperature as the liquid so that

$$h_4 = h_f + \lambda \tag{A5}$$

Using now Eq. 8, A3, A4 and A5 we get

For the condition of stationary equilibrium the total heat flux $\dot{Q}_0 + \dot{Q}_L + \dot{Q}_e$ entering the system at the constant temperature of the boiling liquid has to be exactly equal to the heat of evaporation of the liquid being produced, i.e.

$$\dot{Q}_{o} + \dot{Q}_{L} + \dot{Q}_{e} = \lambda \epsilon \dot{m}_{o}$$
 (A8)

Introducing Eq. A8 into Eq. A7 we finally find

$$\dot{q}_{e} = \dot{m}_{o} (h_{5} - h_{1}) - \dot{Q}_{o} - \dot{Q}_{L} - \dot{m}_{w} (h_{5} - h_{f} - \dot{\lambda})$$
 (A9)

which we can write

$$\dot{\varrho}_{e} = (\dot{\varrho}_{r})_{max} - \dot{\varrho}_{o} - \dot{\varrho}_{L} - \mathcal{A}_{m}$$
(A10)

with

$$(\dot{0}_{r})_{max} = \dot{m}_{o} (h_{5}-h_{1})$$
 (A11)

$$\mathbf{d} = \mathbf{h}_5 - \mathbf{h}_f - \mathbf{\lambda} \tag{A12}$$

If \hat{Q}_1 is a function of \dot{m}_w too, Eq. A9 is identical with Eq. 9.

List of Symbols and Dimensions

- φ₁ total parasitic heat flux entering the system at liquid helium temperature T_{He} ά_L parasitic heat flux due to the W current leads alone φ° parasitic heat flux into cryostat W and transfer lines due to heat conduction, radiation and residual gas in the insulation vacuum heat flux necessary to evaporate liquid at a rate \dot{m}_w
- $\dot{q}_{r}(\dot{m}_{w})$ W cooling capacity of the refrigerator without cryostat

- (Q_r)_{max} W maximum refrigeration power of the unloaded refrigerator
 - W excess refrigeration power of the system refrigerator magnet
 - W excess refrigeration power of a refrigerator with a constant load 0₀
 - g/sec flow rate of high pressure gas passing the Joule-Thomson valve
 - g/sec flow rate of high pressure gas expanded in engines or turbines in order to precool the main gas stream m_o
- \dot{m}_{f} g/sec rate of liquefied gas

Q_e

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$$\dot{m}_{co}$$
 g/sec rate of nonliquefied gas at
T = T_{He}

$$m_c$$
 g/sec rate of cold gas returning to
the refrigerator at T =T_{He}

 $(\dot{m}_c)_{max}$ g/sec maximum rate of cold gas being available in the cryostat

- m_{wo} g/sec optimal cooling gas rate at which \dot{Q}_e has its maximum
 - J/g heat of evaporation
 - fraction of \mathbf{m}_{0} which is liquefied after having passed the JT-valve
- d J/g rate of decrease of refrigeration power with increasing \dot{m}_{u}
 - A electrical current through the leads
 - K temperature
 - K temperature of liquid helium boiling at about atmospheric pressure
- Troom K room temperature
- h J/g specific enthalpy of the gas
- h_f J/g specific enthalpy of the liquid

Reference

 C.P.Parsch, E.Bartosch, F.Böhm, G.Horlitz, G.Knust, S.Wolff, A.Stephan, D.Hauck, The 1.4 m/2.2 Tesla Superconducting Detector Magnet Pluto for the Electron Positron Storage Ring at DESY, Proc. 4th Int. Conf. on Magnet Technology, Brookhaven, 1972.



Fig. 7. Heat load of Pluto \dot{Q}_1 (\dot{m}_w) = \dot{Q}_0 + \dot{Q}_L (\dot{m}_w) as a function of the cooling gas rate in the current leads. Legend: 1, 2, 3: \dot{Q}_1 at I = 0, 600, 1200 A respectively; 4: $\dot{Q}_1 = \dot{Q}_0$; 5: $\dot{Q}_1 = \dot{Q}_v = \dot{M}\dot{m}_w$; \dot{m}_{WS} = stable cooling gas rate for operation without refrigerator; \dot{m}_{W01} , \dot{m}_{W02} , \dot{m}_{W03} = optimal gas cooling rates for operation with refrigerator at magnet currents I = 0, 600, 1200 A respectively.