Three-nucleon forces effects in the electron scattering off ${}^{4}\text{He}$

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Abstract. We report on our study of the inclusive electron scattering off ⁴He, where emphasis on the role of three-nucleon forces is given. A detailed analysis of the longitudinal response function $R_L(\omega, q)$ is done at different kinematics and theoretical results are compared to available experimental data. Calculation are performed with ab-initio techniques where the full fourbody continuum dynamics is considered via the Lorentz integral transform method. At lower momentum transfer ($q \leq 200 \text{ MeV/c}$) three-nucleon forces play an important role and the two three-nucleon force models implemented show differences up to 10%.

1. Introduction

Recently a lot of attention has been payed to the importance of multi-nucleon forces and in particular of the three-nucleon force (3NF). The nuclear potential has clearly an *effective* nature, therefore it is in principle a many-body operator. For the study of three-body potentials or to discriminate among different models one needs to find $A \geq 3$ observables that show sensitivity to 3NFs. An important activity in this direction has taken place in the last years, with accurate calculations of bound-state properties of nuclei of increasing mass number A [1,2] and, more recently, with further studies in the many-body regime (e.g. [3,4]), where 3NF are incorporated in a certain level of approximation. Clearly, also reaction observables may potentially show sensitivity to three-nucleon forces. Theoretical results on hadronic scattering observables involving four nucleons [5,6] and five nucleons [7] have already shown that three-body effects are rather large. We follow a complementary approach and direct our attention towards electromagnetic reactions in the continuum. Many years of electron scattering experiments have demonstrated the power of electro-nuclear reactions, and in particular of the inelastic ones, in providing important information on nuclear dynamics. In our recent work we studied the inelastic inclusive electron scattering off ⁴He and found regions where the searched three-nucleon effects are sizable [8,9,10]. Here we would like to summarize our main findings and put them under a different light to help motivating future theoretical and experimental work.

In the one-photon-exchange approximation, the inclusive cross section for electron scattering

off a nucleus is given in terms of two response functions, i.e.

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{q^4} R_L(\omega, q) + \left(\frac{Q^2}{2q^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, q) \right]$$
(1)

where σ_M denotes the Mott cross section, $Q^2 = -q_{\mu}^2 = q^2 - \omega^2$ the squared four momentum transfer with ω and **q** as energy and three-momentum transfers, respectively, and θ the electron scattering angle. The longitudinal and transverse response functions, $R_L(\omega, q)$ and $R_T(\omega, q)$, are determined by the transition matrix elements of the Fourier transforms of the charge and the transverse current density operators. The longitudinal and transverse responses are given by

$$R_L(\omega,q) = \sum_f |\langle \Psi_f | \hat{\rho}(\mathbf{q}) | \Psi_0 \rangle|^2 \delta\left(E_f + \frac{q^2}{2M} - E_0 - \omega \right), \qquad (2)$$

and

$$R_T(\omega,q) = \oint_f |\langle \Psi_f | \hat{J}_T(\mathbf{q}) | \Psi_0 \rangle|^2 \delta \left(E_f + \frac{q^2}{2M} - E_0 - \omega \right), \qquad (3)$$

where M is the target mass, $|\Psi_{0/f}\rangle$ and $E_{0/f}$ denote initial and final state wave functions and energies, respectively. The δ -function ensures energy conservation.

Due to the low atomic number of ⁴He it is possible to study the longitudinal and the transverse responses separately, without the ambiguities created by the Coulomb distortions affecting heavier systems. Experimentally a Rosenbluth separation is typically carried out. From the theoretical point of view we concentrated our attention to the longitudinal response function, since in a non relativistic approach no meson exchange currents are involved. In fact, the charge density operator $\hat{\rho}$ is defined as

$$\hat{\rho}(\mathbf{q}) = \frac{e}{2} \sum_{k} \left(1 + \tau_k^3 \right) \exp\left[i \mathbf{q} \cdot \mathbf{r}_k \right],\tag{4}$$

where e is the proton electric charge, \mathbf{r}_k and τ_k^3 are the k-th nucleon position and the third isospin component. The coordinates are given with respect to the center of mass.

From Eq. (2) it is evident that in principle one needs the knowledge of all possible final states excited by the electromagnetic probe, including of course states in the continuum. Thus, in a straightforward evaluation one would have to calculate both bound and continuum states. The latter constitute the major obstacle for a many-body system, since the full many-body scattering wave functions are not yet accessible for A > 3. To circumvent this problem we make use of the Lorentz Integral Transform (LIT) method [11,12], which allows us to access the response function even beyond the three-body disintegration threshold by solving the bound state equation

$$(\hat{H} - E_0 - \sigma) |\tilde{\Psi}_{\sigma,q}\rangle = \hat{O}(q) |\Psi_0\rangle, \qquad (5)$$

where $\hat{O}(q)$ is the excitation operator, in this case $\hat{\rho}$, and σ is the complex parameter of the LIT (see Ref. [12]). In Eq. (5) \hat{H} denotes the nuclear Hamiltonian

$$\hat{H} = \hat{T} + \hat{V}_{NN} + \hat{V}_{3N}, \tag{6}$$

which is the same used to calculate the ground state $|\Psi_0\rangle$ and includes two-nucleon forces \hat{V}_{NN} augmented by three-nucleon forces \hat{V}_{3N} . The bound-state techniques that we use for solving Eq. (5) is the effective interaction hyperspherical harmonics (EIHH) method [13,14]. The use of the LIT in conjunction with the EIHH method enables us to obtain an exact calculation of the longitudinal response function.



Figure 1. Longitudinal response function for q = 200 and 100 MeV/c with the AV18 (dashed), AV18+UIX (solid) and MT (dashed-dotted) potentials. Data at $q \simeq 200 \text{ MeV}/c$ from [19].

2. Results

A great advantage of electron scattering is the possibility to vary independently the energy ω and the momentum q transferred by the electron to the nucleus. This allows one to focus on different dynamical aspects. Here, we concentrate on studying the effect of the three-nucleon forces in different kinematical regions for different constant values of q and varying ω . To this purpose we calculate R_L by using either the Hamiltonian in Eq. (6) or the one where we switch off the three-nucleon forces ($V_{3N} = 0$). Our calculations are then compared with the available experimental data.

We start by presenting the case of R_L at q = 200 and 100 MeV/c in Fig. 1. Calculations are performed using the realistic two-body potential AV18 [15] and the three-nucleon force UIX [16]. Also, we show a curve where a simpler semirealistic two-body potential, the Malfliet-Tjon [17] (MT), was used [18]. We do not present the very low threshold theoretical results, since in our present calculation we are not able to resolve the narrow monopole resonance situated there. It is very interesting to compare the theoretical curves in the quasi-elastic peak. The kinematics at q = 200 MeV/c is compared also to data from [19]. As already shown in [8] one has a large quenching effect due to the 3NF, which is strongest at lower q. One should notice that such an effect is not simply correlated to the under-binding of the AV18 potential (binding energy $E_B = 24.35$ MeV). In fact, if this was the case, the results with the MT potential, which gives a slight over-binding of ⁴He ($E_B = 30.56$ MeV), would lay even below those obtained with AV18+UIX ($E_B = 28.40$). For q = 200 MeV/c the experimental error bars are pretty large, but seem to favor the calculation with two-body forces only.

Given the large 3NF effect at lower q it is interesting to see whether there is a dependence of the results on the 3NF model itself. To this end we have performed the calculation using also the Tucson Melbourne (TM') [20] three-nucleon force. While the UIX force contains a two-pion exchange and a short range phenomenological term, with two 3NF parameters fitted to the triton binding energy and on nuclear matter density (in conjunction with the AV18 twonucleon potential), the TM' force is not adjusted in this way. It includes two pion exchange terms where the coupling constants are taken from pion-nucleon scattering data consistently with chiral symmetry. Our results with the TM' force are obtained using the same model space as for the UIX potential. The cutoff of the TM' force has been adjusted on triton binding energy, when used in conjunction with the AV18 NN force (see [9] for more details). It is worth noticing that the ⁴He binding energy with the AV18+TM' potential is practically the same $(E_0 = 28.46 \text{ MeV})$ as for the AV18+UIX case. Figure 2 shows that the increase of 3NF effects



Figure 2. Longitudinal response function for q = 200, 150, 100 and 50 MeV/c with the AV18 two-nucleon force only (dashed), and with the addition of the UIX (solid) or the TM' (dashed-dotted) three-nucleon force.

with decreasing q is confirmed by the TM' force as well. Moreover it becomes evident that also the difference between the results obtained with two 3NF models increases with decreasing q. One actually finds that the shift of the peak to higher energies in the case of UIX generates for R_L a difference up to about 10% on the left hand sides of the peaks. This is a very interesting result and it represents the first case of an electromagnetic observable considerably dependent on the choice of the 3NF. The difference between the AV18+UIX and AV18+TM' curves give also an idea of the accuracy of the experimental data that is needed in order to discriminate among different 3NF models.

In light of these results it would be very interesting to repeat the calculation with EFT twoand three-body potentials [21,22]. If precise measurements of R_L were available at low q, one could think of using them to fix the low-energy constants (LEC) of the effective field theory 3NF. To the best of our knowledge, no data have been published yet for the kinematics at q = 50,100and 150 MeV/c. New measurements have been taken in Mainz at MAMI [23] and the analysis is ongoing.

In Fig. 3, we show some results obtained for larger q [8,9] in comparison with existing experimental data from Bates [24] and Saclay [25] and Carlson *et al.* [26]. One sees that the 3NF results are closer to the data, this is particularly evident at q = 350 MeV/c. However, the 3NF effect is generally not as large as for the lower momentum transfers. In some cases the quenching of the strength due to the 3NF is comparable to the size of the error bars, particularly for the data from Bates.

In order to motivate future work both in theory and experiment, we would like to present a comparison of our semirealistic calculation of [18] with experimental data from [27] in Fig. 4.



Figure 3. Longitudinal response function for q = 350 and 300 MeV/c: calculations with AV18 (dashed) and AV18+UIX (solid). Data from [24] (squares), [25] (circles), [26] (triangles down).



Figure 4. R_{θ} function as in Eq.(7) for $q \approx 200 \text{ MeV/c:}$ calculated with the MT potential in comparison with data from Buki *et al.* [27].

Buki and collaborators in [27] measured the quantity

$$R_{\theta} = \frac{d^2\sigma}{d\Omega d\omega} \Big/ \sigma_M \tag{7}$$

at $\theta = 160^{\circ}$ and constant $q = 1 \text{ fm}^{-1} (\approx 200 \text{ MeV/c})$. Due to the kinematical factors in front of $R_{L/T}$ in the cross section of Eq. (1), for this specific choice of θ and q one has the ratio $R_L/R_{\theta} = 1/15$, so that R_{θ} is dominated by the transverse response. In Ref. [18] we had calculated R_L and R_T with the MT potential. Here we present a comparison of the resulting R_{θ} with experimental data, which we did not show in [18]. It is interesting to note that the semirealistic calculation seems in pretty good agreement with the measurements. The calculation includes the effect of consistent meson exchange current, which was though found to amount only to 2% in such simplified potential and current model (see [18] for details). More realistic calculations, previously done with the Laplace transform [28], had instead found a 20% effect of meson exchange current at higher momentum transfer values ($q \sim 300 \text{ MeV/c}$). In light of these facts, the agreement between theory and experiment found in Fig. 4 seems quite accidental. We therefore think that both more precise experimental measurements and more realistic calculations are needed at such a low q value and would be very interesting to clarify the picture and study the role of both three-nucleon forces and exchange currents. We plan to calculate R_T with meson exchange currents and realistic two and three-body potentials in the near future.

2.1. Acknowledgments

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