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GRAVITATIONAL FORM FACTORS AND INTERNAL FORCES IN HADRONS

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Abstract

Matrix elements of the energy momentum tensor (EMT) bear fundamental information like mass, spin and D-term of a particle which is the "last unknown global property." Recent progress on EMT form factors of hadrons, their interpretations and applications as well as the experimental status is given.

1 Introduction

Matrix elements of the EMT [1] yield fundamental particle properties like mass and spin as well as the D-term [2] which is related to the stress-tensor components of the EMT and gives access to mechanical properties of the system [3, 4]. EMT form factors can be accessed through studies of generalized parton distributions (GPDs) in hard exclusive reactions [5, 6]. While a model-independent extraction of GPDs is a challenging long-term task, accessing information on the D-term may be possible sooner thanks to its relation to the subtraction constant in fixed-t dispersion relations in deeply virtual Compton scattering (DVCS) [3, 7]. The physics of EMT form factors has important applications. The purpose of this article is to provide an overview of the latest developments and experimental status.

2 EMT form factors of hadrons

The nucleon form factors of the symmetric EMT $\hat{T}_{\mu\nu} = \hat{T}^Q_{\mu\nu} + \hat{T}^G_{\mu\nu}$ are defined as

$$\langle p', s'|\hat{T}_{\mu\nu}(0)|p, s\rangle = \bar{u}'\left[A(t)\,\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4m} + D(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m}\right]u\,, \tag{1}$$

with $P=\frac{1}{2}(p'+p)$, $\Delta=p'-p$, $t=\Delta^2$, $a_{\{\mu}b_{\nu\}}=a_\mu b_\nu+a_\nu b_\mu$ and a covariant normalization of the states is used with the nucleon spinors $\bar{u}(p,s)\,u(p,s)=2\,m$. Spin-0 hadrons like the pion have only the 2 total EMT form factors $A(t)$ and $D(t)$. Hadrons with spin $J\geq 1$ have more form factors [8, 9].

The quark and gluon QCD operators $\hat{T}^a_{\mu\nu}$ $(a=Q,\,G)$ are each gauge invariant. Their form factors $A^a(t,\mu)$, etc depend on renormalization scale μ and additional form factors appear, e.g. as the structure $m\,\bar{c}^a(t,\mu)\,g_{\mu\nu}$ in (1), with $\sum_a \bar{c}^a(t,\mu)=0$.

3 Relation to GPDs and 2D interpretation

GPDs provide a practical way to access EMT form factors through the DVCS process $eN\to e'N'\gamma$ or hard exclusive meson production. For the nucleon the second Mellin moments of unpolarized GPDs yield

$$\int \mathrm{d}x\; x\,H^a(x,\xi,t)=A^a(t)+\xi^2 D^a(t),\quad \int \mathrm{d}x\; x\,E^a(x,\xi,t)=B^a(t)-\xi^2 D^a(t). \tag{2}$$

The Fourier transform $H^a(x,b_\perp)=\int d^2\Delta_\perp/(2\pi)^2\,e^{-i\vec{\Delta}_\perp\vec{b}_\perp}\,H^a(x,\xi,-\vec{\Delta}_\perp^2)|_{\xi=0}$ is the probability to find a parton carrying the momentum fraction x and located at the distance $b_\perp$ from the hadron's (transverse) center-of-mass on the lightcone [10]. The 2D interpretation of EMT form factors was also discussed [11].

4 The static EMT and 3D interpretation

In the Breit frame characterized by $P=(E,0,0,0)$ and $\Delta=(0,\vec{\Delta})$ with $t=-\vec{\Delta}^2$ and $E=\sqrt{m^2+\vec{\Delta}^2/4}$ one can define the static EMT [3]

$$T^a_{\mu\nu}(\vec{r},\vec{s})=\int\frac{d^3\vec{\Delta}}{(2\pi)^3 2E}\;e^{-ir\vec{\Delta}}\langle p'|\hat{T}^a_{\mu\nu}(0)|p\rangle, \tag{3}$$

where $\vec{s}$ is the polarization vector of the states $|p\rangle$, $|p'\rangle$ in the respective rest frames. The 00-component of (3) is the energy density which only can be defined for the total system, and yields $\int d^3r\,T_{00}(r)=m$. Decomposition of the nucleon mass in terms of quark and gluon contributions was discussed in [12]. The $0k$-components yield the spatial distribution of the nucleon spin density $J^i_a(\vec{r},\vec{s})=\epsilon^{ijk}r^j T^{0k}_a(\vec{r},\vec{s})$. This 3D density has a monopole term [3], and a quadrupole term [13] which are related to each other [14]. The ij-components of (3) define the stress tensor which can be decomposed in contributions from shear forces $s(r)$ and pressure $p(r)$ as follows [3]

$$T^{ij}(\vec{r})=\left(\frac{r^ir^j}{r^2}-\frac{1}{3}\,\delta^{ij}\right)s(r)+\delta^{ij}\,p(r)\,. \tag{4}$$

For the nucleon the 3D interpretation is subject to small relativistic corrections [15] and becomes exact in the large-N_c limit. The shear forces can be defined separately for quarks and gluons in terms of $D^{Q,G}(t)$. For the "partial" pressures from quarks and gluons one also needs $\bar{c}^{Q,G}(t,\mu)$.

EMT conservation relates $s(r)$ and $p(r)$ as $\frac{2}{3}\,s'(r)+\frac{2}{r}\,s(r)+p'(r)=0$. Notice that $s(r)=0$ would imply $p(r)=$ constant and isotropic matter, cf. (4). Thus, the shear forces are responsible for structure formation [11]. Another consequence of EMT conservation is the von Laue condition [17],

$$\int_0^\infty \mathrm{d}r\,r^2 p(r)=0, \tag{5}$$

implying that $p(r)$ must have at least one node. In all model studies so far $p(r)$ was found positive in the inner region (repulsion towards outside) and negative in the outer region (attraction towards inside).

The D-term can be expressed in two equivalent ways as $D = -\frac{4}{15}\, m \int d^3r\; r^2\, s(r) = m \int d^3r\; r^2\, p(r)$. The stress tensor in (4) has two eigenvalues related to normal (dF_r) and tangential (dF_ϕ, dF_θ) forces

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r), \quad \frac{dF_\theta}{dS_\theta} = \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r) \tag{6}$$

with eigenvectors $\vec{e}_r$ and $\vec{e}_{\theta,\phi}$. The degeneracy is lifted for spin $J \geq 1$. In a stable system the normal force $dF_r/dS_r = \frac{2}{3}s(r) + p(r) > 0$. Otherwise the system would collapse. This mechanical stability requirement can be written as $\int_0^R dr\, r^2 p(r) > 0$ (for any R), thus complementing the von Laue condition (5). It also determines the D-term of a stable system to be negative [18], $D < 0$. The positivity of $\frac{2}{3}s(r) + p(r)$ allows us to define the mechanical radius [19]

$$\langle r^2 \rangle_{\rm mech} = \frac{\int d^3r\; r^2\; \left[\frac{2}{3}s(r) + p(r)\right]}{\int d^3r\; \left[\frac{2}{3}s(r) + p(r)\right]} = \frac{6D}{\int_{-\infty}^0 dt\; D(t)}\,. \tag{7}$$

Interestingly the mechanical radius is given by an "anti-derivative" of $D(t)$ at $t = 0$ unlike e.g. the proton mean charge square radius $\langle r^2 \rangle_{\rm charge} = 6G'_E(0)/G_E(0)$ given in terms of the electric form factor $G_E(t)$.

One can also consider forces in lower-dimensional subsystems [4]. The 2D pressure $p^{(2D)}(r) = -\frac{1}{3}s(r) + p(r)$ satisfies $\int_0^\infty dr\, r\, p^{(2D)}(r) = 0$ and corresponds to the tangential forces in (6). Similarly the 1D pressure $p^{(1D)}(r) = -\frac{4}{3}s(r) + p(r)$ satisfies $\int_0^\infty dr\, p^{(1D)}(r) = 0$. Generically, for a spherically symmetric mechanical system in nD one can express its pressure and shear forces in terms of pressure in kD spherical subsystem as follows:

$$p^{(nD)}(r) = \frac{k}{n}\; p^{(kD)}(r) + \frac{k(n-k)}{n}\;\frac{1}{r^k}\int_0^r dr'\; r'^{\,k-1} p^{(kD)}(r'), \tag{8}$$

$$s^{(nD)}(r) = -\frac{k}{n-1}\; p^{(kD)}(r) + \frac{k^2}{n-1}\;\frac{1}{r^k}\int_0^r dr'\; r'^{\,k-1} p^{(kD)}(r'). \tag{9}$$

Such relations can be useful, e.g. in holographic approaches to QCD. The concepts can be generalized to higher spins [9]. The energy density and pressure in the center of a hadron are given by [4]

$$T_{00}(0) = \frac{m}{4\pi^2}\int_{-\infty}^0 dt\; \sqrt{-t}\; \left[A(t) - \frac{t}{4m^2}D(t)\right], \quad p(0) = \frac{1}{24\pi^2 m}\int_{-\infty}^0 dt\; \sqrt{-t}\;\; tD(t). \tag{10}$$

5 The *D*-term in theory and experiment

In contrast to the constraints $A(0) = 1$ and $B(0) = 0$ resulting from properties of the states under Lorentz transformations [20], the form factor $D(t)$ is not constrained (not even at $t = 0$) by general principles. It is not related to external properties like Lorentz transformations but reflects internal dynamics inside the hadron. The value $D = D(0)$ is therefore not known for (nearly) any particle.

In free field theories one finds $D = -1$ for free Klein Gordon fields [1, 15], and $D = 0$ for free Dirac fields [21]. For Goldstone bosons of spontaneous chiral symmetry breaking it is predicted in the chiral limit that $D_{\rm Goldstone} = -1$ from soft pion theorems for EMT form factors [22] or pion GPDs [2]. Corrections due to finite masses are expected to be small for pions and larger for kaons and η [23, 15].

For large nuclei in the liquid drop model [3] $p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R)$ and $s(r) = \gamma\delta(r-R)$ with nucleus radius $R = R_0 A^{1/3}$ and surface tension γ related by the Kelvin relation $p_0 = 2\gamma/R$ [24]. It is predicted $\langle r^2 \rangle_{\rm mech} = \frac{3}{5}R^2$ and $D = -\frac{4}{5}\, m\, \gamma\, \frac{4\pi}{3}\, R^4 \propto A^{7/3}$ which is supported in Walecka model [25].

The D-term of the nucleon was studied in the chiral quark soliton model [26] which predicts $D \approx -3.5$ and $\langle r^2 \rangle_{\rm mech} \approx 0.75\, \langle r^2 \rangle_{\rm charge}$. Studies were also reported in Skyrme models including nuclear

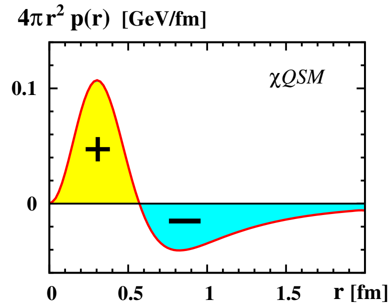

Figure 1: Pressure in chiral quark soliton (χQSM) [26] and realization of the von Laue condition (5).

medium corrections [27], bag model [28], a Nambu–Jona-Lasinio diquark approach [29], using dispersion relations [30], chiral perturbation theory [31], lattice QCD [32], and QCD lightcone sum rules [33]. D-terms of mesons [34], Q-balls [35], photons [36] and Δ-resonance [18] were also studied.

A first extraction of the quark contribution to the pion D-term from the BELLE data [37] on $\gamma^*\gamma \to 2\pi^0$ gave [38] $D^Q(0) \approx -0.75$ with unestimated uncertainties. For the D-term of the nucleon phenomenological fits indicate that $D^Q < 0$ with large uncertainties [39]. The D-term can be accessed in DVCS with help of fixed-t dispersion relations [3, 7] which relate the real and imaginary parts of the complex DVCS Compton form factors with a subtraction constant $\Delta(t,\mu)$ related to $D^Q(t,\mu) = \frac{2}{5}\,\Delta(t,\mu)/(e_u^2+e_d^2) = \frac{18}{25}\,\Delta(t,\mu)$ under certain assumptions (large-N_c limit, $\mu \to \infty$). An analysis of the JLab data [40] performed under such assumptions and additional constraints gave a first insight on $\Delta(t,\mu)$ of the nucleon [41]. Relaxing these assumptions and constraints at the current stage yields much larger uncertainties [42] though the method in principle works.

6 Applications and Conclusions

The EMT form factors have important applications including hard exclusive reactions, the description of hadrons in strong gravitational fields, hadronic decays of heavy quarkonia [22], and the description of exotic hadrons with hidden charm as hadroquarkonia [43, 18].

Unlike the EMT form factors $A(t)$ and $B(t)$ related to the generators of the Poincaré group and ultimately to the mass and spin of a particle, the form factor $D(t)$ is related to the internal forces and opens a new window for studies of the hadron structure and visualization of internal hadronic forces.

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