

RARE SEMILEPTONIC $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ DECAY WITHIN QCD SUM RULES

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In this article, the three-point QCD sum rules approach is used to investigate the rare $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay. The form factors relevant to this decay are calculated, considering the gluon condensate corrections to the correlation function. The total decay width of this decay is also evaluated. The predictions can be confirmed by the experimental data in future.

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1. Introduction

In particle physics, the flavorless mesons containing a heavy b or c quark and its own antiquark are called quarkonia. Quarkonia, especially the bottomonium $b\bar{b}$ states do not contain light quarks, so they are considered as approximately non-relativistic systems. They are used for the investigation of the hadronic dynamics and perturbative as well as non-perturbative characteristics of the QCD. Previous studies investigated the nature of these states by theoretical calculations, mainly based on potential models or their extensions such as the Coulomb gauge model [1]. In 2010, Lucha *et al.* [2] demonstrated that the potential models as well as QCD sum rules approach can be used to obtain the ground state decay constants of the mesons containing heavy b quark. It has been revealed that applying the QCD sum rules method, and tuning the continuum threshold parameter, results in a more accurate and reliable determination of the bound-state characteristics, compared to the potential models.

In order to test the P -wave spin-spin (or hyperfine) interaction, the spin-singlet P -wave bound states of $b\bar{b}$ including $h_b(1p)$ could be applied. Thus, it would be beneficial to calculate the physical parameters of this meson theoretically, and compare the results to the experimental data [3].

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Many attempts have been made to study the semileptonic decay of heavy mesons in literature, such as semileptonic decay of the scalar, pseudoscalar and the vector mesons, using three-point QCD sum rules [1, 4–12].

This research has concentrated on the investigation of the semileptonic decay of the axial vector P -wave bottomonium $h_b(1p)$ meson with the quantum numbers $J^{PC} = 1^{+-}$ into the pseudoscalar B_s meson. We calculate the two gluon condensates as the first non-perturbative contribution to the correlation function, because the heavy quark condensates are suppressed by the inverse powers of the heavy quark mass. In 2011, Belle Collaboration have observed the $h_b(1p)$ spin-singlet bottomonium state produced in $e^+e^- \rightarrow h_b(1p)\pi^+\pi^-$ reaction, with significances of 5.5σ . It was discovered that $m_{h_b(1p)} = 9898.3 \pm 1.1_{-1.1}^{+1.0}$ MeV [13].

The outline of this paper is as follows. In Sec. 2, we calculate the sum rules for the related form factors, considering the two gluon condensates contributions to the correlation function. The light quark condensates contributions are eliminated by applying double Borel transformations with respect to the momentum of the initial and final states. Section 3 contains numerical analysis of the form factors and the estimation of the decay width. The last section discusses the conclusions.

2. $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ form factors using QCD sum rules

In the Standard Model, $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay is demonstrated by $b \rightarrow s \nu \bar{\nu}$ at quark level (Fig. 1). Considering Z penguin and box diagrams, the effective Hamiltonian for $b \rightarrow s \nu \bar{\nu}$ decay can be written as

$$H_{\text{eff}} = \frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* C_{10} \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu, \quad (1)$$

G_F stands for the Fermi constant, α is the fine structure constant at the Z mass scale, V_{tb} and V_{ts} are elements of the CKM matrix, and C_{10} is the Wilson coefficient. The amplitude of the $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay is obtained by sandwiching Eq. (1) between the initial and final meson states

$$M = \frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* C_{10} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \langle B_s(p') | \bar{s} \gamma_\mu (1 - \gamma_5) b | h_b(1p)(p, \varepsilon) \rangle. \quad (2)$$

We should calculate the matrix element $\langle B_s(p') | \bar{s} \gamma_\mu (1 - \gamma_5) b | h_b(1p)(p, \varepsilon) \rangle$ in Eq. (2). This matrix element is parameterized in terms of the form factors as follows:

$$\langle B_s(p') | \bar{s} \gamma_\mu (1 - \gamma_5) b | h_b(1p)(p, \varepsilon) \rangle = -\epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta \frac{2V(q^2)}{m_{B_s} + m_{h_b(1p)}}$$

$$\begin{aligned}
& + i \left[\varepsilon_\mu^* (m_{B_s} + m_{h_b(1p)}) A_1(q^2) - (\varepsilon^* q) P_\mu \frac{A_2(q^2)}{m_{B_s} + m_{h_b(1p)}} \right. \\
& \left. - (\varepsilon^* q) \frac{2m_{B_s}}{q^2} [A_3(q^2) - A_0(q^2)] q_\mu \right] \quad (3)
\end{aligned}$$

in which $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$, and ε is the polarization vector of the axial vector meson. In order to have finite results at $q^2 = 0$, we should have $A_3(0) = A_0(0)$. The form factor $A_3(q^2)$ can be written as a linear combination of A_1 and A_2 as follows:

$$A_3(q^2) = \frac{m_{h_b(1p)} + m_{B_s}}{2m_{B_s}} A_1(q^2) - \frac{m_{h_b(1p)} - m_{B_s}}{2m_{B_s}} A_2(q^2). \quad (4)$$

Considering the three-point QCD sum rules, the form factors V , A_1 and A_2 can be evaluated from the following correlation function

$$\Pi_{\mu\nu} = i^2 \int d^4x d^4y e^{-ipx} e^{i\bar{p}y} \langle 0 | T \left\{ J_{B_s}(y) J_\mu(0) J_{h_b(1p)}^\dagger(x) \right\} | 0 \rangle. \quad (5)$$

In Eq. (5), $J_{h_b(1p)}(x) = \bar{b}\gamma_\nu\gamma_5 b$ and $J_{B_s}(y) = \bar{b}\gamma_5 s$ are the interpolating currents of the axial vector and pseudoscalar mesons. $J_\mu(0) = \bar{s}\gamma_\mu(1 - \gamma_5)b$ is the transition current. We calculate the above correlation function in two approaches: first, in hadron language which results in phenomenological or physical part and then, the QCD or theoretical approach, obtained in the quark gluon language. We find the sum rules expressions for the form factors by equating the corresponding coefficients of the two sides. In order to eliminate the contributions of higher states and continuum, we use double Borel transformation with respect to p and p' . A complete set of intermediate states with the same quantum numbers is inserted in Eq. (5) to get the physical part. Therefore, we find the following result

$$\begin{aligned}
& \Pi_{\mu\nu}(p^2, p'^2, q^2) \\
& = \frac{\langle 0 | J_{B_s} | B_s(p') \rangle \langle B_s(p') | J_\mu(0) | h_b(1p) \rangle \langle h_b(1p) | J_{\nu, h_b(1p)}^\dagger | 0 \rangle}{(p'^2 - m_{B_s}^2)(p^2 - m_{h_b(1p)}^2)} + \dots \quad (6)
\end{aligned}$$

In the above equation, $\dots$ stand for the contributions of higher states and continuum. We write the matrix elements in the above equation in terms of the leptonic decay constants of B_s and $h_b(1p)$ mesons in the following way

$$\langle 0 | J_{B_s} | B_s(p') \rangle = \frac{i f_{B_s} m_{B_s}^2}{m_b + m_s}, \quad \langle 0 | J_{\nu, h_b(1p)} | h_b(1p)(p) \rangle = f_{h_b(1p)} m_{h_b(1p)} \varepsilon_\nu. \quad (7)$$

Equations (3) and (7) are substituted into Eq. (6) and the summation is performed over the polarization of $h_b(1p)$ meson, resulting in the following relation for the physical part

$$\begin{aligned}
 \Pi_{\mu\nu}(p^2, p'^2, q^2) = & \frac{f_{B_s} f_{h_b(1p)} m_{B_s}^2 m_{h_b(1p)}}{(m_b + m_s) (p'^2 - m_{B_s}^2) (p^2 - m_{h_b(1p)}^2)} \\
 & \times \left\{ i \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \frac{2V(q^2)}{m_{B_s} + m_{h_b(1p)}} - (m_{B_s} + m_{h_b(1p)}) \right. \\
 & \times \left(-g_{\mu\nu} + \frac{(P+q)_\mu (P+q)_\nu}{4m_{h_b(1p)}^2} \right) A_1(q^2) + \frac{1}{m_{B_s} + m_{h_b(1p)}} \\
 & \times P_\mu \left(-q_\nu + \frac{pq(P+q)_\nu}{2m_{h_b(1p)}^2} \right) A_2(q^2) + \frac{2m_{B_s}}{q^2} q_\mu \\
 & \left. \times \left(-q_\nu + \frac{pq(P+q)_\nu}{2m_{h_b(1p)}^2} \right) [A_3(q^2) - A_0(q^2)] \right\}. \quad (8)
 \end{aligned}$$

To extract the form factors V, A_1 and A_2 , we need the coefficients of the Lorentz structures $i\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta$, $g_{\mu\nu}$ and $P_\mu q_\nu$. Using the Lorentz structures, the correlation function can be written as follows:

$$\Pi_{\mu\nu}(p^2, p'^2, q^2) = \Pi_V i \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + \Pi_{A_1} g_{\mu\nu} + \Pi_{A_2} P_\mu q_\nu. \quad (9)$$

The three-point correlator is evaluated by the help of the operator product expansion method (OPE) in the deep Euclidean region $p^2 \ll 4m_b^2$, $p'^2 \ll (m_b^2 + m_s^2)$, to calculate the QCD part of the correlation function. For this purpose, each Π_i function can be written in terms of the perturbative and non-perturbative parts as follows:

$$\Pi_i(p^2, p'^2, q^2) = \Pi_i^{\text{per}}(p^2, p'^2, q^2) + \Pi_i^{\text{non-per}}(p^2, p'^2, q^2), \quad (10)$$

where i stands for V, A_1 and A_2 . We consider the bare loop diagram (Fig. 1, (a)) for the perturbative part. Diagrams (b), (c), (d) in Fig. 1, which shows the light quark condensates contributing to the correlation function, are not considered because the double Borel transformations eliminate their part, so only the gluon condensate diagrams are taken into account as the first non-perturbative part (Fig. 2 (a)–(f)). The bare-loop contribution by double dispersion representation is written as

$$\Pi_i^{\text{per}} = -\frac{1}{(2\pi)^2} \int ds' \int ds \frac{\rho_i^{\text{per}}(s, s', q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}. \quad (11)$$

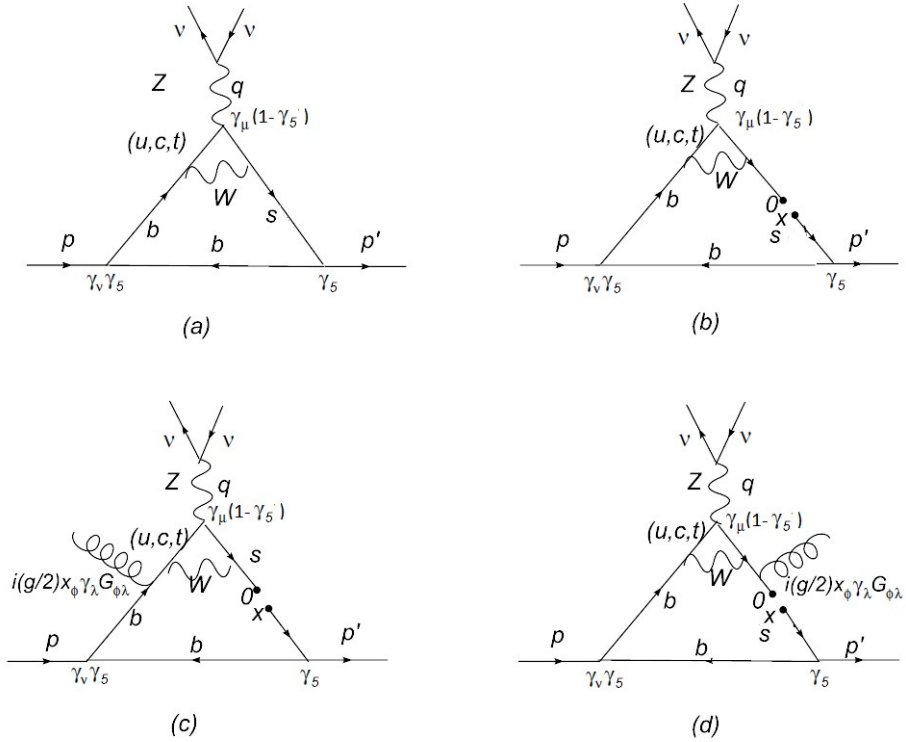


Fig. 1. (a) Bare loop; (b) Light quark condensates; (c), (d) Light quark condensates with one gluon emission for $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay.

In the Gukovskiy rules, the quark propagators are replaced by the Dirac function, i.e., $\frac{1}{p^2 - m^2} \rightarrow -2\pi i \delta(p^2 - m^2)$. This condition results in the inequality, such as

$$-1 \leq \frac{2ss' + (s + s' - q^2)(-s) + 2s(m_b^2 - m_s^2)}{\lambda^{1/2}(s, s', q^2) \lambda^{1/2}(m_b^2, m_s^2, s)} \leq 1. \quad (12)$$

Here, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The lower limit of s' is $(m_b + m_s)^2$, inserting this into Eq. (12), we obtain s_L for the lower limit of s . Performing calculations, the following expressions for the spectral densities are found:

$$\begin{aligned} \rho_V(s, s', q^2) &= 4N_c((m_s - m_b)B_2 - 2m_b B_1 - m_b I_0), \\ \rho_{A_1}(s, s', q^2) &= 2N_c(4(-m_s + m_b)A_1 + 2m_b \Delta' I_0 + (-m_s + m_b)I_0 \Delta \\ &\quad + 2m_b^2(-2m_b + m_s)I_0 - m_b(u - 2m_b m_s)I_0), \\ \rho_{A_2}(s, s', q^2) &= 2N_c((-m_s + m_b)(-A_5 + A_2) - m_b B_2 - m_s B_1). \end{aligned} \quad (13)$$

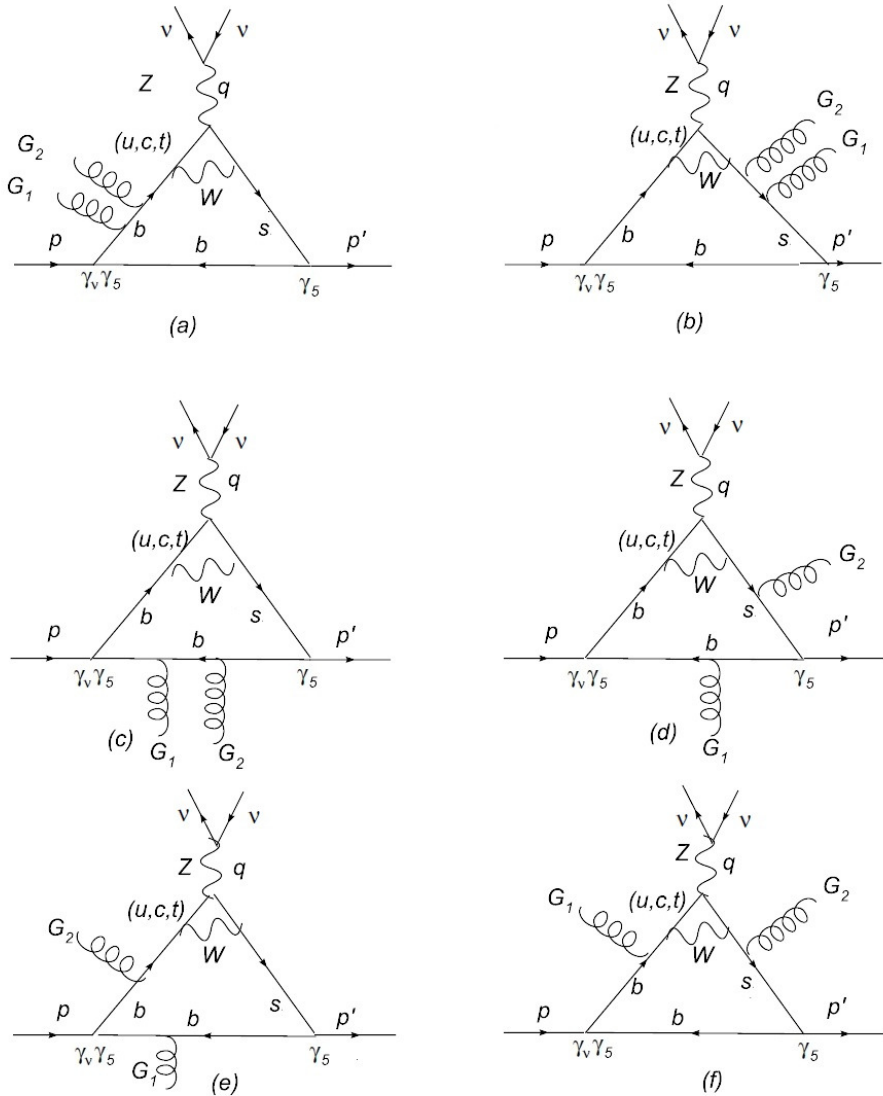


Fig. 2. Gluon condensate contributions to $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay.

Here, $u = s + s' - q^2$, $\Delta = s$, $\Delta' = s' + m_b^2 - m_s^2$, and $N_c = 3$ is the number of colors. B_1 , B_2 , A_1 , A_2 , A_5 and I_0 are as follows:

$$I_0(s, s', q^2) = \frac{1}{4\lambda^{1/2}(s, s', q^2)},$$

$$\lambda(s, s', q^2) = s^2 + s'^2 + q^4 - 2sq^2 - 2s'q^2 - 2ss',$$

$$B_1 = \frac{1}{4\lambda^{3/2}}(2s'\Delta - \Delta'u),$$

$$\begin{aligned}
 B_2 &= \frac{1}{4\lambda^{3/2}} (2s\Delta' - \Delta u) , \\
 A_1 &= \frac{1}{8\lambda^{3/2}} \left(\Delta'^2 s + \Delta^2 s' - 4m_b^2 ss' - \Delta\Delta' u + m_b^2 u^2 \right) , \\
 A_2 &= \frac{1}{4\lambda^{5/2}} \left(2\Delta'^2 ss' + 6\Delta^2 s'^2 - 8m_b^2 ss'^2 - 6\Delta\Delta' s'u + \Delta'^2 u^2 + 2m_b^2 s'u^2 \right) , \\
 A_5 &= \frac{1}{4\lambda^{5/2}} \left(-6\Delta\Delta' s'u + 6s^2 \Delta'^2 - 8s^2 s' m_b^2 + 2u^2 s m_b^2 + u^2 \Delta^2 + 2ss' \Delta^2 \right) .
 \end{aligned} \tag{14}$$

In the evaluation of gluon condensate contributions as the first correction to the non-perturbative part of the correlator, we encounter integrals which are discussed next [4, 8, 10, 14]. We use the Fock–Schwinger fixed-point gauge, $x^\mu G_\mu^a = 0$, where G_μ^a is the gluon field [8, 15–17]. Integrals, such as given in the following, are needed to calculate these diagrams:

$$I_0[a, b, c] = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[k^2 - m_b^2]^a [(p+k)^2 - m_b^2]^b [(p'+k)^2 - m_s^2]^c} , \tag{15}$$

$$I_\mu[a, b, c] = \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_b^2]^a [(p+k)^2 - m_b^2]^b [(p'+k)^2 - m_s^2]^c} , \tag{16}$$

$$I_{\mu\nu}[a, b, c] = \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2 - m_b^2]^a [(p+k)^2 - m_b^2]^b [(p'+k)^2 - m_s^2]^c} . \tag{17}$$

The Schwinger representation is used for propagators as follows:

$$\frac{1}{p^2 + m^2} = \frac{1}{\Gamma(\alpha)} \int_0^\infty d\alpha \alpha^{n-1} e^{-\alpha(p^2 + m^2)} . \tag{18}$$

This representation proves to be very convenient for applying the Borel transformation as

$$\hat{B}_p (M^2) e^{-\alpha p^2} = \delta (1 - \alpha M^2) . \tag{19}$$

Solving integrals and applying double Borel transformations over p^2 and p'^2 , the transformed form of the integrals are as follows:

$$\begin{aligned}
 \hat{I}_0(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{2-a-c} \\
 &\times U_0(a+b+c-4, 1-c-b) ,
 \end{aligned}$$

$$\begin{aligned}
\hat{I}_\mu(a, b, c) &= \frac{1}{2} \left[\hat{I}_1(a, b, c) + \hat{I}_2(a, b, c) \right] P_\mu + \frac{1}{2} \left[\hat{I}_1(a, b, c) - \hat{I}_2(a, b, c) \right] q_\mu, \\
\hat{I}_{\mu\nu}(a, b, c) &= \hat{I}_6(a, b, c) g_{\mu\nu} + \frac{1}{4} \left(2\hat{I}_4 + \hat{I}_3 + \hat{I}_5 \right) P_\mu P_\nu + \frac{1}{4} \left(-\hat{I}_5 + \hat{I}_3 \right) P_\mu q_\nu \\
&\quad + \frac{1}{4} \left(-\hat{I}_5 + \hat{I}_3 \right) P_\nu q_\mu + \frac{1}{4} \left(-2\hat{I}_4 + \hat{I}_3 + \hat{I}_5 \right) q_\mu q_\nu, \quad (20)
\end{aligned}$$

knowing that

$$\begin{aligned}
\hat{I}_1(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{3-a-c} \\
&\quad \times U_0(a+b+c-5, 1-c-b), \\
\hat{I}_2(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{2-a-c} \\
&\quad \times U_0(a+b+c-5, 1-c-b), \\
\hat{I}_3(a, b, c) &= i \frac{(-1)^{a+b+c}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{4-a-c} \\
&\quad \times U_0(a+b+c-6, 1-c-b), \\
\hat{I}_4(a, b, c) &= i \frac{(-1)^{a+b+c}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \\
&\quad \times U_0(a+b+c-6, 1-c-b), \\
\hat{I}_5(a, b, c) &= i \frac{(-1)^{a+b+c}}{16 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{4-a-b} (M_2^2)^{2-a-c} \\
&\quad \times U_0(a+b+c-6, 1-c-b), \\
\hat{I}_6(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{32 \pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \\
&\quad \times U_0(a+b+c-6, 2-c-b). \quad (21)
\end{aligned}$$

In Eqs. (20) and (21), M_1^2 and M_2^2 are the Borel parameters in the s and s' channels, respectively, and the function $U_0(\alpha, \beta)$ is defined as follows:

$$U_0(\alpha, \beta) = \int_0^\infty dy (y + M_1^2 + M_2^2)^\alpha y^\beta \exp \left[-\frac{B_{-1}}{y} - B_0 - B_1 y \right], \quad (22)$$

$$B_{-1} = \frac{1}{M_1^2 M_2^2} [m_s^2 M_1^4 + m_s^2 M_2^4 + M_1^2 M_2^2 (m_b^2 + m_s^2 - q^2)], \quad (23)$$

$$B_0 = \frac{1}{M_1^2 M_2^2} [(m_s^2 + m_b^2) M_1^2 + 2m_b^2 M_2^2], \quad (24)$$

$$B_1 = \frac{m_b^2}{M_1^2 M_2^2}. \quad (25)$$

The following expressions are used for gluon condensate contributions

$$\Pi_i^{\langle G^2 \rangle} = i \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_i}{24}. \quad (26)$$

Expressions for C_i are shown in Appendix A. To find the form factors, we apply double Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_2^2)$ on the physical side and QCD side of the correlation function. Then, we match the coefficient of the Lorentz structures of these two representations of the correlator, and perform continuum subtraction to suppress the higher states and continuum. The following sum rules for the form factors A_1 and A_2 are calculated as follows:

$$\begin{aligned} V &= \frac{(m_b + m_s)(m_{h_b(1p)} + m_{B_s})}{2f_{B_s} f_{h_b(1p)} m_{B_s}^2 m_{h_b(1p)}} e^{\frac{m_{B_s}^2}{M_2^2}} e^{\frac{m_{h_b(1p)}^2}{M_1^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_s)^2}^{s'_0} ds' \int_{s_L}^{s_0} ds \rho_V(s, s', q^2) e^{-\frac{s'}{M_2^2}} e^{-\frac{s}{M_1^2}} + i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_V}{24} \right\}, \\ A_1 &= \frac{(m_b + m_s)}{f_{B_s} f_{h_b(1p)} m_{B_s}^2 m_{h_b(1p)} (m_{h_b(1p)} + m_{B_s})} e^{\frac{m_{B_s}^2}{M_2^2}} e^{\frac{m_{h_b(1p)}^2}{M_1^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_s)^2}^{s'_0} ds' \int_{s_L}^{s_0} ds \rho_{A_1}(s, s', q^2) e^{-\frac{s'}{M_2^2}} e^{-\frac{s}{M_1^2}} + i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{A_1}}{24} \right\}, \\ A_2 &= \frac{4m_{h_b(1p)}(m_b + m_s)(m_{h_b(1p)} + m_{B_s})}{f_{B_s} f_{h_b(1p)} m_{B_s}^2 (3m_{h_b(1p)}^2 + m_{B_s}^2 - q^2)} e^{\frac{m_{B_s}^2}{M_2^2}} e^{\frac{m_{h_b(1p)}^2}{M_1^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_s)^2}^{s'_0} ds' \int_{s_L}^{s_0} ds \rho_{A_2}(s, s', q^2) e^{-\frac{s'}{M_2^2}} e^{-\frac{s}{M_1^2}} + i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{A_2}}{24} \right\}. \end{aligned} \quad (27)$$

In the above relations, s_0 and s'_0 are the continuum thresholds in $h_b(1p)$ and B_s channels, respectively, and s_L is as follows:

$$s_L = \frac{m_b^2 (q^2 - s')^2}{(q^2 - m_b^2) (m_b^2 - s')}. \quad (28)$$

To subtract the contributions of the higher states and continuum in Eq. (27), the quark-hadron duality assumption is considered as follows:

$$\rho^{\text{higher states}}(s, s') = \rho^{\text{OPE}}(s, s') \Theta(s - s_0) \Theta(s - s'_0). \quad (29)$$

We also used the following form of Borel transformation

$$\hat{B}_p(M^2) \left\{ \frac{1}{p^2 - m^2} \right\} = \frac{1}{M^2} e^{-m^2/M^2}. \quad (30)$$

In the following, we obtain the differential decay width $d\Gamma/dq^2$ for the process $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ in terms of the form factors:

$$\frac{d\Gamma}{dq^2 d\cos\theta} = \frac{\sqrt{\lambda}}{256 \pi^3 m_{h_b(1p)}^3} |M|^2, \quad (31)$$

$$M = \frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* C_{10} L^\mu H_\mu, \quad (32)$$

$$|M|^2 = \frac{G_F^2 \alpha^2}{8 \pi^2} |V_{tb} V_{ts}^*|^2 C_{10}^2 L^{\mu\nu} H_\mu H_\nu^\dagger, \quad (33)$$

$$\begin{aligned} L^{\mu\nu} H_\mu H_\nu^\dagger = & \frac{1}{3} \left\{ \frac{(12m_{h_b(1p)}^2 q^2 + \lambda \sin^2 \theta) (m_{h_b(1p)} + m_{B_s})^2}{m_{h_b(1p)}^2} A_1^2 - \frac{2}{m_{h_b(1p)}^2} \right. \\ & \times \left(-m_{B_s}^2 + m_{h_b(1p)}^2 + q^2 \right) \lambda \sin^2 \theta A_1 A_2 + \frac{1}{m_{h_b(1p)}^2 (m_{B_s} + m_{h_b(1p)})^2} \\ & \left. \times \lambda^2 \sin^2 \theta A_2^2 + 16\sqrt{\lambda} q^2 \cos \theta A_1 V \right\}, \end{aligned} \quad (34)$$

where $\lambda = m_{h_b(1p)}^4 + m_{B_s}^4 + q^4 - 2m_{h_b(1p)}^2 m_{B_s}^2 - 2m_{h_b(1p)}^2 q^2 - 2m_{B_s}^2 q^2$. The total decay width is obtained from the integration of Eq. (31) on q^2 in the interval $0 < q^2 < (m_{h_b(1p)} - m_{B_s})^2$.

3. Numerical calculations and results

This section contains our numerical results of the form factors. The input parameters entering our calculations namely, gluon condensate, Wilson coefficient C_{10} , elements of the CKM matrix V_{tb} , V_{ts} , leptonic decay constants, $f_{h_b(1p)}$ and f_{B_s} , quark and meson masses, continuum thresholds s_0 and s'_0 , as well as the Borel parameters M_1^2 and M_2^2 are chosen to be: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4$ [18], $C_{10} = -4.669$ [19, 20], $|V_{tb}| = 0.77_{-0.24}^{+0.18}$, $|V_{ts}| = (40.6 \pm$

$2.7) \times 10^{-3}$ [21], $f_{B_s} = 232.5 \pm 18.6_{(\text{OPE})} \pm 2.4_{\text{syst}}$ MeV [22], $f_{h_b(1p)} = 0.86 \pm 0.01 \pm 0.11 \pm 0.26$ GeV [23], $m_b = 4.6$ GeV [4], $m_s(1 \text{ GeV}) \simeq 142$ MeV [24], $m_{B_s} = (5366.3 \pm 0.6)$ MeV [25], $m_{h_b(1p)} = (9898.3 \pm 1.1_{-1.1}^{+1.0})$ MeV, $m_{h_b(2p)} = (10259.8 \pm 0.6_{-1.0}^{+1.4})$ MeV [13]. Form factors contain four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum thresholds, s_0 and s'_0 . These are mathematical objects, so the physical quantities, form factors, should not depend upon them. The parameters s_0 and s'_0 , the continuum thresholds of $h_b(1p)$ and B_s mesons are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 region. The value of the continuum thresholds s'_0 calculated from the two-point QCD sum rules is taken to be $s'_0 = (34.2 \pm 2)$ GeV² [26] and the interval of s_0 must be between the mass square of $h_b(1p)$ and $h_b(2p)$ *i.e.*, $m^2[h_b(1p)] < s_0 < m^2[h_b(2p)]$ [3]. We choose the interval $98 \text{ GeV}^2 \leq s_0 \leq 105 \text{ GeV}^2$ for s_0 . We should choose the regions for M_1^2 and M_2^2 , considering that contributions of the higher states and continuum are effectively suppressed and the gluon condensate contributions are small, assuring that the contributions of the higher dimensional operators are small. Both conditions are satisfied in the regions $5 \text{ GeV}^2 \leq M_1^2 \leq 15 \text{ GeV}^2$ and $4 \text{ GeV}^2 \leq M_2^2 \leq 10 \text{ GeV}^2$. The values of the form factors at $q^2 = 0$ are shown in Table I.

TABLE I

The values of the form factors at $q^2 = 0$, for $M_1^2 = 10 \text{ GeV}^2$, $M_2^2 = 7 \text{ GeV}^2$.

	$h_b(1p) \rightarrow B_s \nu \bar{\nu}$
$V(0)$	0.015
$A_1(0)$	-0.010
$A_2(0)$	0.145

Since the sum rules for the form factors are truncated at some points, in order to extend our calculations to the full physical range, *i.e.*, the region $0 \leq q^2 \leq 20.54 \text{ GeV}^2$, we should use proper parametrization for the form factors. Our numerical calculations show that the best parametrization of the form factors with respect to q^2 is as follows:

$$f_i(q^2) = \frac{a}{\left(1 - \frac{q^2}{m_{\text{fit}}^2}\right)} + \frac{b}{\left(1 - \frac{q^2}{m_{\text{fit}}^2}\right)^2}. \quad (35)$$

The values of the parameters, a and b are given in Table II.

Performing the integration over q^2 in Eq. (31) in the interval $0 < q^2 < (m_{h_b(1p)} - m_{B_s})^2$, we obtain the expression for the total decay width. The numerical value of the decay width is presented in Table III.

TABLE II

Parameters in the fit function of the form factors, for $M_1^2 = 10 \text{ GeV}^2$, $M_2^2 = 7 \text{ GeV}^2$.

	a	b	m_{fit}
V	-0.506	0.478	7.5
A_1	-0.137	0.125	7.5
A_2	-30.8	31.33	4.6

TABLE III

Total decay width of $h_b(1p) \rightarrow B_s \nu \bar{\nu}$.

	$\Gamma \text{ [GeV]}$
$h_b(1p) \rightarrow B_s \nu \bar{\nu}$	2.82×10^{-15}

4. Conclusion

In this work, we investigated the rare $h_b(1p) \rightarrow B_s \nu \bar{\nu}$ decay within the framework of the three-point QCD sum rules. We considered the gluon corrections to the correlation function as a first non-perturbative contribution and derived the form factors. Considering the calculated values, we used proper parametrization for the form factors to calculate the total decay width. The present predictions can be confirmed by the experimental data in the future.

Appendix A

$$\begin{aligned}
 C_V = & 16m_b \hat{I}_0(1, 2, 2) - 96m_b^3 \hat{I}_0(1, 4, 1) + 64m_b^3 \hat{I}_0(2, 3, 1) + 16m_b \hat{I}_0(3, 1, 1) \\
 & - 16m_b^3 \hat{I}_0(3, 1, 2) - 16m_b \hat{I}_0^{[0,1]}(3, 1, 2) - 16m_b \hat{I}_0^{[1,0]}(3, 2, 1) \\
 & - 32m_b^3 \hat{I}_0^{[0,1]}(3, 2, 2) - 16m_b^3 \hat{I}_0^{[1,0]}(3, 2, 2) + 16m_b \hat{I}_0^{[1,1]}(3, 2, 2) \\
 & + 32m_b \hat{I}_1(1, 2, 2) - 96m_b \hat{I}_1(1, 3, 1) - 192m_b^3 \hat{I}_1(1, 4, 1) - 32m_b \hat{I}_1(2, 2, 1) \\
 & + 32m_b \hat{I}_1^{[1,0]}(2, 3, 1) + 16m_b \hat{I}_1(3, 1, 1) - 48m_b^3 \hat{I}_1(3, 1, 2) \\
 & - 16m_b \hat{I}_1^{[0,1]}(3, 1, 2) - 32m_b \hat{I}_1^{[1,0]}(3, 2, 1) - 64m_b^3 \hat{I}_1^{[0,1]}(3, 2, 2) \\
 & - 32m_b^3 \hat{I}_1^{[1,0]}(3, 2, 2) + 32m_b \hat{I}_1^{[1,1]}(3, 2, 2) + 16m_b \hat{I}_2(1, 2, 2) \\
 & - 96m_b^3 \hat{I}_2(1, 4, 1) + 64m_b \hat{I}_2^{[0,1]}(2, 3, 1) - 16m_b^3 \hat{I}_2(3, 1, 2) \\
 & - 16m_b \hat{I}_2^{[0,1]}(3, 1, 2) - 64m_b^3 \hat{I}_2(3, 2, 1) - 32m_b^3 \hat{I}_2^{[0,1]}(3, 2, 2) \\
 & - 16m_b^3 \hat{I}_2^{[1,0]}(3, 2, 2) + 16m_b \hat{I}_2^{[1,1]}(3, 2, 2),
 \end{aligned}$$

$$\begin{aligned}
C_{A_1} = & -8m_b \hat{I}_0(1, 1, 2) - 16m_b \hat{I}_0(1, 2, 1) - 16m_b^3 \hat{I}_0(1, 2, 2) \\
& + 8m_b \hat{I}_0^{[0,1]}(1, 2, 2) + 96m_b^3 \hat{I}_0(1, 3, 1) - 48m_b \hat{I}_0^{[0,1]}(1, 3, 1) \\
& + 96m_b^5 \hat{I}_0(1, 4, 1) - 48m_b^3 \hat{I}_0^{[0,1]}(1, 4, 1) + 16m_b^3 \hat{I}_0(2, 2, 1) \\
& + 16m_b \hat{I}_0^{[0,1]}(2, 2, 1) + 32m_b^3 \hat{I}_0^{[0,1]}(2, 3, 1) + 16m_b^3 \hat{I}_0^{[1,0]}(2, 3, 1) \\
& - 16m_b \hat{I}_0^{[1,1]}(2, 3, 1) + 8m_b \hat{I}_0^{[0,1]}(3, 1, 1) - 8m_b \hat{I}_0^{[1,0]}(3, 1, 1) \\
& + 24m_b^5 \hat{I}_0(3, 1, 2) + 8m_b^3 \hat{I}_0^{[0,1]}(3, 1, 2) + 8m_b^3 \hat{I}_0^{[1,0]}(3, 1, 2) \\
& - 8m_b \hat{I}_0^{[1,1]}(3, 1, 2) + 32m_b^3 \hat{I}_0^{[0,1]}(3, 2, 1) - 24m_b \hat{I}_0^{[1,1]}(3, 2, 1) \\
& + 8m_b \hat{I}_0^{[2,0]}(3, 2, 1) + 32m_b^5 \hat{I}_0^{[0,1]}(3, 2, 2) - 16m_b^3 \hat{I}_0^{[0,2]}(3, 2, 2) \\
& + 16m_b^5 \hat{I}_0^{[1,0]}(3, 2, 2) - 24m_b^3 \hat{I}_0^{[1,1]}(3, 2, 2) + 8m_b \hat{I}_0^{[1,2]}(3, 2, 2) \\
& + 32m_b \hat{I}_6(1, 2, 2) - 192m_b^3 \hat{I}_6(1, 4, 1) - 64m_b \hat{I}_6^{[0,1]}(2, 3, 1) \\
& - 64m_b \hat{I}_6^{[1,0]}(2, 3, 1) - 64m_b^3 \hat{I}_6^{[0,1]}(3, 2, 2) - 32m_b^3 \hat{I}_6^{[1,0]}(3, 2, 2) \\
& + 32m_b \hat{I}_6^{[1,1]}(3, 2, 2), \\
C_{A_2} = & -16m_b^3 \hat{I}_0(3, 1, 2) + 16m_b^3 \hat{I}_0(3, 2, 1) - 4m_b \hat{I}_1(1, 2, 2) \\
& + 24m_b \hat{I}_1(1, 3, 1) + 24m_b^3 \hat{I}_1(1, 4, 1) - 8m_b \hat{I}_1(2, 2, 1) \\
& - 16m_b^3 \hat{I}_1(2, 3, 1) + 8m_b \hat{I}_1^{[1,0]}(2, 3, 1) - 4m_b \hat{I}_1(3, 1, 1) \\
& - 8m_b^3 \hat{I}_1(3, 1, 2) + 16m_b^3 \hat{I}_1(3, 2, 1) + 8m_b \hat{I}_1^{[1,0]}(3, 2, 1) \\
& + 8m_b^3 \hat{I}_1^{[0,1]}(3, 2, 2) + 4m_b^3 \hat{I}_1^{[1,0]}(3, 2, 2) - 4m_b \hat{I}_1^{[1,1]}(3, 2, 2) \\
& - 4m_b \hat{I}_2(1, 2, 2) + 24m_b \hat{I}_2(1, 3, 1) + 24m_b^3 \hat{I}_2(1, 4, 1) - 8m_b \hat{I}_2(2, 2, 1) \\
& - 16m_b^3 \hat{I}_2(2, 3, 1) + 8m_b \hat{I}_2^{[1,0]}(2, 3, 1) - 4m_b \hat{I}_2(3, 1, 1) - 8m_b^3 \hat{I}_2(3, 1, 2) \\
& + 16m_b^3 \hat{I}_2(3, 2, 1) + 8m_b \hat{I}_2^{[1,0]}(3, 2, 1) + 8m_b^3 \hat{I}_2^{[0,1]}(3, 2, 2) \\
& + 4m_b^3 \hat{I}_2^{[1,0]}(3, 2, 2) - 4m_b \hat{I}_2^{[1,1]}(3, 2, 2) + 8m_b \hat{I}_3(1, 2, 2) \\
& - 48m_b^3 \hat{I}_3(1, 4, 1) - 16m_b \hat{I}_3^{[0,1]}(2, 3, 1) - 16m_b \hat{I}_3^{[1,0]}(2, 3, 1) \\
& - 16m_b^3 \hat{I}_3^{[0,1]}(3, 2, 2) - 8m_b^3 \hat{I}_3^{[1,0]}(3, 2, 2) + 8m_b \hat{I}_3^{[1,1]}(3, 2, 2) \\
& - 8m_b \hat{I}_5(1, 2, 2) + 48m_b^3 \hat{I}_5(1, 4, 1) + 16m_b \hat{I}_5^{[0,1]}(2, 3, 1) \\
& + 16m_b \hat{I}_5^{[1,0]}(2, 3, 1) + 16m_b^3 \hat{I}_5^{[0,1]}(3, 2, 2) + 8m_b^3 \hat{I}_5^{[1,0]}(3, 2, 2) \\
& - 8m_b \hat{I}_5^{[1,1]}(3, 2, 2),
\end{aligned}$$

where

$$\hat{I}_n^{[i,j]}(a, b, c) = (M_1^2)^i (M_2^2)^j \frac{d^i}{d(M_1^2)^i} \frac{d^j}{d(M_2^2)^j} \left[(M_1^2)^i (M_2^2)^j \hat{I}_n(a, b, c) \right].$$

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