# The Materialization of the Superghosts


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**Abstract**

An **off-shell BV description** (with materialized superghosts ) of 10-dimensional Super-Yang-Mills is presented. This construction involves 8 auxiliary scalar fields and 8 superghosts, taking values in a certain manifold, having non-trivial topology. Both the manifold and the BV action are manifestly symmetric w.r.t the SO(8) rotations. Integrating out one of these auxiliary fields, say $G_8$, one obtains the action and supersymmetry transformations suggested in [3] with the remaining SO(7) group of symmetry.

On the other hand, recently, a paper [2], claiming the possibility to obtain the on-shell BV action of 10-*d* SYM, coupled to superghosts, from the certain «Fundamental Theory», has appeared. We show that integrating out the remaining 7 auxiliary fields, one obtains precisely this on-shell BV action.


## 1 Introduction

The main technical result of the present note is that the **"off-shell BV"** action of SYM can be built using the classical action

$$S^{\rm cl} = \int d^{10}x \, {\rm Tr}\Big( -\frac{1}{2}F_{\mu\nu}^2 + i\psi\gamma^\mu D_\mu\psi \ + \ G_A^2 \ \Big) \tag{1}$$

which is obtained from the 'standard' on-shell one by addition of 8 auxiliary fields $G_A$ (index $A = 1..8$). While the symmetry part of this BV action is constructed using 8 even spinor superghosts $U_A^\alpha$, taking values in a manifold defined by

$$U^A\gamma^\mu U^B \ = \ \frac{1}{8}\delta^{AB}(U^C\gamma^\mu U_C) \tag{2}$$

This manifold is symmetric w.r.t. the $SO(8)$ rotations. The fact that these ghosts variables are not free parameters but lie on a manifold defined by (2) reflects the phenomenon which we call **the materialization of the superghosts** [1].

Remarkably, integrating out one of auxiliary fields, say $G_8$, one obtains precisely the BV action with the symmetry transformations, suggested in [3]. The ghosts variables are decomposed as

$$U_A \ \rightarrow \ \big( \, \varepsilon, \ v_i(\varepsilon) \, \big) \tag{3}$$

Here $\varepsilon^\alpha$ is conventional ghost for supersymmetry, while the functions $v_i(\varepsilon)$ $(i = 1..7)$, used to define the transformations of auxiliary fields $G_i$, are some non-linear expressions in $\varepsilon^\alpha$. This integration of the field $G_8$ breaks $SO(8)$ symmetry of the problem down to SO(7). The $SO(7)$ symmetric action is believed to describe 9 supersymmetries, closed off-shell [3].

The next observation is that integrating out the remaining 7 fields $G_i$ one can obtain the on-shell BV action of SYM, constructed in [2]. This on-shell BV action was obtained from the 'superfield-like' BV action

$$S^S = \int \mathrm{Tr}\Big( < \mathcal{P}, (Q+\Phi)\mathcal{A} > + g < \mathcal{P}, \mathcal{A}^2 > + < \mathcal{P}, \varepsilon^\alpha Q^s_\alpha \mathcal{A} > + < \mathcal{P}, \eta^\mu P^s_\mu \mathcal{A} > + i\eta^*_\mu (\varepsilon\gamma^\mu\varepsilon)\Big) \quad (4)$$

integrating out auxiliary fields and applying the $Z_2$ projection [2]. The auxiliary fields are the elements of the complement to the space of $Q$-cohomologies.

We tried to find out an explicit parametrization of the manifold defined by (2). The results are presented in the appendix.

## 2 BV description of MSG approach

The classical action (1) is invariant under two symmetries: gauge symmetry and supersymmetry. In BV language this invariance is formulated in the form that the action

$$\begin{aligned} S^{MSG} = \int d^{10}x\, \mathrm{Tr}\Big( &-\frac{1}{2}F^2_{\mu\nu} + i\psi\gamma^\mu D_\mu\psi + G^2_A - (D_\mu c)A^*_\mu + g\{\psi, c\}\psi^* - g[G^A, c]G^*_A + \\ +gccc^* + iV^A(U^A\gamma^\mu\psi)A^*_\mu &- \frac{1}{2}V^A(U^A\gamma^{\mu\nu}\psi^*)F_{\mu\nu} + G_A T^{AB}(U_B\psi^*) - i(U_A\gamma^\mu D_\mu\psi)T^{AB}G^*_B + \\ + \eta^\mu[(\psi^*\partial_\mu\psi) + A^*_\nu\partial_\mu A^\nu &+ c^*\partial_\mu c + G^*_A T^{AB}\partial_\mu G_B] + \frac{i}{8}\eta^*_\mu(U\gamma^\mu U) + \frac{i}{8}c^* A_\mu(U\gamma^\mu U)\Big) \end{aligned} \quad (5)$$

satisfies classical BV equation. The last term reflects [2] the fact that the SUSY algebra is closed only up to the gauge transformation with the parameter $(\varepsilon\gamma^\mu\varepsilon)A_\mu$. This is a common feature of off-shell formulation in a certain supergauge. Quite similar term appears in the BV action of $\mathcal{N}=1$, $d=4$ SYM in the Wess-Zumino gauge. This term disappears from the BV action if one restores all the auxiliary fields vanishing in the WZ gauge, i.e. write BV action using superfields [2].

In this action the ghosts for the supersymmetry are represented by even spinors $U^\alpha_A$. These ghosts take value in the SO(8) invariant manifold defined by (2). The geometrical meaning of this equation is orthogonality of basis elements in the space of $SO(8)$ spinors. It is straightforward to derive [1] the completeness property which can be written as

$$U^\alpha_A U^\beta_A = \frac{1}{16}(U\gamma^\mu U)\gamma^{\alpha\beta}_\mu \quad (6)$$

Equation (2) and (6) describe the ghost manifold in a manifestly $SO(8)$ invariant way. To achieve this invariance at the level of BV action one should introduce the global compensator field $V^A$, with the SO(8) invariant constraint

$$V^A V_A = 1 \quad (7)$$

and symmetric 2-tensor $T^{AB} = \delta^{AB} - V^A V^B$. These objects satisfy the following relations

$$V^A T^{AB} = 0, \qquad T^{AB}T^{BC} = T^{AC}$$

Using these properties one can check that the action (5) indeed satisfies classical BV equation. Moreover, both this action and the ghost manifold are manifestly invariant under $SO(8)$ rotations. We would like to emphasize that this action is linear in antifields and has the structure of classical BV action, hence describes **off-shell BV invariant** system. The fact that the ghosts are not free parameters but take value in the ghost manifold (2) reflect the phenomenon of materialization.

## 3 Berkovits patch

In this section we demonstrate how it is possible to break the $SO(8)$ invariance described in the previous section down to $SO(7)$. This is done integrating out auxiliary field $G_8$ on the lagrangian submanifold $G_8^* = 0$. Integration of BV action over a lagrangian submanifold results in BV action [4]. Remarkably, the classical action and supersymmetry transformations extracted from this effective BV action are exactly those suggested in [3].

For the following convenience we introduce the notations [3]

$$U^8 = \varepsilon, \qquad U^i = v_i, \qquad (i = 1...7)$$

Integrating out the field $G^8$ on the lagrangian submanifold $G_8^* = 0$ and choosing the frame $V^8 = 1$ and $V^i = 0$, one can obtain effective BV action

$$\begin{aligned}S^{SO(7)} = \int d^{10}x \, \mathrm{Tr}\Big( -\frac{1}{2}F_{\mu\nu}^2 + i\psi\gamma^\mu D_\mu\psi + G_i^2 - D^\mu c A_\mu^* + g\{\psi, c\}\psi^* - g[G^i, c]G_i^* + \\ + gccc^* + i(\varepsilon\gamma^\mu\psi)A_\mu^* - \frac{1}{2}(\varepsilon\gamma^{\mu\nu}\psi^*)F_{\mu\nu} + G_i v_i \psi^* - i v_i \gamma^\mu D_\mu \psi G_i^* + \\ + \eta^\mu[(\psi^*\partial_\mu\psi) + A_\nu^*\partial_\mu A^\nu + c^*\partial_\mu c + G_i^*\partial_\mu G_i] + i\eta_\mu^*(\varepsilon\gamma^\mu\varepsilon) + ic^* A_\mu(\varepsilon\gamma^\mu\varepsilon)\Big)\end{aligned} \tag{8}$$

The classical invariant action is given by the first three terms in the first line. The supersymmetry transformations can be extracted from the last fourth terms of the second line. They are exactly those suggested in [3].

Making decomposition (3) of equations (2) and (6), one can come to the expressions [3]

$$\begin{gathered} v^i\gamma^\mu\varepsilon = 0 \\ v^i\gamma^\mu v^k - \delta^{ik}(\varepsilon\gamma^\mu\varepsilon) = 0 \\ \sum_i v_i^\alpha v_i^\beta = \tfrac{1}{2}(\varepsilon\gamma^\mu\varepsilon)\gamma_\mu^{\alpha\beta} - \varepsilon^\alpha\varepsilon^\beta \end{gathered} \tag{9}$$

Here $v_i$ should be considered as functions depending on $\varepsilon$. These functions turn out to be non-linear. Let us see this even without direct solution of the system.

In the paper [4] it was shown that the $Q$-cohomologies can be calculated using the tower of fundamental relations. In case of 10-dimensional quadrics these relations are given by

$$\begin{aligned} f^\mu(\varepsilon) &= \varepsilon\gamma^\mu\varepsilon \\ G_\alpha^\mu(\varepsilon) &= (\varepsilon\gamma^\mu)_\alpha \\ R^{\alpha\beta} &= \varepsilon^\alpha\varepsilon^\beta - \tfrac{1}{2}(\varepsilon\gamma^\mu\varepsilon)(\gamma^\mu)^{\alpha\beta} \\ G_\alpha^\mu(\varepsilon) &= (\varepsilon\gamma^\mu)_\alpha \\ f^\mu(\varepsilon) &= \varepsilon\gamma^\mu\varepsilon \end{aligned}$$

This means that the following relations take place: $G_\alpha^\mu \, f_\mu = 0$, $R^{\alpha\beta} \, G_\beta^\mu = 0$, etc. These relations are valid without imposing pure spinor constraints $f^\mu = 0$. Appearance of 5 relations results into 6 well known representatives of cohomologies.

Be there a linear solution to the system (9), the first equation

$$v^i(\varepsilon)\gamma^\mu\varepsilon = 0$$

states that there is another relation on $G^\mu_\alpha$, which is linear in $\varepsilon^\alpha$, hence does not coincide with $R^{\alpha\beta}$. If that is true, the cohomologies of SYM would be different. Thus, there is no such linear solution.

The situation is different in $d = 4, 6$, where we do have a linear dependence $v^i(\varepsilon)$ but the structure of cohomologies for SYM is also different [5].

## 4 From the Fundamental Theory down to on-shell

This section is a recapitulation of our results [2], were we claimed the possibility to obtain the on-shell supersymmetric action of SYM from the 'superfield-like' action (4).

Starting with 16 even variables $\lambda^\alpha$ and the equal number of odd variables $\theta^\alpha$ it is possible to construct the space of polynomials $C[\lambda,\theta]$. Using 10 dimensional gamma-matrices define the pure spinor constraints $f^\mu = (\lambda\gamma^\mu\lambda)$. Let us call $C[\lambda,\theta|f^\mu]$ the space of polynomials factorized with respect to ideal spanned by $\{f^\mu\}$. The superfield $\mathcal{A}$ is build out of matrix valued component fields with polarizations, being the basis elements in $C[\lambda,\theta|f^\mu]$. The superfield $\mathcal{P}$ is an element of the dual superspace and $<\ ,\ >$ denotes the canonical pairing. In the action (4) the following notations are used

$$Q = \lambda^\alpha\frac{\partial}{\partial\theta^\alpha}, \quad \Phi = (\lambda\gamma^\mu\theta)\frac{\partial}{\partial x^\mu}, \quad \varepsilon^\alpha Q^s_\alpha = \varepsilon^\alpha\frac{\partial}{\partial\theta^\alpha} - (\varepsilon\gamma^\mu\theta)\frac{\partial}{\partial x^\mu}, \quad P^s_\mu = \frac{\partial}{\partial x^\mu}$$

The fields $\varepsilon^\alpha$ and $\eta^\mu$ are global ghosts for the supersymmetry and translations respectively, while operators $Q^s_\alpha$ and $P^s_\mu$ generate these transformations.

The physical fields in this action are cohomologies of $Q$. All other fields are auxiliary. The main statement of [2] was that if we integrate out all auxiliary fields from this action and apply a $Z_2$ projection [2] on the space of cohomologies, effective action will be nothing but on-shell supersymmetric BV action of 10-dimensional SYM theory. This action is given by

$$\begin{aligned} S^{\text{on-shell}} = \int d^{10}x\ \mathrm{Tr}\Big(&-\frac{1}{2}F^2_{\mu\nu} + i\psi\gamma^\mu D_\mu\psi - D^\mu c A^*_\mu + g\{\psi,c\}\psi^* + gcc c^* + \\ &+ i(\varepsilon\gamma^\mu\psi)A^*_\mu - \frac{1}{2}(\varepsilon\gamma^{\mu\nu}\psi^*)F_{\mu\nu} + \eta^\mu[(\psi^*\partial_\mu\psi) + A^*_\nu\partial_\mu A^\nu + c^*\partial_\mu c] + \\ &+ i\eta^*_\mu(\varepsilon\gamma^\mu\varepsilon) + ic^*A_\mu(\varepsilon\gamma_\mu\varepsilon) - \frac{1}{8}(\varepsilon\gamma^\mu\varepsilon)(\psi^*\gamma_\mu\psi^*) + \frac{1}{4}(\varepsilon\psi^*)^2\Big) \end{aligned} \tag{10}$$

This action was obtained by summation of all possible Feynman diagrams arising in the expansion of (4). The last three terms in this action reflect the appearance of corrections to the commutator of supersymmetry transformations proportional to the gauge shift with parameter $(\varepsilon\gamma_\mu\varepsilon)A_\mu$ and to the equations of motion for fermions (the last two terms). As it was explained in [2] this action satisfies BV equation and describes on-shell supersymmetric Yang-Mills theory (with control of corrections to the commutator of SUSY transformations proportional to the equations of motion and to the gauge transformation).

## 5 From Berkovits patch to on-shell

In this section we show that integrating out the remaining 7 fields $G^i$ from the action (8) one obtains precisely the action (10). To see this one should integrate out the fields $G^i$ on the lagrangian submanifold $G_i^* = 0$. This integration results in appearance of the term

$$-\frac{1}{4}(v^i\psi^*)^2$$

in the lagrangian. This term can be easily transformed to the last two terms of (10) using the last (completeness) identity of (9). This is another way to see that the action (10), found in [2], indeed describes the on-shell supersymmetric Yang-Mills theory.

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