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Transverse Lattice Construction of 2 + 1 Chern-Simons Theory

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Abstract

A transverse lattice model, with one lattice dimension and two continuum dimensions, is constructed by introducing Wess-Zumino terms into the gauged $1 + 1$ dimensional non-linear sigma model action of the link fields. Its continuum limit is the pure Chern-Simons gauge theory in $2 + 1$ dimensions. The lattice model is quantized, and some simple expectation values for Wilson loops on $M^2 \times S^1$ are evaluated. This construction provides an explicit connection between Chern-Simons theory and the gauged Wess-Zumino-Witten model.

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1. Introduction

We wish to apply the mathematical tools developed in the context of string theory directly to physics in higher dimensions. A two-dimensional model should be able to describe higher-dimensional dynamical systems with perturbatively small interactions in the transverse directions. This is the currently intractable (and therefore interesting) regime of the non-linear dynamics in physical non-abelian gauge theories. The goal of this type of application of two-dimensional models is to solve a significant subset of the higher-dimensional theory before resorting to numerical simulations.

These ideas were realized in the context of QCD by Bardeen and Pearson[1] over ten years ago, and in subsequent numerical analysis[2]. The physics of two spatial 'transverse' dimensions is described by a lattice gauge theory of link variables. The link variables are functions of two longitudinal coordinates and gauged with respect to longitudinal gauge transformations by introducing a 2-D gauge field. The partition function is given by a set of coupled 2-D gauged non-linear sigma models describing the dynamics of the link fields.

One unsatisfying aspect of their analysis is the approximation of the 2-D non-linear sigma model dynamics of link fields with the dynamics of a linear sigma model action. This is required because the solution of the 2-D non-linear sigma model[3] is not developed to the point where it can be applied in their context. We will explore the possibility of avoiding the need for this approximation by introducing a 2-D Wess-Zumino[4] term into the sigma model action, and describing higher-dimensional gauge theories in the basis of operators given by the well-studied Wess-Zumino-Witten[5] (WZW) model. In this case, the WZW currents are the linear variables which describe exactly the dynamics of the non-linear sigma model.

In this letter, the simple case of one lattice dimension and two continuum dimensions will be used to study the effect of adding Wess-Zumino terms to the link actions. We will find that, unlike the case where the regular gauged sigma model is used as the link action, the gauged sigma model action for link fields with the Wess-Zumino term can describe a three dimensional gauge theory – the pure non-abelian Chern-Simons model which was solved in the continuum by Witten[6]. In the subsequent analysis, it will become clear that the primary benefit of the lattice approach is that the connection between 2-D conformal field theory and Chern-Simons theory is made explicitly at the Lagrangian level.

2. Chern-Simons Transverse Lattice Model

In this section, we construct the local link action for the matrix-valued chiral field $U_n(x^+, x^-)$, which belongs to the fundamental representation of $SU(N)$, and which lies on the link between sites n and $n+1$. The chain of sites has lattice spacing a, and is either periodic (S^1 topology) or infinite (R^1 topology). The link fields U_n are continuous functions of the light-cone coordinates $x^{\pm} = (x^0 \pm x^1)/\sqrt{2}$, so that the 1-D lattice describes a $2+1$ dimensional field theory. The continuous 2-D manifold is specified as Minkowski space M^2, although the analysis below generalizes trivially to the 2-sphere, and (with some extra work) to arbitrary Riemann surfaces.

Consider the sigma model action with Wess-Zumino term[5],

$$I_{1,n}(U_n, \lambda, k) = \frac{1}{4\lambda^2} \int d^2x \mathrm{Tr}\ \partial_\mu U_n \partial^\mu U_n^{-1} + k_n \Gamma(U_n)\ . \tag{2.1}$$

Because the Wess-Zumino term is defined only up to $\Gamma \to \Gamma + 2\pi$, the coupling constant k_n must be an integer to make the Minkowski path integral well-defined. The action is invariant under the global symmetry $U \to AUB$, where A and B are constant (independent of $x^{\pm}$) unitary matrices. We wish to promote this to a local gauge symmetry at connecting sites,

$$\delta_G U_n = \Lambda_n(x^\mu) U_n - U_n \Lambda_{n+1}(x^\mu)\ . \tag{2.2}$$

It is useful to define the currents

$$\begin{aligned} J_{-,n} &= -\frac{2\pi}{\lambda^2}(\partial_- U_n)U_n^{-1}, \quad J_{+,n} = -\frac{2\pi}{\lambda^2} U_n^{-1}(\partial_+ U_n), \\ \tilde{J}_{+,n} &= -\frac{2\pi}{\lambda^2}(\partial_+ U_n)U_n^{-1}, \quad \tilde{J}_{-,n} = -\frac{2\pi}{\lambda^2} U_n^{-1}(\partial_- U_n)\ . \end{aligned} \tag{2.3}$$

The gauge variations of the J currents are

$$\begin{aligned} \delta_G J_{+,n} &= \frac{2\pi}{\lambda^2}\partial_+ \Lambda_{n+1} + [\Lambda_{n+1}, J_{+,n}] - \frac{2\pi}{\lambda^2} U^{-1} \partial_+ \Lambda_n U\ , \\ \delta_G J_{-,n} &= -\frac{2\pi}{\lambda^2}\partial_- \Lambda_n + [\Lambda_n, J_{-,n}] + \frac{2\pi}{\lambda^2} U \partial_+ \Lambda_{n+1} U^{-1}\ , \end{aligned} \tag{2.4}$$

where the currents J are specified with respect to site n. The variations of the $\tilde{J}$ currents are similar. For the critical coupling $\lambda^2 = \frac{4\pi}{k_n}$, k_n positive, the $J_{\pm}$ currents are the 'right' conformal dimension one operators which generate the Kac-Moody algebra of the WZW model, and the $\tilde{J}_{\pm}$ currents are the 'wrong' currents with conformal dimension greater than one[7]. For the cases with opposite parity, $\lambda^2 = -\frac{4\pi}{k_n}$, k_n negative, the 'right' and 'wrong' labels are reversed.

These definitions are useful because the gauge variation of the action (2.1) is linear in the currents,

$$\begin{aligned}\delta_G I_{1,n} =& \frac{-1}{2\pi}\int d^2x \mathrm{Tr}\Big\{\Big(\frac{1}{2}+\frac{k_n\lambda^2}{8\pi}\Big)[\Lambda_n\partial_+ J_{-,n} - \Lambda_{n+1}\partial_- J_{+,n}] \\ &+\Big(\frac{1}{2}-\frac{k_n\lambda^2}{8\pi}\Big)[\Lambda_n\partial_-\tilde{J}_{+,n} - \Lambda_{n+1}\partial_+\tilde{J}_{-,n}]\Big\} .\end{aligned} \tag{2.5}$$

To construct a gauge invariant action, we define $SU(N)$ gauge fields $A_{\pm,n} = iA^a_{\pm,n}T^a$ at each site n, where the adjoint representation matrices T^a are normalized according to Tr $T^aT^b = \frac{1}{2}\delta^{ab}$. The transformation law for the gauge fields is

$$\delta_G A_{\pm,n} = \partial_\pm\Lambda_n + [\Lambda_n, A_{\pm,n}] . \tag{2.6}$$

Denote $I_n(U_n, A, \lambda, k_n) = I_{1,x_\perp} + I_{2,x_\perp}$ as the gauged sigma model action, where

$$\begin{aligned}I_{2,n} =& \frac{1}{2\pi}\int d^2x \mathrm{Tr}\Big\{\Big(\frac{1}{2}+\frac{k_n\lambda^2}{8\pi}\Big)[A_{-,n+1}J_{+,n} - A_{+,n}J_{-,n} \\ &+\frac{2\pi}{\lambda^2}A_{+,n}U_nA_{-,n+1}U_n^{-1}] + \Big(\frac{1}{2}-\frac{k_n\lambda^2}{8\pi}\Big)[A_{+,n+1}\tilde{J}_{-,n} \\ &- A_{-,n}\tilde{J}_{+,n} + \frac{2\pi}{\lambda^2}A_{-,n}U_nA_{+,n+1}U_n^{-1}] \\ &- \frac{\pi}{\lambda^2}[A_{-,n}A_{+,n} + A_{+,n+1}A_{-,n+1}]\Big\} .\end{aligned} \tag{2.7}$$

Even with this additional term, the action for each individual link is not gauge invariant,

$$\delta_G I_n = \frac{k_n}{8\pi}\int d^2x \mathrm{Tr}\Big\{\Lambda_n\epsilon^{\mu\nu}\partial_\mu A_{\nu,n} - \Lambda_{n+1}\epsilon^{\mu\nu}\partial_\mu A_{\nu,n+1}\Big\} , \tag{2.8}$$

where $\epsilon^{+-} = 1$. The lack of gauge invariance is proportional to the Wess-Zumino couplings k_n for each link field, and has the same form as the non-abelian anomaly in two dimensions. From eqn. (2.8) the 'anomaly' cancellation requirement is satisfied by an infinite chain, or by a finite chain with periodic boundary conditions, if the Wess-Zumino coupling is site independent,

$$k_n = k \ , \ \forall n . \tag{2.9}$$

Note that the simplest solution is a lattice consisting of a single link with left and right sites identified. This is equivalent to gauging the anomaly-free vector subgroup of the left and right global chiral symmetries of the sigma model[8].

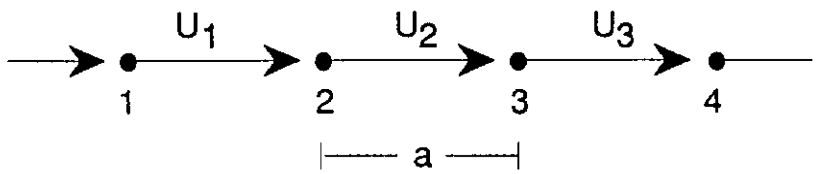

Fig. 1. The Chern-Simons lattice model. The links on the one-dimensional lattice, with lattice spacing a, have sigma-model actions with equal Wess-Zumino coupling constants k.

Now consider the gauge invariant one-dimensional lattice configuration described locally by figure 1, with link action

$$I = \sum_n I_n(\lambda, k) \ . \tag{2.10}$$

The continuum limit of this model is obtained by representing the link field as a slowly varying function on the transverse lattice,

$$U_n = \exp\left[-aA_\perp(x_\perp + \tfrac{1}{2}a)\right] \ , \tag{2.11}$$

where $x_\perp = na$ will become a continuous coordinate as $a \to 0$, and then expanding equation (2.10) order by order in lattice spacing. The sign in the exponent in (2.11) is fixed by eqns. (2.2) and (2.6). The volume term is the leading $\mathcal{O}(a)$ contribution which comes from the $I_{2,n}$ contribution to the action of each link,

$$\begin{aligned} I =& \frac{k}{8\pi} \sum_{x_\perp} a \int d^2x \mathrm{Tr} \ \epsilon^{ijk} \left[A_i \partial_j A_k - \tfrac{2}{3} A_i A_j A_k\right] + \mathcal{O}(a^2) \\ & \text{where } i,j,k = \{+,-,\perp\} \ . \end{aligned} \tag{2.12}$$

This is the pure Chern-Simons action for the $2+1$ dimensional gauge fields A_i. It is an exactly solvable quantum theory[6] with non-local degrees of freedom consisting of topological classes of Wilson loops. The coupling constant k is quantized in this theory by requiring gauge invariance under 'large' gauge transformations disconnected from the identity. These are precisely the values recovered by taking the limit of the lattice model (2.10), for which the coupling constant was also quantized.

Away from the critical point $\lambda^2 = \frac{4\pi}{|k|}$, the quantum theory of the sigma model, which describes the link actions of the lattice model, has a mass gap. But since the coupling constant λ of the sigma model action appears only to order a^2, massive states should decouple from the low energy spectrum in the continuum limit. This agrees with the conformal invariance property of the continuum Chern-Simons theory. Therefore it should be possible to analyze the Chern-Simons theory with the critical lattice models.

3. Quantization and Gauge Fixing

At criticality, the lattice model consists of a system of coupled gauged WZW models. We now briefly discuss the current algebraic solution of the quantum WZW model in light-cone coordinates, which was first given by Dashen and Frishman[9] for level one in the context of the Thirring model, and was later generalized to arbitrary level by Knizhnik and Zamolodchikov[7]. After this discussion, the effect of the coupling of the currents to gauge fields will be considered.

It is insufficient to specify initial data on a single light-like hypersurface for the case of massless fields. Instead[10], in the infinite volume limit of the 1+1 Minkowski space, the right-movers (functions of x^-) are specified on the line $x^+ = 0$ and the left-movers (functions of x^+) are specified on $x^- = 0$. The momenta are given by

$$P^\mu = \int dx^- T^{+\mu}(x^+ = 0) + \int dx^+ T^{-\mu}(x^- = 0) \ . \tag{3.1}$$

The commutation relations below specify the current algebra of the WZW model for the right-movers; there are equivalent expressions for the left-movers[1]

$$[J^a_-(v), J^b_-(v')] = if^{abc}J^c_-(v) + \frac{i}{2\pi}k\delta^{ab}\delta'(v - v') \ . \tag{3.2}$$

The mode expansion for the currents is obtained by imposing periodic boundary conditions for the appropriate light-cone coordinates $x^\pm$[11]. For the right-movers,

$$J^a_n = \int_0^L J^a(v) z^n dv \ , \tag{3.3}$$

where

$$z = e^{2\pi i v/L} \ . \tag{3.4}$$

The modes of the left-moving currents are denoted as $\bar{J}^a_n$, and both sets of modes satisfy Kac-Moody algebras. The space of states for the right-mover sector are built up from vacuum states satisfying

$$\begin{aligned} J^a_n|\mu_0\rangle &= 0 \ , \ \ n > 0 \\ E^\alpha_0|\mu_0\rangle &= 0 \ , \end{aligned} \tag{3.5}$$

where $E^{\pm\alpha}, H^i_0$ denote the zero mode currents J^a_0 in the Cartan-Weyl basis of the algebra. These vacuum states are the highest weights of the finite dimensional

[1] We discuss the positive k case; the negative k case has identical expressions with $J_\pm \to \tilde{J}_\pm$.

representations $|\mu\rangle$ generated by acting the zero modes on the highest weights $|\mu_0\rangle$. These highest weight representations for $SU(N)$ level k are unitary if they have Young tableau with the number of columns less than or equal to k[12]. We will consider diagonal representations,which consist of products of highest weights which transform in the same representation under the left and right isospins. These unitary highest weight representations correspond to the primary fields $\phi_\mu(v)$ of the chiral algebra, which have simple commutation relations with the currents,

$$[J_-^a(v), \phi_\mu(v')] = t_\mu^a \phi_\mu \delta(v - v') \ , \tag{3.6}$$

where t_μ^a is a generator of $SU(N)$ in the representation μ. The highest weight field in the fundamental representation is identified with the classical link field.

The gauged WZW model includes the interaction of WZW currents and gauge fields given by eqn. (2.7). In the gauge $A_- = 0$, the interaction reduces to

$$I_{2,n} = \frac{-1}{2\pi} \int d^2x \mathrm{Tr}\ A_{+,n} J_{-,n} \tag{3.7}$$

The path integral over $A_{+,n}$ therefore imposes the Gauss law constraint $J_{-,n} = 0$. In the quantum theory, the constraint on the Hilbert space $\mathcal{H}_{\mathbf{phys}}$ of physical states is

$$J_{m,n}^a |\mathrm{phys}\rangle = 0 \ , \ m > 0, \ \forall n \ , \tag{3.8}$$

where $J_{m,n}^a$ is the m^{th} mode of the right-mover current algebra of the n^{th} link theory. This gauge fixing is incomplete, since the gauge constraint does not fix x^+ dependent gauge transformations. On the remaining states in the Hilbert space, these gauge transformations are generated by the $J_{+,n}$ currents. Therefore, we keep states in the quantum theory which are in the gauge invariant subspace and satisfy the constraint

$$\bar{J}_{m,n}^a |\mathrm{phys}\rangle = 0 \ , \ m > 0, \ \forall n \ . \tag{3.9}$$

A more complete discussion of the gauge fixing can be found in refs. [8],[13]. The states which satisfy these constraints are the finite dimensional representations $|\mu\rangle$ for each sector.

The remaining constraints from the gauge theory are the projections onto states which are invariant under $x^\pm$ independent gauge transformations. In the continuum limit, these are the gauge transformations that are functions of the third coordinate $x_\perp$. The generators of these gauge transformations for the lattice are the zero modes of the currents. We therefore require that physical states satisfy

$$(E^{\alpha}_{0,n} - \bar{E}^{\alpha}_{0,n-1})|\mathrm{phys}\rangle = 0 \ , \ \forall n \ , \tag{3.10}$$

and

$$(H^{i}_{0,n} - \bar{H}^{i}_{0,n-1})|\mathrm{phys}\rangle = 0 \ , \ \forall n \ , \tag{3.11}$$

where again the Cartan-Weyl decomposition is used for the zero modes of the left and right mover currents. The important feature of these constraints is that they are non-local and relate states of the n link theory with nearest neighbor states.

To specifically solve the constraints, consider a periodic lattice with L sites. This is the discrete version of the manifold $M^2 \times S^1$. The solutions to the constraints for this geometry are

$$|R_{\mu}\rangle = \prod_{n=1}^{L} |\mu_0\rangle_n |\overline{\mu_0}\rangle_n \ . \tag{3.12}$$

The constraint equation (3.10) requires the physical states to be products of the highest weight states of each link theory, and the final constraint (3.11) fixes the nearest neighbor highest weight states to be of the same representation.

4. Path Integral with Wilson Loops

The quantization approach outlined above yields the excitation spectrum of the lattice Chern-Simons theory quantized about the vacuum state $|0\rangle$. We now wish to compare this analysis with the path integral of the continuum Chern-Simons theory in the presence of Wilson loops.

Each state given by eqn. (3.12) is the lattice version of the degree of freedom of a Wilson loop (carrying flux in representation μ and winding once about S^1), which is not eliminated by gauge fixing or by satisfying the Gauss constraints. In fact, a state corresponds in this way to an infinite number of Wilson loops in each topological class.

To test this relationship further, we will calculate the correlation functions of operators which generate physical states on the discrete version of the manifold $M^2 \times S^1$, and compare these correlators with results from the continuum Chern-Simons theory. These Wilson loop operators are constructed from the local operators which generate the highest weight states for each link theory. The local operators are in turn constructed from the zero modes of the primary fields ϕ_{λ}

of the WZW model. When acting on a highest weight representation $|\mu\rangle$, the zero modes generate the tensor product of the primary field representation and the highest weight representation. The irreducible representations occurring in the tensor product are given by the operator product algebra of the primary fields associated with each representation. This tensor product is therefore the ordinary $SU(N)$ tensor product, with the additional constraint that the irreducible $SU(N)$ representations appearing in the product correspond to unitary Kac-Moody representations. These local operators are denoted as Φ_λ,

$$\Phi_\lambda|\mu_0\rangle = P_{\rm hw}[|\lambda\rangle \otimes |\mu\rangle] = N^{\nu}_{\lambda\mu}|\nu_0\rangle \ , \tag{4.1}$$

where the operator $P_{\rm hw}$ projects onto the highest weight states of each irreducible representation in the tensor product. The left-mover representations are implicit in eqn. (4.1), and $P_{\rm hw}$ is defined to project onto diagonal product of left and right-mover representations. The operator product coefficient $N^{\nu}_{\lambda\mu}$ is the multiplicity that counts the number of independent ways the representations λ, μ can fuse onto ν. These coefficients satisfy the condition[14]

$$\sum_{\lambda} N^{\lambda}_{\mu\nu} N^{\sigma}_{\lambda\rho} = N^{\lambda}_{\mu\rho} N^{\sigma}_{\lambda\nu} \ , \tag{4.2}$$

which describes the associativity of the operator product expansion, in the context of the four point function of primary fields in the WZW model. This relation, and the symmetry $N^{\lambda}_{\mu\nu} = N^{\lambda}_{\nu\mu}$, implies that the (associative) algebra of the Φ operators is

$$\Phi_\mu \Phi_\nu = N^{\lambda}_{\mu\nu} \Phi_\lambda \ . \tag{4.3}$$

To construct the operators which generate Wilson loops on the lattice, the Φ operators are glued together,

$$R_\mu = \mathrm{Tr} \prod_{n=1}^{L} \Phi_{\mu,n} \ , \quad R_\mu|R_\nu\rangle = N^{\lambda}_{\mu\nu}|R_\lambda\rangle \ . \tag{4.4}$$

The 'Tr' projects the products of the Φ operators onto configurations which satisfy the non-local constriants (3.11).

The correlators of Wilson loops are products of the correlation functions of local operators for each link field theory, and therefore their expectation value is determined by the operator algebra eqn. (4.3), and the singlet selection rules of the conformal field theory for each link theory. The singlet selection rule requires that the total product of operators Φ_μ at each link fuses onto the identity representation. The one, two and three point correlation functions are

$$
\begin{aligned}
\langle R_\mu \rangle &= \delta_{\mu,0} \ , \\
\langle R_\mu R_\nu \rangle &= N^0_{\mu\nu} \ , \\
\langle R_\lambda R_\mu R_\nu \rangle &= N^\rho_{\lambda\mu} N^0_{\rho\nu} \ .
\end{aligned}
\tag{4.5}
$$

These expectation values are independent of the lattice spacing a. Therefore, unlike other transverse lattice models[1], the coupling constants do not scale as the continuum limit is approached. This should be the case when the physical states are of a purely topological nature, and when the coupling constants are quantized. All physical quantities of the lattice theory calculated for finite a are therefore also the continuum results.

The calculation of correlation functions for lattice Wilson loop operators on the topology $S^2 \times S^1$ is the same, since the Hilbert spaces of Kac-Moody states for M^2 and S^2 are isomorphic, and the above results agree with Wilson loop path integrals given for this topology in the continuum theory by Witten[6].

5. Discussion

The transverse lattice construction has led to a very concrete connection between 2-D conformal field theory and $2+1$ Chern-Simons theory. Previous connections which were made by relating structures in the quantized Chern-Simons theory to stuctures in 2-D conformal field theory. The construction discussed here is a lattice regularized version of the Chern-Simons theory, which turns out to be a system of coupled conformal field theories. The topological nature of the physical states makes the scaling behavior of the transverse lattice model trivial, so that the lattice model exactly describes the continuum theory.

Although the analysis above describes a relatively trivial case of the Chern-Simons field theory, the transverse lattice construction clearly shows that the Chern-Simons model is probing the physics of the 2-D WZW model. For 3-manifolds with more complicated topology (see [6][15] and refs. therein), the Chern-Simons model will probe more features of the underlying WZW model physics, and the lattice picture may serve as a useful tool in unifying our understanding of the two systems. Also, as an intermediate step in the construction of transverse lattice models in the physical dimension, it displays the utility of the transverse lattice approach advocated by Bardeen and others.

The more phenomenologically interesting $3+1$ transverse lattice models with Wess-Zumino terms in the link actions have additional conceptual twists, which will be addressed in a separate publication. In particular, one can 'stagger' the Wess-Zumino terms on the lattice to assure that in the naive continuum limit, the leading terms are order a^2. There exist lattice configurations which have this property and are anomaly free.

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