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A 12 Deep Inelastic Scattering beyond the Leading Order in Asymptotically Free Gauge Theories

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I would like to discuss higher order effects in the running coupling constant of QCD in deep inelastic lepton–hadron scatterings. I shall mainly talk about the work performed at Fermilab[1] by Bardeen, Buras, Duke and myself and also mention some related results obtained by other groups.

The moments of structure functions in deep inelastic leptoproductions are related to the Wilson coefficients in the lightcone operator expansions of the current product, $C_n(Q^2/\mu^2, g)$, where g is a renormalized coupling constant. For simplicity we neglect the complication due to the operator mixing (or we consider only a nonsinglet combination of structure functions). The function $C_n(Q^2/\mu^2, g)$ satisfies the renormalization group equation for large Q^2, the solution of which is given by

$$C_n\left(\frac{Q^2}{\mu^2}, g\right)=C_n(1, \bar{g}(Q^2))\exp\int_{\bar{g}(Q^2)}^{g} d\tau \frac{\gamma_n(\tau)}{\beta(\tau)}, \tag{1}$$

with $g=\bar{g}(\mu^2)$ and μ the renormalization scale, where the running coupling constant $\bar{g}(Q^2)$ is given by $Q^2 d\bar{g}/dQ^2=\beta(\bar{g})$, $\beta(\bar{g})$ and $\gamma_n(\bar{g})$ being the Gell-Mann–Low–Callan–Symanzik function and the anomalous dimension of the composite operator respectively.

We now wish to expand $C_n(Q^2/\mu^2, g)$ in powers of $\bar{g}(Q^2)$. For this purpose we first expand β, γ_n and $C_n(1, \bar{g})$ in the following way, $\beta(g)=-\beta_0\bar{g}^3-\beta_1\bar{g}^5+0(\bar{g}^7)$, $\gamma_n(\bar{g})=\gamma_0^n\bar{g}^2+\gamma_1^n\bar{g}^4+0(\bar{g}^6)$, $C_n(1, \bar{g})=1+c_n\bar{g}^2+0(\bar{g}^4)$. Putting these expansions into eq. (1) and expanding the result in powers of $1/\ln Q^2$ we find

$$C_n\left(\frac{Q^2}{\mu^2}, g\right)\propto\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_0^n/2\beta_0}\times\left[1+\frac{1}{\beta_0\ln(Q^2/\Lambda^2)}\left(f_1^n-f_2^n\ln\ln\frac{Q^2}{\Lambda^2}\right)+\cdots\right], \tag{2}$$

where f_1^n and f_2^n are given by the combinations of $c_n, \beta_0, \beta_1, \gamma_0^n$ and γ_1^n, and the scale parameter Λ is given by

$$\Lambda^2=\mu^2 e^{-1/\beta_0 g^2}(\beta_0 g^2)^{-\beta_1/\beta_0^2}. \tag{3}$$

As is expected by the renormalization group property, $C_n(Q^2/\mu^2, g)$ is expressed in terms of the single parameter Λ which is a combination of two parameters μ and g.[1,2] In fact this is true to all orders of the expansion (2).[3]

Once coefficients β_0, β_1, γ_0^n, γ_1^n and c_n are known, one can use eq. (2) to do phenomenology with one free parameter Λ. The coefficients β_0 and γ_0^n are well-known in QCD[4] and β_1 was calculated in two loops.[5] They are all independent of the renormalization prescription used. The value of γ_1^n and c_n depends on the way how one renormalize the theory. This dependence on the renormalization prescription, however, should cancel out in the physical result $C_n(Q^2/\mu^2, g)$ as far as we make consistent use of the renormalization prescription in calculating γ_1^n and c_n.[6] The two-loop anomalous dimension γ_1^n was calculated recently[6] only for nonsinglet operators *in the minimal subtraction scheme of 't Hooft*.[7] Thus it is needed to calculate c_n in the same prescription as above. This is precisely what we have done: we completed the evaluation of the next-to-the leading order in $C_n(Q^2/\mu^2, g)$ by calculating c_n in the minimal subtraction scheme[7] both for the non-singlet and singlet parts of electroproduction and neutrino (charged current) reactions.[1] The final expression for c_n can be found in ref. 1. We applied the results to the phenomenological fit of F_3 in the neutrino data.[8]

The coefficient c_n has been calculated also by other groups in renormalization schemes different from ours. Since their renormalization schemes differ from ours, their results on c_n cannot be compared directly with ours. We can, however, find out a prescription which converts one renormalization scheme to the other. We show the results of such comparisons among those results on c_n. *Nonsinglet part (quark contributions)*: Calvo uses a

mass-shell renormalization with massive quarks and works in the Feynman gauge.[9] DeRujula, Georgi and Politzer (DGP) calculate c_n in an off-shell renormalization with massless quarks for the Landau gauge.[10] Altarelli, Ellis and Martinelli (AEM) use the same scheme as in DGP and work in the Feynman gauge.[11] We (B²DM) work in an arbitrary covariant gauge using the minimal subtraction scheme* with massless quarks.[1] By carefully taking into account the difference of schemes we find that our result is consistent with that of AEM,[11] but disagrees in a minor term with DGP.[10] (The agreement is possible if $10/(n+1)$ is replaced by $6/(n+1)$ in c_n of DGP. There, however, remains a problem of gauge dependence.) The comparison between B²DM and Calvo can be done easily, but I simply did not have time to check it. *Singlet part (gluon contribution)***: A set of the relevant amplitudes is free of ultraviolet divergences and no renormalization prescription is needed. However, the mass singularity causes some complication. After taking into account the difference of the ways to handle mass singularities we find that our result agrees with Kingsley[12] and AEM,[11] but disagrees with Ahmed–Ross,[13] Witten (by overall factor 1/4),[14] Calvo[9] and Hinchliffe–Llewellyn–Smith.[15]

Next we consider sum rules. Since we calculated the full $\bar{g}^2$ correction to the moments of structure functions, it is possible to figure out the correction term to sum rules like the Alder, Gross–Llewellyn–Smith and Bjorken sum rules. For such moments anomalous dimensions vanish because vector and axial-vector currents have no anomalous dimension. Our findings are as follows.[1,9,11] The Adler sum rule receives no correction as is expected. The Gross–Llewellyn–Smith and Bjorken backward sum rules are modified:

$$\int_0^1 dx(F_3^{\bar{\nu}}+F_3^{\nu})=-6\left(1-\frac{12}{33-2f}\frac{1}{\ln Q^2/\Lambda^2}\right),$$

$$\int_0^1 dx(F_1^{\bar{\nu}}-F_1^{\nu})=2\left(1-\frac{8}{33-2f}\frac{1}{\ln Q^2/\Lambda^2}\right),$$

where f is the number of flavors. These corrections are of the order of 10–20% for a wide range of Q^2 and should be detected by accurate neutrino data. It is also possible to calculate the $\bar{g}^2$ correction to the Bjorken sum rule for polarized electroproductions.

Finally I would like to make a comment on the problem associated with the experimental determination of scale parameter Λ. Let us remember the expression for $C_n(Q^2/\mu^2, g)$ in the leading order: $C_n\propto(\ln Q^2/\mu^2)^{-\gamma_0^n/2\beta_0}$. If we redefine the scale Λ by $\Lambda'=\kappa\Lambda$ with numerical factor κ, we find

$$C_n=\left(\ln\frac{Q^2}{\Lambda'^2}\right)^{-\gamma_0^n/2\beta_0}\times\left(1-\frac{\gamma_0^n}{2\beta_0}\frac{\ln\kappa^2}{\ln Q^2/\Lambda'^2}+\cdots\right), \tag{4}$$

As is seen in eq. (4) the redefinition of Λ generates higher order terms. Since we work in the power series expansion of C_n in $1/\ln Q^2$, *i.e.*, eq. (2), the redefinition of $\Lambda(\Lambda'=\kappa\Lambda)$ modifies higher order terms in eq. (2):

$$C_n\propto\left(\ln\frac{Q^2}{\Lambda'^2}\right)^{-\gamma_0^n/2\beta_0}\times\left[1+\frac{1}{\beta_0\ln Q^2/\Lambda'^2}\left(f_1^n-\gamma_0^n\ln\kappa-f_0^n\ln\ln\frac{Q^2}{\Lambda'^2}\right)+\cdots\right]. \tag{5}$$

We see that in the second order the effect of the redefinition of Λ is equivalent to the change of f_1^n (or c_n) by a term proportional to γ_0^n. If we truncate the series (2) and (5) at the second order (as we do in the phenomenological application), we have two different expressions. This fact leaves some uncertainty of determining Λ by experimental data. Since the range of Q^2 in practical data is limited, $1/\ln Q^2/\Lambda^2$ may not be small for some large Λ. Then the perturbation expansion (2) or (5) is meaningless in this range of Q^2. Hence the range of the freedom of "rescaling" is restricted by the fast-convergence condition of the series (5) in a given range of Q^2. The restriction is not very strong and there still remains the problem of

* It is very important to note that, in the minimal subtraction scheme, the matrix element of the composite operator is not normalized and should be subtracted from the current correlation function to get the coefficient function. No groups except for us performed this procedure and their results can not be used together with the two-loop anomalous dimension obtained by Floratos, Ross and Sachrajda.

** The quark contribution is the same as that of the non-singlet part.

an uncertainty of order 1 in determining Λ by experimental data.[1,16] This freedom of rescaling for Λ is related to the ambiguity in defining the running coupling constant $\bar{g}$.

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A 12 Jets

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Some History and Introduction

At the time of the previous conference of this series (Tbilisi, 1976) data were presented by Hansen *et al.*[1] (SLAC) that constituted the first indication for the existence of "quark" jets. Hadrons produced in e^+e^- annihilation at energies ranging from $Q=3$ GeV to $Q=7$ GeV were analyzed in two extreme models. One of them was a "phase space" model wherein hadrons were assumed to be produced in random directions. This model fits the lower energy data but disagrees significantly with the higher energy data. The second model was a "two step" (quark production plus soft hadronization) model that consistently described all the data. In the first step the virtual photon is assumed to materialize into two pointlike spin 1/2 quarks. This implies a distribution of outgoing energy $\sim 1+\cos^2\theta$, with θ the angle between the lepton beam and the quarks. In a second step or longer time scale, the outgoing quarks were assumed to materialize into hadrons with an exponentially damped transverse momentum ($\exp -\{4p_T^2\}$ or $\exp -\{6p_T\}$) relative to the quark axis: the would-be jet axis. This model was successful in describing the data in all the available energy range and, moreover, the measured jet axis distribution (found by somehow minimizing event per event, the transverse momentum) agreed with the expected $1+\cos^2\theta$. This is evidence for the spin 1/2 nature of quarks (scalar quarks, for instance, would give a distribution $\sim\sin^2\theta$). To the eyes of an optimist, this is evidence for QCD, the only realistic asymptotically free theory we know where it makes sense to draw a quark-production diagram as the leading contribution in perturbation theory. We may also conclude that whatever incompletely understood process turns quarks into hadrons, it is soft enough not to obliterate at sufficiently high energy the quark (jet) axis