FREEZE-OUT PROPERTIES OF HOT NUCLEAR MATTER CREATED IN HEAVY ION COLLISIONS

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The study of properties of nuclei under extreme conditions of temperature and density has been the subject of many investigations in recent decades, since they are very important in the study of the process of supernovae, neutron stars and nuclei. Heavy-ion collision experiments are often employed to determine these properties. We present a short and limited review of the theoretical and experimental status of determining the temperature and density of the disassembling hot nucleus from ratios of the yields of emitted fragments.

1. Introduction

The decay of highly excited nuclear matter produced in the laboratory in heavy ion collisions is a complex dynamic process. One simple approach is the freeze-out concept in which the hot and dense matter in the initial stage is assumed to reach thermal equilibrium. Often the description of the nuclear matter, in particular the distribution of clusters, is calculated within a statistical multifragmentation model assuming nuclear statistical equilibrium (NSE). A simple method for extracting the temperature of the fragmenting hot system was given by Albergo, Costa, Costanzo and Rubbino (ACCR) [1]. In the context of a grand canonical model [2, 3], ACCR exploit the assumed existence of thermal and chemical equilibrium and the additional assumption that the fragments are formed in a 'freeze-out' volume large enough so that they can be treated as noninteracting. The method is based on selecting double ratios, R_2 of the yields $Y(A,Z)$ of emitted fragments, such that the nucleon chemical potentials are eliminated leading to a relation between R_2, T and the binding energies of the selected fragments. This method has been used in the analysis of a large number of experiments [4, 5]. In these experiments, the dependence of the excitation energy of the decaying system on the temperature (i.e. the caloric curve) was found to show irregularities which is interpreted as a possible signal for the occurrence of a phase transition in finite nuclei.

In the following we discuss the ACCR method and the extensions necessary [5] to account for the effects of:

(i) The long range Coulomb interactions among fragments in the freeze-out volume [6]. Here we employ the Wigner-Seitz approximation.

(ii) The radial collective flow [7]. An expanding system, in a strict thermodynamic sense, is not in equilibrium. However, if the time scale involved in the expansion is much larger compared to the equilibration times in the expanding complex, i.e. the flow velocity is quite small compared to the average nucleonic velocity, the assumption of thermodynamic equilibrium may not be inappropriate.

(iii) The post emission decay (secondary decay) processes of the fragments emitted from the freeze-out surface [8].

(iv) The effect of the medium on the binding energies of clusters [9].

2. Nuclear temperature and density from ratios of fragments yields

In statistical models describing the decay of a hot nucleus one assumes that at a certain point in the evolution, the excited nucleus reaches a thermal equilibrium at a certain freeze out volume, where the multifragmentation process takes place. In some of the proposed models it is also assumed that the disassembling nucleus reaches a chemical equilibrium, i.e., the chemical potentials of the emitted fragments are directly related to the chemical potentials of the nucleons in the decaying system [1, 10 - 12]. In the following we present a derivation of a relation between the temperature T and the double ratio R_2 of fragment yields (see Ref. [6] for details), which is similar to that proposed in Ref. [1]. We adopt the Wigner-Seitz approximation [13] for the Coulomb interaction among fragments and the Maxwell - Boltzmann statistics [11] and impose only the condition of thermal equilibrium.

In the break-up stage of the multifragmentation we assume the presence of a number of isolated fragments (clusters) in thermal equilibrium at a temperature T within a certain freeze-out volume $V = V_0(1+\kappa)$, where V_0 is the volume of the decaying nucleus corresponding to normal nuclear density $n_0 = 0.17$ fm^{-3} and κ is the expansion parameter [14]. The main assumption of the statistical model of multifragmentation [3, 10, 15] is that the yield of the fragments (N_s, Z_s) with given numbers of neutrons, N_s, and protons, Z_s, is determined by the phase space available for decay. In a macrocanonical ensemble, the statistical properties of the system can be evaluated using the grand partition sum

$$\mathcal{Z} = \sum_{\{f\}} e^{(\mu_n N_f + \mu_p Z_f)/T} Q_f, \quad Q_f = \exp(-F_f / T). \tag{1}$$

In Eq. (1), N_f, Z_f, Q_f and F_f are the total neutron number, the total charge, the partition function and the free energy for a given event f, respectively. The Lagrange multipliers T, μ_n, μ_n are determined by the corresponding conservation laws,

$$\bar{E}_f = E_0 \ , \quad \bar{N}_f = N_0 \ , \quad \bar{Z}_f = Z_0 \ . \tag{2}$$

Here, N_0 and Z_0 are the numbers of neutrons and protons in the decaying system, respectively, E_0 is the excitation energy and the bar denotes an ensemble average of the corresponding quantity. The summation in Eq. (1) extends over all possible events which are characterized by the space position, momenta and internal degrees of freedom of the fragments.

In general, the free energy F_f can be written as the sum of the contributions from the individual fragments s and from their interaction as

$$F_f = \sum_s F_s^{(1)} M_s + F_f^{(2)} \ . \tag{3}$$

The quantity M_s is the number (multiplicity) of clusters with (N_s, Z_s) for a given event f. The individual free energy $F_s^{(1)}$ includes the ground state, translation and internal free energies of a fragment s and $F_f^{(2)}$ is the contribution from inter-fragment interaction. We point out that the (short-range) nuclear inter-fragment interaction at the break-up stage is neglected under the main assumption of a freeze out volume. However, the free energy F_f is not an additive quantity because of the long-range Coulomb interaction between fragments. An essential simplification is achieved by employing the well known Wigner-Seitz approximation [13], so that the interaction free energy $F_f^{(2)}$ can be represented as an additive quantity given by

$$F_f^{(2)} \equiv F_f^{(\text{Coulomb})} = \sum_s E_C(s) M_s + E_C^{(0)} \ . \tag{4}$$

Here $E_C^{(0)}$ is the Coulomb energy associated with the uniform distribution of the Z_0 protons over the freeze out volume. We point out that the constant term $E_C^{(0)}$ does not affect the calculation of any average quantity and therefore is omitted in the following. Note also that the Coulomb self energy $E_C(N_s, Z_s)$ of the fragment is included in $F_s^{(1)}$. The energy $E_C(s)$ in Eq. (4) is given by [14]

$$E_C(s) = -\frac{3}{5} Z_s^2 \frac{e^2}{R_s^C} \ , \tag{5}$$

where R_s^C is the radius of the Wigner-Seitz cell

$$R_s^C = R_s (1+\kappa)^{1/3} \left(\frac{Z_s / A_s}{Z_0 / A_0} \right)^{1/3} \ . \tag{6}$$

In Eq. (6), $A_0 = N_0 + Z_0$ and $A_s = N_s + Z_s$ are the numbers of particles of the decaying system and of the cluster s, respectively, and R_s is the (ground state) radius of cluster $R_s = r_0 A_s^{1/3}$ with $r_0 = 1.2$ fm. One usually adopts the values of $\kappa = 2$ and $Z_0 / A_0 = 1/2$.

Using Eqs. (3) and (4), the partition function Q_f can now be factorized due to the additivity of F_f and can be written in the form [16]

$$Q_f = \prod_{\{s\}} \frac{1}{M_s!} \left(Q_s^{\text{transl.}} Q_s^{\text{intr.}} \right)^{M_s} \ . \tag{7}$$

In Eq. (7) $Q_s^{\text{transl.}}$ is the partition function of the translational motion of the cluster s. It is given by,

$$Q_s^{\text{transl.}} = A_s^{3/2} \frac{V'}{\lambda_T^3} \ , \tag{8}$$

where $\lambda_T = \sqrt{2\pi\hbar^2 / mT}$ the thermal nucleon wave-length and V' is the free volume available for the translational motion of clusters in the freeze-out volume V [14]. The internal partition function $Q_s^{\text{intr.}}$ of fragment s in Eq. (7) is given by

$$Q_s^{\text{intr.}} = \sum_i (2 I_i(s) + 1) e^{-E_i(s)/T} \ , \tag{9}$$

where $I_i(s)$ and $E_i(s)$ are the total angular momenta and energies of the eigenstates of the cluster s, respectively. In the Wigner-Seitz approximation one has

$$E_i(s) = B_s + \varepsilon_i^*(s) + E_C(s) \ , \tag{10}$$

where B_s is the binding energy of cluster s and $\varepsilon_i^*(s)$ is its excitation energy.

Using Eqs. (1), (7) and (10), we obtain that

$$\mathcal{Z} = \sum_{\{M_s\}} \prod_{\{s\}} e^{(\mu_n N_s + \mu_p Z_s - B_s - E_C(s))M_s/T} \frac{1}{M_s!} \left(Q_s^{\text{transl.}} Q_s^*\right)^{M_s} =$$

$$= \prod_{\{s\}} \exp\left(e^{\mu_s M_s/T} Q_s^{\text{transl.}} Q_s^*\right), \tag{11}$$

where

$$Q_s^* = \sum_i \left(2I_i(s)+1\right) e^{-\varepsilon_i^*(s)/T} . \tag{12}$$

For the purpose of simplification we have introduced in Eq. (11) the quantity

$$\mu_s = \mu_n N_s + \mu_p Z_s - B_s - E_C(s) , \tag{13}$$

It is important to note that apart from the Coulomb energy $E_C(s)$ the quantity μ_s defined in Eq. (13), is similar to the cluster chemical potential introduced by ACCR [1] under the condition of chemical equilibrium. We emphasize that, in contrast to Ref. [1], only the thermal equilibrium condition was imposed in the derivation of Eq. (11). Using Eqs. (3), (4), (8) and (11), the average multiplicity $\bar{M}_s$ of clusters s is given by

$$\bar{M}_s = \frac{1}{\mathcal{Z}} \sum_{\{M_s\}} M_s \prod_s e^{\mu_s M_s/T} \frac{1}{M_s!} \left(Q_s^{\text{transl.}} Q_s^*\right)^{M_s} = e^{\mu_s/T} \frac{V'}{\lambda_T^3} A_s^{3/2} Q_s^* . \tag{14}$$

Let us introduce the average density $\bar{n}_s$ of clusters s:

$$\bar{n}_s \equiv \bar{n}(N_s, Z_s) = \frac{\bar{M}_s}{V} . \tag{15}$$

Using Eqs. (12) - (14) one finds for the nucleon densities $\bar{n}_n$ and $\bar{n}_p$ the expressions,

$$\bar{n}_n = \frac{2\chi}{1+\kappa} \frac{1}{\lambda_T^3} e^{\mu_n/T} , \quad \bar{n}_p = \frac{2\chi}{1+\kappa} \frac{1}{\lambda_T^3} e^{\mu_p/T} , \tag{16}$$

where $\chi = V'/V_0$ is the hindrance factor. Note that the nucleon spin-degeneracy factor 2 was taken into account. From Eqs. (13) - (16) one finds for the relative yield of fragments s the expression

$$\frac{\bar{n}_s}{\bar{n}_n^{N_s} \bar{n}_p^{Z_s}} = \frac{1}{2} \left(\frac{1+\kappa}{2\chi}\right)^{A_s-1} A_s^{3/2} \left(\lambda_T^3\right)^{A_s-1} Q_s^* \, e^{-(B_s + E_C(s))/T} . \tag{17}$$

The ACCR method

In the ACCR approach, the chemical equilibrium condition has the form,

$$\mu(A,Z,T) = Z\mu_p(T) + (A-Z)\mu_n(T) + B(A,Z), \tag{18}$$

Here, $\mu(A,Z,T)$, $\mu_p(T)$, and $\mu_n(T)$ are the chemical potentials of the fragment (A,Z), the free proton and neutron at the temperature T, respectively, and $B(A,Z) > 0$ is the ground state binding energy [17] of the fragment (A,Z). Employing Boltzmann statistics, the temperature can be deduced from the double ratio

$$R_2 = \frac{Y(A_1',Z_1')/Y(A_1,Z_1)}{Y(A_2',Z_2')/Y(A_2,Z_2)} = F\left(A_1',Z_1',A_1,Z_1,A_2',Z_2',A_2,Z_2\right) \cdot \exp\left(\Delta B/T\right),$$

$$F\left(A_1',Z_1',A_1,Z_1,A_2',Z_2',A_2,Z_2\right) = \left(\frac{A_1' \cdot A_2}{A_1 \cdot A_2'}\right)^{3/2} \frac{\left(2I(A_1',Z_1')+1\right)\left(2I(A_2,Z_2)+1\right)}{\left(2I(A_1,Z_1)+1\right)\left(2I(A_2',Z_2')+1\right)}, \tag{19}$$

where $I(A,Z)$ and $Y(A,Z)$ are the total angular momentum of the ground state and ground state yield of the fragment (A,Z), respectively. The quantity ΔB is given in terms of the binding energies of the fragments:

$$\Delta B = B(A_1', Z_1') - B(A_1, Z_1) + B(A_2, Z_2) - B(A_2', Z_2'). \tag{20}$$

The fragment yields considered in Eq. (20) must be selected in such a way that

$$\left(N_1' = N_1 + n, N_2' = N_2 + n\right) \text{ and } \left(Z_1' = Z_1 + p, Z_2' = Z_2 + p\right) \tag{21}$$

where $A = N + Z$, n and p are integer numbers. It is important to note that the method is based on selecting double ratios, R_2 of the yields of $Y(A,Z)$ of emitted fragments, such that the nucleon chemical potentials are eliminated leading to a relation between R_2, T and the binding energies of the selected fragments.

Chemical equilibrium

The expression of Eq. (17) is similar to that of ACCR [1], except for the volume correction term $(1+\kappa)/\chi$ and the Wigner-Seitz energy $E_C(s)$.We emphasize, however, that in contrast to Refs. [1, 11, 12, 15], Eq. (17) was derived without imposing the condition of chemical equilibrium. Note that the grand partition sum in the form of Eq. (1) does not imply a macrocanonical description of clusters since the chemical potential of clusters does not enter Eq. (1). Moreover, the partition function of clusters used in the derivation, see Eqs. (7) - (9), is just the canonical partition function. This fact is essential conceptually since it allows the application of the result (14) in the case $\bar{M}_s \le 1$, where the macrocanonical description is doubtful.

Effects of Coulomb interaction

It is seen from Eq. (17) that the ACCR relation of Eq. (19) should be modified. Using Eq. (17), the temperature of the disassembling nucleus can be determined from the double ratio R_2 of the fragment yields $Y(A,Z)$ using the modified relation,

$$R_2 = \frac{Y(A_1', Z_1')/Y(A_1, Z_1)}{Y(A_2', Z_2')/Y(A_2, Z_2)} = \left(\frac{A_1' \cdot A_2}{A_1 \cdot A_2'}\right)^{3/2} \frac{\omega(A_1', Z_1', T)\,\omega(A_2, Z_2, T)}{\omega(A_1, Z_1, T)\,\omega(A_2', Z_2', T)} \cdot \exp\left(\Delta B / T\right) \cdot \exp\left(-\Delta E_C / T\right), \tag{22}$$

where we have adopted the notation of Ref. [1], using $Y(A,Z) = \bar{M}_s(N_s, Z_s)$, $\omega(A,Z,T) = Q_s^*$ and $B(A,Z) = -B_s(N_s, Z_s)$. In Eq. (22) ΔB is given by Eq. (20) and ΔE_C is given in terms of the Wigner - Seitz Coulomb energy, Eq. (5), of the fragments:

$$\Delta E_C = E_C(A_1', Z_1') - E_C(A_1, Z_1) + E_C(A_2, Z_2) - E_C(A_2', Z_2'). \tag{23}$$

The expression of Eq. (22) differs from the ACCR [1] expression in the factor $\exp\left(-\Delta E_C / T\right)$. For the case of double ratios of isotope fragments, i.e. $p = 0$ in (21), one has from Eqi. (5), (6) and (23), that $\Delta E_C = 0$. Since the temperature deduced from Eq. (22) is proportional to $\Delta B - \Delta E_C$, the Coulomb term may affect the value extracted for T for the cases with $p \neq 0$. In most cases, the correction to T is less than 20 %. However, in certain cases the change in T can be as much as 50 %. For example, for the case of $R_2 =$ (($Y(^{16}\mathrm{O})/Y(^{12}\mathrm{C})$)/$Y(^{6}\mathrm{Li})/Y(\mathrm{d})$) we have that $\Delta B = 5.69$ MeV and $\Delta E_C = -2.75$ MeV.

Effects of flow

Within the ACCR approach, one can first determine the temperature from double ratio R_2 of fragment yields, using Eq. (22), and then obtain the free neutron density at freeze-out from Eq. (17), using single ratios of yields of isotopes differing just by one unit of mass. The freeze-out density can then be determined by establishing the initial size of the fragmenting system and adopting the assumptions made by Campi et al [18]. Considering the caloric curve, Campi et al. [19] deduced for $T = 4$ MeV the value of $n/n_0 \approx 0.04$, i.e., a freeze out volume of $25V_0$, where V_0 is the volume of the nucleus at normal density. This value of $n/n_0 \approx 0.04$, is much smaller than the values of $\approx 0.16 - 0.3$ commonly used in the statistical multi fragmentation models [2, 20]. In all these model calculations, the influence of radial collective flow observed [21 - 23] in central nuclear collisions at energies $\approx 50 - 100$ MeV/nucleon or above has been ignored. Below we consider the effect of flow on the freeze-out volume [7].

In Ref. [24], a simple method to simulate the effect of collective radial flow through the inclusion of an external negative pressure in the total thermodynamic potential at freeze-out density was suggested. The external pressure P is then given by $P = -\sum_s P_s$, where P_s is the internal partial pressure exerted by the radial outflowing s fragments at the

freeze out surface. The validity of this model hinges on the assumption that the time scale involved in the expansion is larger compared to the equilibration times in the expanding complex, which works when the flow velocity is smaller compared to the average nucleonic velocity.

It was shown in Ref. [24] that the flow pressure P_s can be related to the kinetic energy of flow E_s for the fragments,

$$P_s = D(\mathrm{v}_{fs}, T) \cdot n_s \cdot E_s \ . \tag{24}$$

In Eq. (24), v_f is the magnitude of the radial flow velocity, and $D(\mathrm{v}_{fs}, T) \approx 4.5$. Experimental measurements [25] indicate that the heavier fragments carry less flow energy per nucleon compared to the lighter ones. Thus E_s, in Eq. (24), is assumed to take a simple parametric form

$$E_s = \varepsilon A_s^{\alpha} \ , \tag{25}$$

where ε is the average flow energy carried by a single nucleon and the value of α is ≈ 0.95. It can be easily seen from Ref. [24] that if the flow effects are taken into account, the chemical potential is modified as

$$\mu_s = \mu_n N_s + \mu_p Z_s - B_s - E_C(s) + P_s / n_s \ , \tag{26}$$

where $P_s / n_s = D \cdot \varepsilon A_s^{\alpha}$. For $\alpha = 1$, the fragment multiplicities remain unaltered with or without flow; only the chemical potentials get renormalized. Using Eq. (17) the expression for the free neutron density is modified as

$$n_n = \left(\frac{A}{A+1}\right)^{3/2} \cdot \frac{g_A}{g_{A+1}} \cdot \frac{2}{\lambda_T^3} \cdot \exp\left((B(A,Z) - B(A+1,Z))/T\right) \times$$

$$\times \exp\left(-(E_C(A,Z) - E_C(A+1,Z))/T\right) \cdot \exp\left(-D\varepsilon((A+1)^{\alpha} - A^{\alpha} - 1)/T\right) \cdot R_1 \ , \tag{27}$$

where $R_1 = Y(A+1,Z)/Y(A,Z)$ is the (single) ratio between the yields of two fragments differing by one neutron. Also, Eq. (22) for the double ratio is modified as

$$R_2 = \frac{Y(A_1', Z_1')/Y(A_1, Z_1)}{Y(A_2', Z_2')/Y(A_2, Z_2)} =$$

$$= \left(\frac{A_1' \cdot A_2}{A_1 \cdot A_2'}\right)^{3/2} \frac{\omega(A_1', Z_1', T)\,\omega(A_2, Z_2, T)}{\omega(A_1, Z_1, T)\,\omega(A_2', Z_2', T)} \cdot \exp\left(\Delta B / T\right) \cdot \exp\left(-\Delta E_C / T\right) \cdot \exp\left(\Delta F / T\right) \ , \tag{28}$$

where ΔF is given as

$$\Delta F = D\varepsilon(A_1'^{\alpha} - A_1^{\alpha} + A_2^{\alpha} - A_2'^{\alpha}) \ , \tag{29}$$

with A_1', A_2', etc. defined through Eq. (21).

Since $(1-\alpha)$ is very small, the last exponential in Eq. (27) can be very well approximated by

$$\exp\left(-D\varepsilon((A+1)^{\alpha} - A^{\alpha} - 1)/T\right) \approx \exp\left(D\varepsilon(1-\alpha)(1+\ln A)/T\right) . \tag{30}$$

We thus have that the neutron density is increased by this factor, which is larger for smaller temperature, heavier isotopes and larger flow energy. For example, from the yield ratio of, ^{4}He/^{3}He at $T \approx 4.0$ MeV, $\alpha \approx 0.95$, $\varepsilon \approx 10.0$ MeV, one finds that the neutron density is increased by a factor of almost 4, and thus the freeze-out volume is decreased by the same factor. With the inclusion of this correction (Eq. (30)), one finds that the freeze-out volume is $\approx 8V_0$ which is closer to the values usually taken in macrocanonical calculations [14, 20] of nuclear multifragmentation. We note that the extracted values of temperature can be modified, depending on the value of ΔF. Considering, for example, the He-Li thermometer, one has from Eq. (29) that $\Delta F \approx 0.11\varepsilon$, compared to the value of $\Delta B \approx 13.32$ MeV. With $\varepsilon \approx 10$ MeV, using Eq. (28), an increase in T of $\approx$ 10 % is obtained. Similar results are found for other thermometers.

Post emission decay

In deriving Eq. (19) it was assumed that ground state populations are the experimentally observed fragment yields. A possible feeding of the ground state populations through particle- and γ-decay of excited fragments which takes place after fragments leave the source (i.e., the freeze-out volume) is, thus, ignored. In order to take into account the feeding into the ground state by γ-decay, Eq. (19) should be replaced by Eq. (22) where the sum in Eq. (12) extends over the ground state and all γ-decaying states with excitation energy $\varepsilon_i(A,Z)$ and angular momentum I_i of the fragment (A,Z) below the particle-decay threshold energy.

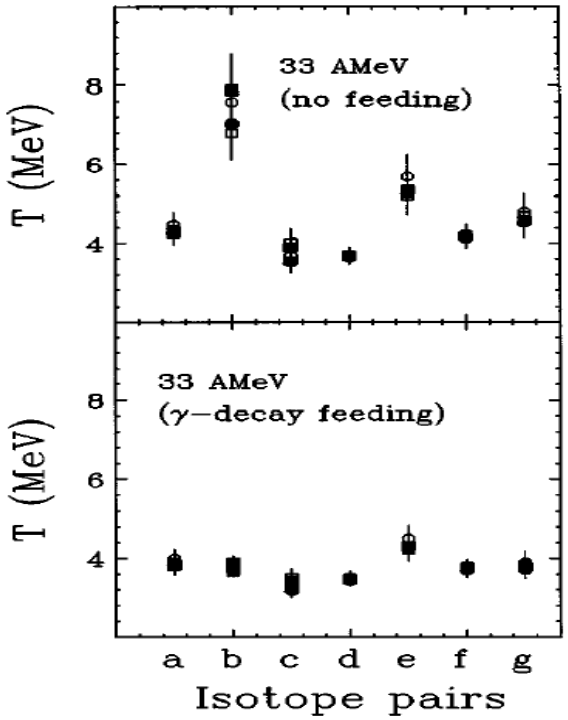

Fig. 1. Temperature extracted from double ratios of (*a*) ^{4}He/^{3}He, ^{7}Li/^{6}Li; (*b*) ^{4}He/^{3}He, ^{10}Be/^{9}Be; (*c*) ^{4}He/^{3}He, ^{11}B/^{10}B; (*d*) ^{4}He/^{3}He, ^{13}C/^{12}C; (*e*) ^{7}Li/^{6}Li, ^{12}C/^{11}C; (*f*) ^{12}C/^{11}C, ^{13}C/^{12}C; (*g*) ^{7}Be/^{6}Li, ^{12}C/^{11}B yield ratios for the reactions ^{40}Ca + ^{58}Ni (filled circles), ^{40}Ar + ^{58}Ni (open circles), ^{40}Ca + ^{58}Fe (filled squares), and ^{40}Ar + ^{58}Fe (open squares) at beam energies o 33 MeV/nucleon before (top) and after (bottom) accounting for poppulations of γ-decaying states (Taken from Ref [8]).

In Ref. [8], the effect of particle-decay of primary fragments emitted from the freeze-out surface on the yields of fragments detected in experiment was taken into account in a similar way. The populations of particle-decaying excited states, calculated in the framework of the statistical model with chemical equilibrium, were also added to the ground state population of the corresponding product fragment, taking into account only nucleon- and α-decay and neglecting multiple-step feeding. Under these assumptions, for a given set of experimental yields of four fragments, one double ratio and two single ratios can be constructed and a system of three independent equations is derived. This system of equations can be solved by iteration to determine the temperature T and chemical potentials μ_p and μ_n. Following Refs. [26, 27], only the dominant decay mode for all excited states was considered. For a specific fragment, the dominant decay mode was taken to be the one with the lowest Q-value among proton-, neutron- and α-decay modes. The data for the lowest particle-decaying excited states that enter Eq. (12) were taken from Refs. [28] and [29]. Contributions from states with higher excitation energies were found in terms of the effective level density $\rho^{\text{eff}}(A,Z)$, see Ref. [26] for details.

The important effect of the γ-decay feeding is nicely demonstrated in Fig. 1. Ratios of isotope yields of fragments from helium through carbon produced in near central collisions from the reactions ^{40}Ca + ^{58}Ni, ^{40}Ar + ^{58}Ni, ^{40}Ca + ^{58}Fe, and ^{40}Ar + ^{58}Fe at 33 MeV/nucleon projectile energy, taken from Ref. [30], were used to extract nuclear temperatures for the emission zone. It is seen from Fig. 1 that with the inclusion of the correction due to post emission decay, the values obtained for the temperature are consistent over various isotope/isotone pairs. The extracted temperature is consistent with values obtained from isotope yields and from relative yields of excited state populations in other measurements. The values of temperature for several combinations of fragment yields obtained with the combined γ- and particle decay feeding are very similar to those obtained by taking into account the γ- decay.

Medium effects

Starting from the nuclear spectral function, an effective wave equation for an A-nucleon cluster embedded in hot low density nuclear matter can be derived [31]. The A-particle wave function and the corresponding eigenvalue are obtained by solving the in-medium Schrödinger equation

$$[E^{qu}(1)+\cdots+E^{qu}(A)-E^{qu}_{A\nu p}(p)]\psi_{A\nu p}(1\ldots A)+$$

$$+\sum_{1'\ldots A'}\sum_{i<j}[1-\tilde{f}(i)-\tilde{f}(j)]\,V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu p}(1'\ldots A')=0\,. \qquad (31)$$

This equation contains the effects of the medium in the single-nucleon quasiparticle energies, $E^{qu}(i)$,as well as in the Pauli blocking terms, $\tilde{f}(i)$. It can be shown that the EoS can be evaluated as in the non-interacting case, except that the number densities of clusters must be calculated with the quasiparticle energies E^{qu},

$$n^{qu}(A,Z) = g_{A,Z}\int \frac{d^3p}{(2\pi)^3} f_{A,Z}[E^{qu}_{A,Z}(p)] \,. \tag{32}$$

In the cluster-quasiparticle approximation, the EoS reads,

$$n^{qu}_p(T,\mu_p,\mu_n) = \sum_{A,Z} Z n^{qu}(A,Z) \,, \tag{33}$$

$$n^{qu}_n(T,\mu_p,\mu_n) = \sum_{A,Z} (A-Z) n^{qu}(A,Z) \,, \tag{34}$$

for the total proton and neutron densities, respectively.

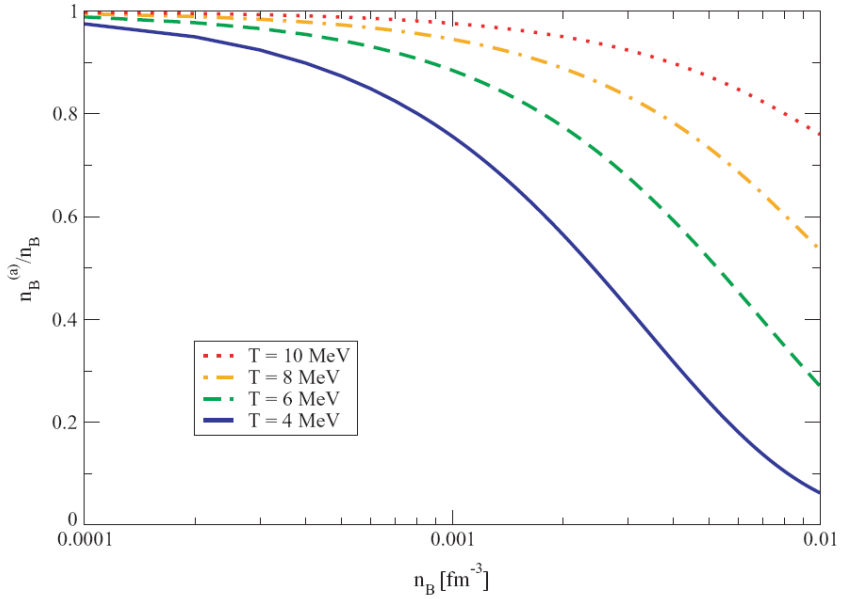

Fig. 2. The ratio between the ACCR baryon density $n_B^{(a)}$ (no medium effects) and n (including medium effects) as a function of the baryon density $n_B = n_p + n_n$ for various values of T (Taken from Ref. [9]).

Comparing the values of the parameters obtained in the full calculation, with inclusion of medium effects on the yields with those deduced in the ACCR approach, we find [9] that moderate deviations in the temperature arise for densities larger than 0.0001 fm^{-3}. However, it is seen from Fig. 2 that determination of the densities is more sensitive to the medium effects.

3. Conclusions

Analysis of heavy-ion collision experiments indicate that hot nuclei exhibit the phenomena of multifragmentation and the saturation of the caloric curve. We emphasize that the main difficulty in the interpretation of experimental data is the separation between the dynamic and the statistical effects. Theoretical description of hot nuclei is very challenging since the nucleus is a finite two-component system of constituents interacting with a short range strong interaction and a long range Coulomb interaction. In this review we limit the discussion to the description of the properties of hot nuclei, assuming the existence of thermal equilibrium in a certain freeze out volume. We have concentrated on the determination of the temperature and density by employing extensions of the method proposed by ACCR [1]. The ACCR method is based on the evaluation of the double ratios R_2 of the yields of the emitted fragments and depends essentially on the existence of both thermal equilibrium and chemical equilibrium in the decaying system. We have derived a relation between the temperature T and R_2 of fragment yields, which is similar to the ACCR method, taking into account the effect of the long range Coulomb interaction and only impose the condition of thermal equilibrium. We have also extend of the method in order to account for the effect of radial flow and the effect of the

population of excited states that γ-decay to the ground state. It was shown that with these modifications of the ACCR method one obtains a reasonable value for the freeze-out volume and extract the same transition temperature using different thermometers (double yields ratios).

We have also considered the effects of medium on the binding energies of clusters embedded in hot low density nuclear matter. We note that a simple statistical model neglecting all medium effects, i.e., treating it as an ideal mixture of non-interacting nuclei, is not applicable for determining the yields of different clusters when the density is larger than 0.0001 fm^{-3}. The success of the simple ACCR method to determine the values for the temperature can be understood from a partial compensation of the effect of the energy shifts of the in medium clusters so that reasonable values for the temperature are obtained also at relatively high densities. More care must be taken in inferring densities from the data.

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