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The charged and neutral pion masses revisited

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ABSTRACT

Results from neutrino mass and oscillation experiments now set the mass of the muon neutrino to less than 2 eV/c^2. This fact, together with our former measurement of the muon momentum in pion decay at rest, $p_{\mu^+} = (29.792\,00 \pm 0.000\,11)$ MeV/c, allows us to directly determine the charged pion mass with 1 ppm precision which constitutes the most precise value of the charged pion mass to date, $m_{\pi^+} = (139.570\,21 \pm 0.000\,14)$ MeV/c^2. This value is within 1.44 σ of the Particle Data Group's compilation of the charged pion mass value, $m_{\pi^\pm} = (139.570\,61 \pm 0.000\,24)$ MeV/c^2. From p_{μ^+} we derive the kinetic energy of the muon, $T_{\mu^+} = (4.119\,84 \pm 0.000\,03)$ MeV and the mass difference, $m_{\pi^+} - m_{\mu^+} = (33.911\,84 \pm 0.000\,14)$ MeV/c^2. From our new m_{π^+} value, assuming CPT invariance ($m_{\pi^-} = m_{\pi^+}$) and our measured mass difference $D_\pi = m_{\pi^-} - m_{\pi^0} = (4.593\,64 \pm 0.000\,48)$ MeV/c^2 we obtain a new value for the neutral pion mass, $m_{\pi^0} = (134.976\,57 \pm 0.000\,50)$ MeV/c^2. One also obtains a new quantitative measure of CPT invariance in the pion sector: $(m_{\pi^+} - m_{\pi^-})/m_\pi(\mathrm{av}) = (-2.9 \pm 2.0) \cdot 10^{-6}$, an improvement by two orders of magnitude.

At the Paul Scherer Institute, PSI, in Villigen, Switzerland, we have measured the muon momentum from pion decay at rest [1–7]:

$$\pi^+ \to \mu^+ + \nu_\mu . \tag{1}$$

Energy and momentum conservation yield

$$\sqrt{m_{\pi^+}^2 + p_{\pi^+}^2} = \sqrt{m_{\mu^+}^2 + p_{\mu^+}^2} + \sqrt{m_{\nu_\mu}^2 + p_{\nu_\mu}^2} \tag{2}$$

$$\vec{p}_{\pi^+} = \vec{p}_{\mu^+} + \vec{p}_{\nu_\mu} . \tag{3}$$

For the special case of a pion at rest, $p_{\pi^+} = 0$, follows

$$p_{\nu_\mu}^2 = p_{\mu^+}^2 . \tag{4}$$

Hence Eq. (2) now reads

$$m_{\pi^+} = \sqrt{m_{\mu^+}^2 + p_{\mu^+}^2} + \sqrt{m_{\nu_\mu}^2 + p_{\mu^+}^2} . \tag{5}$$

For the measurement of the muon momentum, we used a single focusing semicircular magnetic spectrometer [8,9] with a homogeneous field. Details of the apparatus are described in Refs. [3, 7]. The results from the five different experimental periods (Mark I to V) are listed in Table 1 and displayed in Fig. 1.

Table 1
Our results for the muon momentum from pion decay at rest.

Mark	Year	p_{μ^+} [MeV/c]	Reference
I	1979	29.788 5 ± 0.001 9	[3,4]
II$^{*)}$	1984	29.791 39 ± 0.000 83	[4]
III$^{**)}$	1991	29.792 06 ± 0.000 68	[5]
IV	1994	29.792 07 ± 0.000 12	[6]
V$^{***)}$	1996	29.792 00 ± 0.000 11	[7]
weighted mean	2019	29.792 00 ± 0.000 11	this paper

$^{*)}$ This value includes the Mark I result, Refs. [1–3].
$^{**)}$ This value includes the Mark II result, Ref. [4].
$^{***)}$ This value includes the Mark IV result, Ref. [6].

These measurements were originally intended to determine the mass of the muon neutrino m_{ν_μ}, or its upper limit, respectively. This can be obtained from Eq. (5) as

$$m_{\nu_\mu}^2 = m_{\pi^+}^2 + m_{\mu^+}^2 - 2m_{\pi^+}\sqrt{m_{\mu^+}^2 + p_{\mu^+}^2} . \tag{6}$$

For the numerical evaluation we used the validity of the CPT theorem, $m_{\pi^+} = m_{\pi^-}$ and we obtained an upper limit, $m_{\nu_\mu} \le$ 170 keV/c^2 with 95% confidence [7].

Now, with the establishment of neutrino oscillations [10,11], the scenario has changed significantly. It was B. Pontecorvo's prediction that neutrinos might oscillate [12,13] that led to the formulation of the Pontecorvo-Maki-Nakagawa-Sakawa (PMNS) mixing matrix to explain the phenomenon of neutrino oscillations [14].

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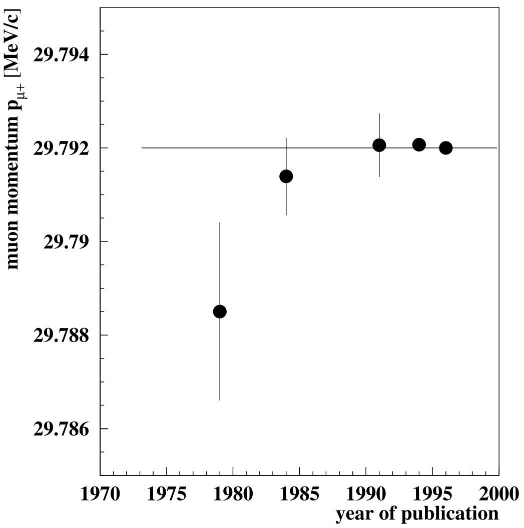

Fig. 1. Our results for the muon momentum from pion decay at rest. The Mark II (1984) result includes the Mark I results (1976-1979). The Mark III (1991) result includes the Mark II result. The Mark V (1996) result includes the Mark IV result (1994). The horizontal line represents the final value, i.e. the weighted mean of the 1991 and 1996 data.

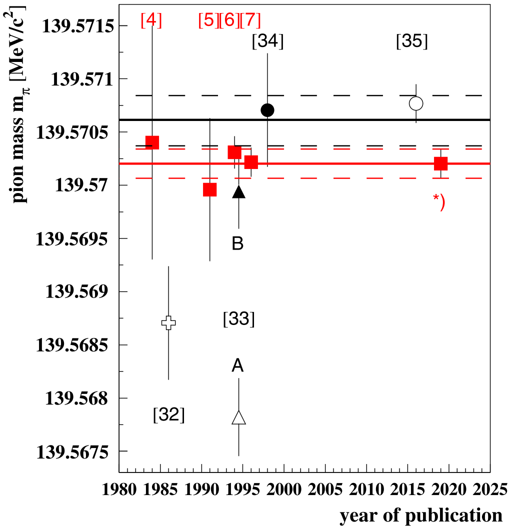

Fig. 2. Chronology plot of the charged pion mass evolution. Black symbols and lines: results for m_{π^-} from pionic atoms. Red symbols and lines: results for m_{π^+} from muon momentum in pion decay at rest. The π^- measurements of Ref. [35] were re-analyzed after the π^+ results of Ref. [6] were published in view of the large discrepancy. The re-analysis resulted in two solutions in Ref. [36] A and B. The continuous and dashed black lines show the PDG average and 1σ band for the charged pion mass which comprises of purely pionic atom measurements: Ref. [36] solution B and Refs. [37,38], as earlier measurements and Ref. [36] solution A may have incorrect K-shell corrections [16]. The continuous and dashed red lines represent the final result, the weighted mean of our 1991 and 1996 values of m_{π^+} together with the 1σ uncertainty band. *) This analysis.

This matrix parameterizes the transformation of mass eigenstates ν_i $(i = 1, 2, 3, \ldots)$ to flavor eigenstates ν_l $(l = e, \mu, \tau \ldots)$.

The number of light neutrinos has been restricted to three [15,16] and so far no hints for heavy sterile neutrinos were found in pion decay [17–23]. Thus, the PMNS matrix reads

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{7}$$

with the numerical values of the matrix elements [24]:

$$\begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.233 \to 0.495 & 0.448 \to 0.697 & 0.639 \to 0.783 \\ 0.287 \to 0.532 & 0.486 \to 0.706 & 0.604 \to 0.754 \end{pmatrix} \tag{8}$$

The upper limit of the electron neutrino mass has been measured at the level of $m_{\nu_e} \leq 2$ eV/c^2 [16,25,26]. This mass value stands for the "effective" electron neutrino mass which is the weighted sum of the mass eigenstates,

$$m_{\nu_e}^2 = \sum_{i=1}^{3} |U_{ei}|^2 m_{\nu_i}^2. \tag{9}$$

In all these experiments, the energy or momentum resolution cannot resolve the mass splitting of the mass eigenstates: the mass differences Δm_{21} and Δm_{32} are experimentally found to be in the range of millielectronvolt [16,27–30].

As can be seen from Eq. (8), m_{ν_e} is predominantly m_{ν_1}. That is, also the masses of m_{ν_2} and m_{ν_3} must be equal or less than $\sim$2 eV/c^2 and consequently the muon and tau neutrinos are equal or less than $\sim$2 eV/c^2. Thus, our measurements of the muon momentum from pion decay at rest should be re-interpreted as a precise direct determination of the mass of the positively charged pion, m_{π^+}.

According to Eq. (5), the uncertainty Δm_{π^+} is limited by the uncertainties of p_{μ^+}, m_{μ^+}, and m_{ν_μ}. Differentiation of Eq. (5) leads to the following sensitivities:

$$\frac{\partial m_{\pi^+}}{\partial p_{\mu^+}} = \frac{p_{\mu^+}}{\sqrt{m_{\mu^+}^2 + p_{\mu^+}^2}} + \frac{p_{\mu^+}}{\sqrt{m_{\nu_\mu}^2 + p_{\mu^+}^2}} = 1.271 \tag{10}$$

$$\frac{\partial m_{\pi^+}}{\partial m_{\mu^+}} = \frac{m_{\mu^+}}{\sqrt{m_{\mu^+}^2 + p_{\mu^+}^2}} = 0.963 \tag{11}$$

$$\frac{\partial m_{\pi^+}}{\partial m_{\nu_\mu}} = \frac{m_{\nu_\mu}}{\sqrt{m_{\nu_\mu}^2 + p_{\mu^+}^2}} = 6.7 \cdot 10^{-8} \tag{12}$$

For the mass of the muon, we use $m_\mu = (105.658\,3745 \pm 0.000\,002\,4)$ MeV/c^2 [16,31,32] and for the mass of the neutrino we use (conservatively) $m_{\nu_\mu} = (2.0 \pm 2.0) \cdot 10^{-6}$ MeV/c^2 [16]. The momentum of the muon from pion decay at rest is $p_{\mu^+} = (29.792\,00 \pm 0.000\,11)$ MeV/c, cf. Table 1. As can be seen, the contributions from the uncertainties of the masses of the muon and the neutrino to the total uncertainty of m_{π^+} are negligible compared to the uncertainty of p_{μ^+}. The result for the mass of the positively charged pion is thus

$$m_{\pi^+} = (139.570\,21 \pm 0.000\,14)\ \text{MeV/c}^2. \tag{13}$$

This is the most precise value for the charged pion mass with a precision of 1 ppm.

The re-analysis of our experimental results, in view of the more stringent limit from the neutrino sector yields practically the same value as published earlier [7]. The new main aspect here is that we no longer, as in our earlier analysis, rely on a value for the neutrino mass based on cosmological considerations [33,34] but rather a value from experimental data from the neutrino sector [16,27–30].

Our new result is now effectively independent of the neutrino mass in the range up to $\sim$2 eV/c^2 and thus constitutes a direct precise measurement of the pion mass and no longer a lower limit. The measured values of m_{π^-} from pionic atoms and m_{π^+} from our measurements are shown in Fig. 2. Our result is more precise than and within 1.44σ of the recent compilation of the Particle Data Group for $m_{\pi^\pm}$ [16]:

$$m_{\pi^\pm} = (139.570\,61 \pm 0.000\,24)\ \mathrm{MeV/c}^2 \tag{14}$$

which uses the three most recent pionic atom experiments [36–38].[2] Our mass value agrees also with those of the single measurements,

$$m_{\pi^-} = (139.569\,95 \pm 0.000\,35)\ \mathrm{MeV/c}^2\ [36]\ \ \text{solution B} \tag{15}$$

$$m_{\pi^-} = (139.570\,71 \pm 0.000\,53)\ \mathrm{MeV/c}^2\ [37] \tag{16}$$

In particular, the agreement with the most precise single measurement of m_{π^-},

$$m_{\pi^-} = (139.570\,77 \pm 0.000\,18)\ \mathrm{MeV/c}^2\ [38] \tag{17}$$

is only fair (2.43σ):

$$m_{\pi^-} - m_{\pi^+} = (0.000\,56 \pm 0.000\,23)\ \mathrm{MeV/c}^2. \tag{18}$$

To summarize, with this negligibly small neutrino mass, we can rewrite Eq. (5) as:

$$m_{\pi^+} = \sqrt{m^2_{\mu^+} + p^2_{\mu^+}} + p_{\mu^+} \tag{19}$$

$$m_{\pi^+} = m_{\mu^+} + T_{\mu^+} + p_{\mu^+} \tag{20}$$

yielding T_{μ^+} the kinetic energy of the muon from the decay of the pion at rest as:

$$T_{\mu^+} = \sqrt{m^2_{\mu^+} + p^2_{\mu^+}} - m_{\mu^+} = (4.119\,84 \pm 0.000\,03)\ \mathrm{MeV}. \tag{21}$$

From Eq. (20) we also obtain the difference of the masses of the pion and muon:

$$m_{\pi^+} - m_{\mu^+} = T_{\mu^+} + p_{\mu^+} = (33.911\,84 \pm 0.000\,14)\ \mathrm{MeV/c}^2. \tag{22}$$

Furthermore, by considering the charged states of the pion mass separately and comparing the PDG value, Eq. (14), which is based solely on π^- measurements, with our π^+-value one has a quantitative measure of the CPT invariance in the pion sector. Using the PDG nomenclature one obtains:

$$\frac{m_{\pi^+} - m_{\pi^-}}{m_{av}} = (-2.9 \pm 2.0) \cdot 10^{-6} \tag{23}$$

This is two orders of magnitude more precise than the best value so far, $(2 \pm 5) \cdot 10^{-4}$ [39]. Our result is consistent within 1.45 σ with the CPT theorem, which predicts that a particle and its antiparticle have equal masses.

Finally, we derive a new value for the neutral pion mass based on our experimentally measured value for the mass difference D_π [40–42] between the negatively charged and neutral pion masses $D_\pi = m_{\pi^-} - m_{\pi^0}$ from the charge exchange reaction

$$\pi^- p \to \pi^0 n. \tag{24}$$

This value is

$$D_\pi = m_{\pi^-} - m_{\pi^0} = (4.593\,64 \pm 0.000\,48)\ \mathrm{MeV/c}^2. \tag{25}$$

Together with the mass of the positively charged pion, m_{π^+}, Eq. (13) and assuming the validity of the CPT theorem ($m_{\pi^+} = m_{\pi^-}$) yields the new value for m_{π^0} with a precision of 3.7 ppm

$$m_{\pi^0} = (134.976\,57 \pm 0.000\,50)\ \mathrm{MeV/c}^2. \tag{26}$$

Using Eq. (14), i.e., the PDG average for m_{π^-} we obtain

$$m_{\pi^0} = (134.97697 \pm 0.000\,54)\ \mathrm{MeV/c}^2. \tag{27}$$

As an outlook we should mention that Hori et al. [43] recently proposed a new method to improve the charged pion mass by using laser spectroscopy of pionic helium atoms.

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