Fermi National Accelerator Laboratory

FERMILAB-Pub-91/52-A
February 1991

# Existence and Stability of Gauged Nontopological Solitons

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February 22, 1991

**Abstract**

Classical nontopological soliton configurations are considered in a theory with a local $U(1)$ symmetry. Their existence, stability against dispersion into free particles are studied numerically. As in the case of Friedberg, Lee and Sirlin with a global $U(1)$ symmetry, in this case also there are two critical charges; $Q_c$ for the existence and $Q_s$ for the stability of the nontopological soliton configurations. Our numerical results show that the magnitudes of both $Q_c$ and $Q_s$ increase as the magnitude of the gauge coupling constant $e$ is increased with the other parameters kept at fixed values.

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## 1. Introduction

Cosmic strings, domain walls and magnetic monopoles attract a great deal of interest because of various possible roles these topological objects may play in cosmology. Athough it had already been demonstrated in the middle of the seventies that there can exist nontopological solitons (NTS's) as well as topological solitons in nature, the cosmological significance of NTS's has been considered only recently. The formation of NTS's in a cosmological phase transition in the early universe has been studied by many groups in the recent years[1]. If NTS's can be formed in the cosmological phase transition, since they may play similar roles to those played by topological solitons, it may be interesting in cosmology as well as in particle physics to study a variety of field theories in which NTS's might arise.

NTS's are extended objects that arise in theories with an unbroken continuous symmetry, and variants on this theme include Q balls[2], cosmic neutrino balls[3], quark nuggets[4], soliton stars[5]. An NTS is a nondissipative solution to the classical equations that, for fixed charge $Q$, represents the field configuration with the lowest energy. Friedberg, Lee and Sirlin[6] (FLS) demonstrated that such a class of complex scalar soliton solutions in three spatial dimensions existed in the theory with a global $U(1)$ symmetry and, for a large enough charge, were in fact stable both classically and quantum mechanically.

In this paper, by localizing the $U(1)$ symmetry of FLS, we study what effect 'electromagnetism' may have on the the formation of NTS's. In Sec. 2 we describe the particle physics setting for the gauged NTS. More details concerning NTS solutions in general can be found in Ref. 6. Section 3 is devoted to our numerical results. The equations of motion are numerically integrated on computer and some solutions are obtained. Finally, in Sec. 4 we discuss our results.

## 2. General Setting

The kinetic part of the Lagrangian density we consider is

$$\mathcal{L}_{KIN} = -(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where $\phi$ and $\sigma$ are respectively complex and real scalar fields, $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ; $e$ is the gauge coupling constant and $A_\mu$ is the $U(1)$ gauge field.

The Lagrangian density also has a scalar potential, which must have nonlinear couplings in the fields for the existence of soliton solutions. We consider the same potential as was considered in Ref. 6;

$$U(\phi,\sigma) = f^2\sigma^2 \mid \phi \mid^2 + \frac{g^2}{8}(\sigma^2 - \sigma_0^2)^2, \tag{2}$$

where $f$ and $g$ are dimensionless coupling constants, and $\sigma_0$ has the dimension of energy and sets the scale of spontaneous symmetry breaking. The Lagrangian is symmetric under the transformations,

$$\phi \longrightarrow e^{i\alpha}\phi \quad \text{and} \quad \sigma \longrightarrow -\sigma.$$

The range of the coupling constants of interest to us is $f^2 > 0$ and $g^2 > 0$. In this range, the minimum of the potential is at $<\sigma>=\sigma_0$ and $<\phi>=0$, so that the local $U(1)$ symmetry is unbroken, and $A_\mu$ may be identified with the electomagnetic field.

The $U(1)$ invariance of the theory gives the conserved current,

$$J_\mu = -i(\phi^\dagger \overleftrightarrow{\partial_\mu} \phi + 2ieA_\mu \mid \phi \mid^2), \tag{3}$$

and the corresponding charge,

$$Q = \int d^3x J^0. \tag{4}$$

Consider a coherent configuration of $\phi$, $\sigma$ and $A_\mu$ with a given charge $Q$. The lowest energy state will have no 'electric' current and therefore no 'magnetic' field. Furthermore, we assume spherical symmetry for the lowest energy configuration. For this configuration with a nonzero $Q$, one can choose a gauge such that $\phi \sim e^{-iwt}$ with $w$ constant, $\vec{\mathbf{A}} = 0$ and $A_t(r) \longrightarrow 0$ as $r \longrightarrow \infty$. For definiteness, we assume that $w > 0$. Scaling away the physical dimensions, we introduce field variables $A, B$ and $C$ defined by

$$A_t = \sigma_0 \frac{e}{g} A(\rho), \quad \phi = \frac{\sigma_0}{\sqrt{2}} B(\rho) e^{-iwt}, \quad \sigma = \sigma_0 C(\rho), \tag{5}$$

where $\rho \equiv g\sigma_0 r$. The Lagrangian, with the substitution of the field variables defined above, becomes

$$\begin{aligned} L \;\; = \;\; & 4\pi \frac{\sigma_0}{g} \int d\rho \rho^2 \{ \frac{1}{2}(\nu - \alpha^2 A)^2 B^2 - \frac{1}{8}(C^2-1)^2 - \frac{k^2}{2} C^2 B^2 - \frac{1}{2}(\partial_\rho B)^2 \\ & - \frac{1}{2}(\partial_\rho C)^2 + \frac{\alpha^2}{2}(\partial_\rho A)^2 \}, \end{aligned} \tag{6}$$

where $\alpha \equiv \frac{|e|}{g}$, $k \equiv \frac{f}{g}$ and $\nu \equiv \frac{w}{g\sigma_0}$.

By varying $L$ with respect to $A$, $B$ and $C$, we find the equations of motion;

$$\frac{1}{\rho^2}\partial_\rho(\rho^2 \partial_\rho \chi) - \alpha^2 \chi B^2 = 0, \tag{7}$$

$$\frac{1}{\rho^2}\partial_\rho(\rho^2 \partial_\rho B) + (\chi^2 - k^2 C^2) B = 0, \tag{8}$$

$$\frac{1}{\rho^2}\partial_\rho(\rho^2 \partial_\rho C) - k^2 C B^2 - \frac{1}{2} C(C^2-1) = 0, \tag{9}$$

where $\chi \equiv \nu - \alpha^2 A$.

The total energy and charge are given by

$$E = 4\pi\frac{\sigma_0}{g}\int d\rho\rho^2\mathcal{E}, \tag{10}$$

where $\mathcal{E} = \frac{1}{2\alpha^2}(\partial_\rho\chi)^2 + \frac{1}{2}(\partial_\rho B)^2 + \frac{1}{2}(\partial_\rho C)^2 + \frac{1}{2}(\chi^2 + k^2C^2)B^2 + \frac{1}{8}(C^2-1)^2$,

$$Q = \frac{4\pi}{g^2}\int d\rho\rho^2\chi B^2. \tag{11}$$

Eq's (7), (8) and (9) can also be obtained by keeping $Q$ fixed and setting the functional derivatives,

$$\frac{\delta E}{\delta\chi(\rho)} = \frac{\delta E}{\delta B(\rho)} = \frac{\delta E}{\delta C(\rho)} = 0. \tag{12}$$

Once an NTS solution of the equations of motion is found, comparison the energy of the solution with that of the free field solution determines stability of the NTS against decay into free particles.

Some qualitative features of the NTS solutions are readily apparent. From Eq's (7) and (11) we see that

$$e^2Q = \lim_{\rho\to\infty} 4\pi\rho^2\partial_\rho\chi. \tag{13}$$

For large $\rho$, $\chi \to \nu - \frac{e^2Q}{4\pi\rho}$. Because the fields configuration is at true vacuum for large $\rho$, Eq. (8) then becomes

$$\frac{1}{\rho^2}\partial_\rho(\rho^2\partial_\rho B) + (\nu^2 - k^2)B = 0. \tag{14}$$

For the existence of a localized solution without oscillation with respect to $\rho$, we require $\nu < k$ and then it follows that $B \propto \frac{1}{\rho}exp(-\sqrt{k^2-\nu^2})$.

We now show that $\chi(\rho)$ obeys the inequalities

$$0 \le \chi(0) \le \chi(\rho) \le \chi(\infty) = \nu < k. \tag{15}$$

It is convenient to write Eq.(7) in the form

$$\partial_\rho(\rho^2\partial_\rho\chi) = \alpha^2\chi B^2\rho^2. \tag{16}$$

Suppose that $\chi(0) < 0$. Eq. (16) then implies that $\rho^2\partial_\rho\chi$ is a decreasing function of $\rho$ so that $\partial_\rho\chi$ goes negative and $\chi(\rho) < 0$ for all $\rho$. This possibility is not acceptable given that $w > 0$ and $\chi(\rho) \to \nu$ for $\rho \to \infty$. The only acceptable possiblity is that $\chi(0) \ge 0$. We then see that $\partial_\rho\chi$ is positive and therefore $\chi(\rho)$ is a monotonically increasing function of $\rho$.

Let us consider a different form of the energy integral. Demanding that the Lagrangian (6) is stationary at $\lambda = 1$ under the rescaling of the form $\rho \to \lambda\rho$ leads to the relation,

$$3\int d\rho\,\rho^2[\frac{1}{2}\chi^2B^2 - \frac{1}{8}(C^2-1)^2 - \frac{1}{2}C^2B^2k^2]$$
$$= \int d\rho\,\rho^2[\frac{1}{2}(\partial_\rho B)^2 + \frac{1}{2}(\partial_\rho C)^2 - \frac{1}{2\alpha^2}(\partial_\rho\chi)^2]. \tag{17}$$

Using Eq's (17) and (7) in Eq. (10), one obtains

$$E = 4\pi\frac{\sigma_0}{g}\int d\rho[\chi\partial_\rho(\rho^2\partial_\rho\chi) + \rho^2\{\frac{1}{3}(\partial_\rho B)^2 + \frac{1}{3}(\partial_\rho C)^2 + \frac{2}{3\alpha^2}(\partial_\rho\chi)^2\}] \quad (18)$$

If the first term in the integrand of Eq. (18) is partially integrated and then Eq. (13) is used, $E_{NTS}$ can be written as

$$E_{NTS} = wQ + \frac{4\pi}{3}\frac{\sigma_0}{g}\int d\rho\,\rho^2[(\partial_\rho B)^2 + (\partial_\rho C)^2 - \frac{1}{\alpha^2}(\partial_\rho\chi)^2]. \quad (19)$$

The energy of the free field solution is

$$E_{free} = mQ + \text{'}electrostatic\text{' } energy, \quad (20)$$

where $m(= f\sigma_0)$ is the mass of $\phi$ field in true vacuum. The 'electrostatic' energy is roughly proportional to $\frac{Q^2}{R}$ for charges uniformly distributed on scale $R$ and therefore, for large $R$, is negligible.

If $w < m$ (or $\nu < k$) and the integrl on the right hand side of Eq. (19) is smaller than $(m - w)Q$, stability of the gauged NTS solution is ensured.

## 3. Numerical Results

In this section we present our numerical solutions of gauged NTS's. Numerical integrations of the differential equations (7), (8) and (9) were carried out with a fourth-order Runge-Kutta method at intervals of $10^{-3}$. We restrict ourselves to studying the lowest energy soliton solutions (zero node solutions), subject to the boundary conditions,

$$\frac{\partial A}{\partial\rho} = \frac{\partial B}{\partial\rho} = \frac{\partial C}{\partial\rho} = 0 \quad at \;\; \rho = 0,$$
$$and \;\; A = 0, \;\; B = 0 \;\; and \;\; C = 1 \;\; at \;\; \rho = \infty. \quad (21)$$

The former is necessary so that the terms $\frac{2}{\rho}(\frac{\partial A}{\partial\rho})$, $\frac{2}{\rho}(\frac{\partial B}{\partial\rho})$ and $\frac{2}{\rho}(\frac{\partial C}{\partial\rho})$ do not become singular at $\rho = 0$, and the latter is necessary because of the requirement that $\mathcal{E}$ (in Eq. (10)) and $\chi B^2$ (in Eq. (11)) be integrable over the infinite volume.

It is convenient to introduce the dimensionless quantities,

$$\hat{E} \equiv \frac{g^2 E}{4\pi m} = \frac{1}{k}\int_0^\infty d\rho\rho^2\mathcal{E},$$
$$\hat{Q} \equiv \frac{g^2 Q}{4\pi} = \int_0^\infty d\rho\rho^2\chi B^2. \quad (22)$$

which can be calculated directly once the appropriate solutions of the differential equations are found. On the basis of Eq's (20) and (22), we see that

$$\hat{E}_{free} \equiv \frac{g^2 E_{free}}{4\pi m} = \hat{Q}. \quad (23)$$

For given values of $k$, $\alpha$ and $\nu(< k)$, we select a set of tentative values for A(0), B(0) and C(0) and integrate numerically the differential equations up to the lowest value $\rho$ at which any of the following possibilities occurs;

$$\text{(i) } A(\rho) < 0, \quad \text{(ii) } B(\rho) < 0, \quad \text{(iii) } C(\rho) > 1,$$
$$\text{(iv) } \partial_\rho A > 0, \quad \text{(v) } \partial_\rho B > 0, \quad \text{(vi) } \partial_\rho C < 0.$$

When any of these six possibilities occurs, the solution is rejected, a new set of initial values are selected, and the integration is repeated. Using the solution corresponding to the correct initial values, $\hat{E}$ and $\hat{Q}$ are calculated according to Eq. (22).

Fig's 1 and 2 give the $\hat{E}_{NTS}$ vs. $\hat{Q}$ curves for $k = 1$, $\alpha = 0.1$ and $k = 1$, $\alpha = 0.2$ respectively. The arrows in the figures represent the direction along which $\nu$ increases. We denote the limiting value of $\hat{Q}$ below which no soliton solution exists by $\hat{Q}_c$, and the value of $\hat{Q}$ at which the solid line crosses with the dashed line in the figures by $\hat{Q}_s$. From our numerical results, we obtain $\hat{Q}_c \simeq 6.29$ and $\hat{Q}_s \simeq 7.5$ for the case of $k = 1$ and $\alpha = 0.1$, and $\hat{Q}_c \simeq 7.83$ and $\hat{Q}_s \simeq 10.1$ for the case of $k = 1$ and $\alpha = 0.2$.

Finally, Fig. 3 illustrates the solution configurations of $A$, $B$ and $C$ for the particular case of $k = 1$, $\alpha = 0.1$ and $\nu = 0.88$.

## 4. Discussion

NTS's occur in theories with a continous symmetry and therefore a conserved Noether charge. Previous investigations of NTS's have, for the most part, concentrated on theories with global symmetries, while a few investigations[7,8] of NTS's have considered theories with local symmetries. In this work, we have considered the model of FLS but with a local $U(1)$ symmetry.

The general features of the $\hat{E}$ vs. $\hat{Q}$ diagrams representing our results are similar to those of FLS in that there exist two critical values $\hat{Q}_c$ and $\hat{Q}_s$. In the case of FLS where $k = 1$ and $\alpha = 0$, the values of $\hat{Q}_c$ and $\hat{Q}_s$ are 6.06 and 6.94 respectively. We can see that, as the value of $\alpha$ is increased, the values of both $\hat{Q}_c$ and $\hat{Q}_s$ increase.

As can be seen from Fig. 3, an NTS solution corresponds to the localized $\phi$-field trapped in the false vacuum. The NTS solutions with $\hat{E} > \hat{Q}$ in Fig's 1 and 2 may be stable classically but not quantum mechanically. The existence of the NTS solutions with $\hat{E} < \hat{Q}$ in the figures shows that the gauged nontopological soliton which is stable both classically and quantum mechanically is a possibility in nature.

## Acknowledgements

We would like to thank Professors Hyun Kyu Lee and Bum-Hoon Lee for helpful discussions. One of us (SUY) was supported by the Daewoo Foundation. This work was also supported in part by the Korea Ministry of Education through the Institute of Basic Science of Hanyang University and in part by the DOE and by the NASA (NAGW-1340 at Fermilab).

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## Figure Caption

Fig. 1. $\hat{E}_{NTS}$ vs. $\hat{Q}$ curve for $\alpha = 0.1$, $k = 1$ in the neighborhood of critical charges. The dashed line is $\hat{E} = \hat{Q}$.

Fig. 2. $\hat{E}_{NTS}$ vs. $\hat{Q}$ curve for $\alpha = 0.2$, $k = 1$ in the neighborhood of critical charges. The dashed line is $\hat{E} = \hat{Q}$.

Fig. 3. Ground-state solution of Eq's (7), (8) and (9) for $\alpha = 0.1$, $k = 1$, $\nu = 0.88$.

Fig. 1

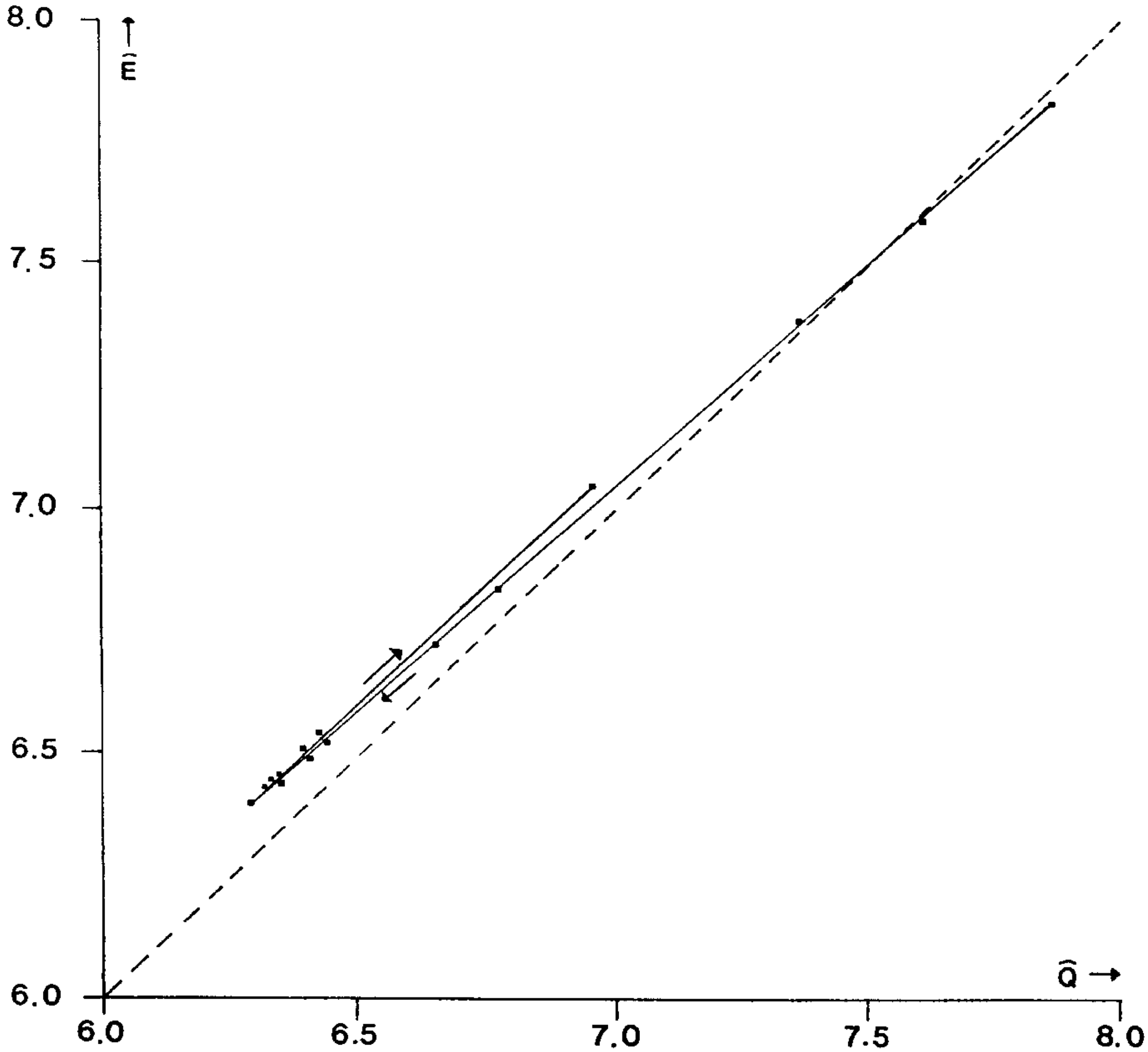


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FIG. 1.

Fig. 2

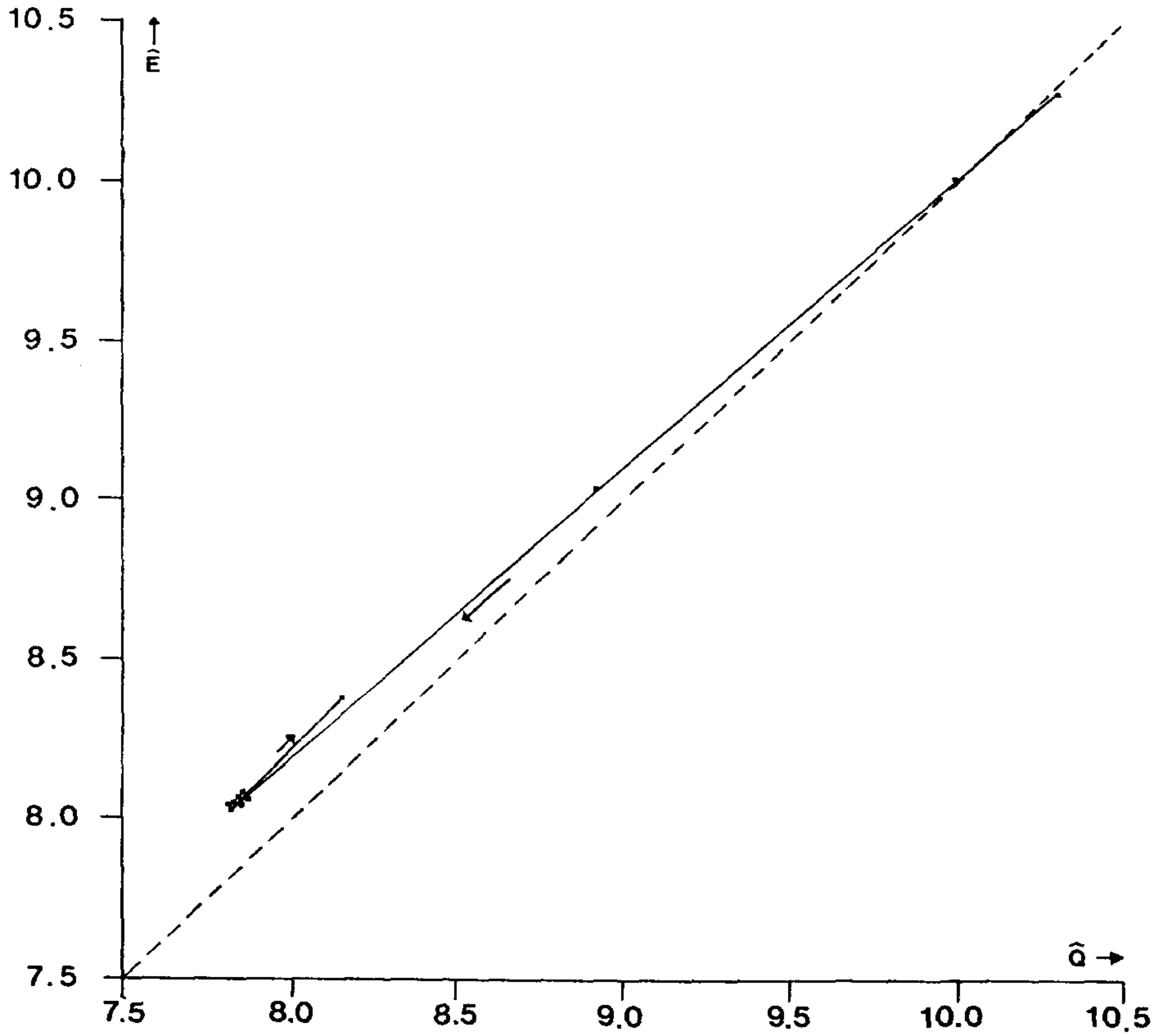


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FIG. 2.

Fig. 3

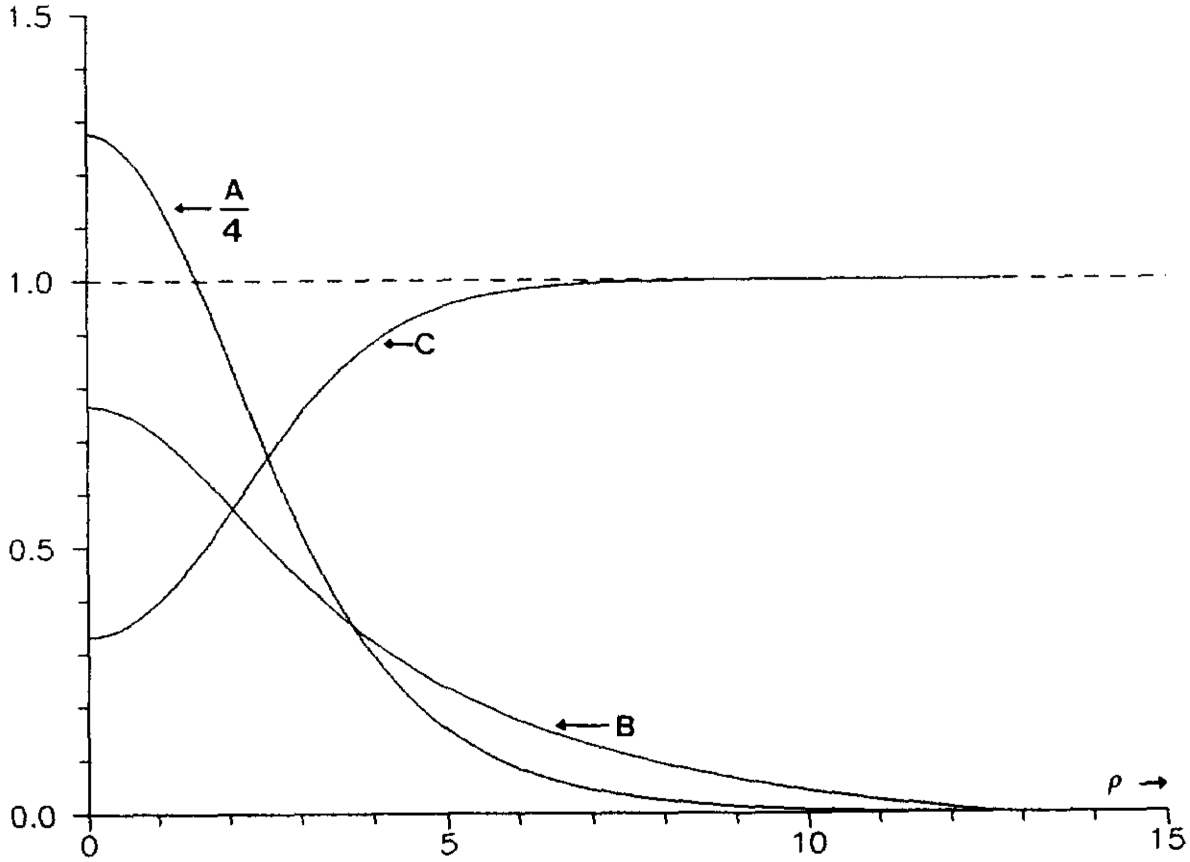


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FIG. 3.