

# QCD phase diagram with chiral imbalance in NJL model: duality and lattice QCD results

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**Abstract.** In addition to temperature and baryon chemical potential there exist other parameters that are essential in the real quark matter. One of them is isospin asymmetry, which does exist in nature, for example, in the compact stars or in heavy ion collisions. Chiral imbalance, the difference between left- and right-handed quarks, is another phenomenon that could occur in quark matter. We have shown that lattice QCD results support the existence of the approximate duality (found in effective models) in real QCD. The rise of pseudo-critical temperature with increase of chiral chemical potential  $\mu_5$  (the much debated effect recently, a lot of studies on this contradict each other) has been established in terms of just duality notion, hence reinforce confidence in this result and put it on the considerably more solid ground.

## 1. Introduction

The QCD phase diagram is one of the open questions of modern elementary particle physics that yet to be answered. In addition to temperature and baryon chemical potential  $\mu_B$ , there are other quantities that describe real quark matter. One of them is isospin asymmetry, i. e. non-zero isospin chemical potential  $\mu_I$ . Isospin asymmetry does exist in nature, for example, in the case of neutron stars and heavy-ion collisions. Using the chiral perturbation theory, it was shown that there is a phase transition at  $\mu_I^c = m_\pi \approx 140$  MeV to the charged pion condensation (PC) phase [1, 2]. This result was ultimately proved in lattice simulations [3, 4, 5]. There is another imbalance, so called chiral imbalance – different densities of right-handed  $n_R$  and left-handed  $n_L$  quarks. It is expected to appear in heavy-ion collisions on event-by-event basis [6]. In addition, media with chiral imbalance can be created in various physical systems (in Dirac and Weyl semimetals, in Early Universe, in neutron stars and supernovae).

Previously, chiral imbalance were considered in NJL type model and other effective models in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. In particular, it was shown in [12] that in the large- $N_c$  limit there is a duality between chiral symmetry breaking (CSB) and charged PC phenomena.

In this paper we explore phase structure of hot quark matter with only chiral or isospin asymmetry at the physical point, in particular, the dependence of the (pseudo-)critical temperature on the chiral isospin chemical potential  $\mu_{I5}$ ,  $\mu_5$  in the framework of the NJL<sub>4</sub> model at  $m_0 \neq 0$ . In this case at zero baryon chemical potential,  $\mu_B = 0$ , there is no sign-problem and we have solid results from lattice simulations [17]. The comparison with LQCD



results is performed and we discuss the possible use of duality in order to get or reinforce some features of the phase diagram.

## 2. The model and its thermodynamic potential

### 2.1. Lagrangian and symmetries

Baryonic (quark) matter, composed of  $u$  and  $d$  quarks, can be described by the following NJL Lagrangian

$$\bar{L} = \bar{q} \left[ \gamma^\nu i \partial_\nu - m_0 + \frac{\mu_B}{3} \gamma^0 + \frac{\mu_I}{2} \tau_3 \gamma^0 + \frac{\mu_{I5}}{2} \tau_3 \gamma^0 \gamma^5 \right] q + \frac{G}{N_c} \left[ (\bar{q}q)^2 + (\bar{q}i\gamma^5 \vec{\tau}q)^2 \right]. \quad (1)$$

Here  $q$  is a flavor doublet,  $q = (q_u, q_d)^T$ , where  $q_u$  and  $q_d$  are Dirac spinors of the  $u$  and  $d$  quark fields;  $\tau_k$  ( $k = 1, 2, 3$ ) are Pauli matrices;  $m_0$  is the bare quark mass;  $\mu_B$ ,  $\mu_I$  and  $\mu_{I5}$  are chemical potentials, which are introduced in order to study quark matter with nonzero baryon, and isospin and chiral (more specifically chiral isospin (or chiral isotopic)) densities, respectively.

In the present paper we study the dynamical generation of  $\langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle$  and  $\langle \pi_a(x) \rangle \sim \langle \bar{q}i\gamma^5 \tau_a q \rangle$  in the ground state of the system and assume the following ansatz

$$\langle \sigma(x) \rangle = M - m_0, \quad \langle \pi_1(x) \rangle = \Delta, \quad \langle \pi_2(x) \rangle = 0, \quad \langle \pi_3(x) \rangle = 0. \quad (2)$$

One can show that in the chiral limit, the TDP is invariant with respect to the so-called duality transformation:

$$\mathcal{D} : \quad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5. \quad (3)$$

We use the following, widely used parameters:  $m_0 = 5, 5$  MeV;  $G = 15.03$  GeV<sup>-2</sup>;  $\Lambda = 0.65$  GeV.

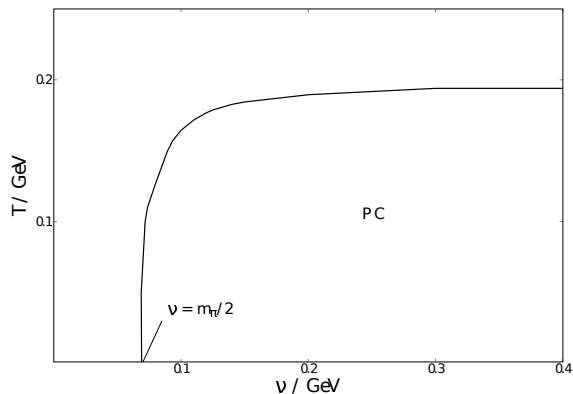
## 3. Phase structure in NJL model and lattice QCD

Let us first consider the case of only one non-zero isospin chemical potential  $\mu_I \equiv 2\nu$ . One can see  $(\nu, T)$ -phase diagrams of the model at  $\mu = 0$  in figure 1. One can see that at zero temperature and small isospin chemical potential the system shows the so-called Silver Blaze phenomenon (the ground state of the system is not affected by increase of chemical potential). When the value of isospin chemical potential crosses the threshold,  $\mu_I = \frac{m_\pi}{2}$ , charged pions can be created and pion condensation appears. At the figure 2 the same phase diagram obtained in LQCD (in case there is no sign problem) is depicted and one can see that the NJL model results are in agreement with first principle lattice calculations [3, 4, 5].

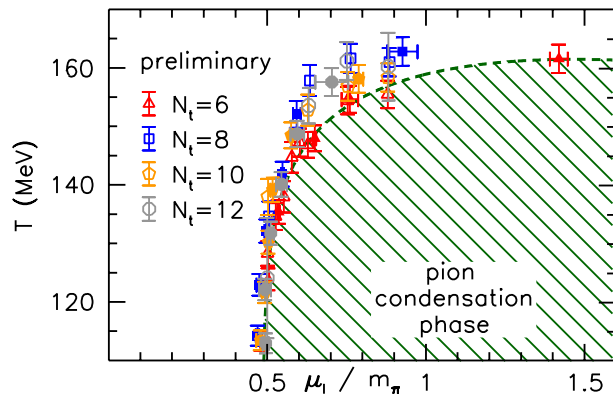
Now let us consider the dual section  $\nu = 0$ ,  $\mu = 0$  of the full phase diagram  $(\mu, \nu, \nu_5)$ , more specifically the behavior (at  $\mu = 0$  and  $\nu = 0$ ) of the pseudo-critical temperature  $T_c(\nu_5)$ . In the most general case, the chiral asymmetry of dense quark matter is described by two chemical potentials, chiral  $\mu_5$  and chiral isospin  $\mu_{I5} \equiv 2\nu_5$  chemical potential. The case of only non-zero chiral chemical potential  $\mu_5$  can be considered on the lattice because in this case there is no sign problem. We plan to compare our NJL model results with lattice simulations of QCD phase diagram with  $\mu_5$  but we considered the NJL model at non-zero  $\mu_{I5}$ .

In the recent paper [11] it has been established that in addition to the duality (3) the TDP of the NJL<sub>4</sub> model is invariant with respect to a transformation  $\mathcal{D}_M : \mu_5 \longleftrightarrow \nu_5$ , if  $\Delta = 0$ . So the phase diagrams  $(\mu_5, T)$  and  $(\nu_5, T)$  are exactly the same due to this duality and this enables us to compare our results with lattice QCD ones.

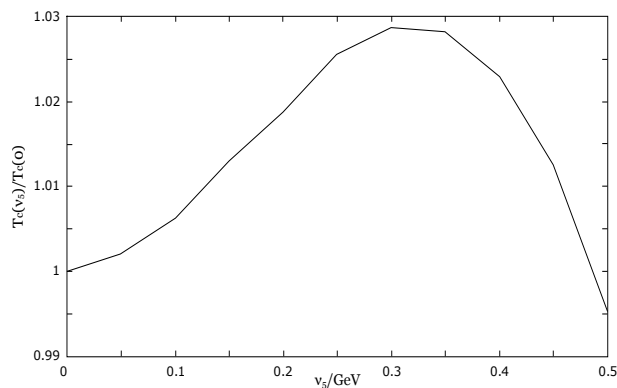
The plot of the *pseudo*-critical temperature  $T_c(\nu_5)$  ( $T_c(\mu_5)$ ) is depicted in figure 3, the lattice QCD result for  $T_c(\mu_5)$  is shown at figure 4. It is easy to see from the plot of figure 3, that the pseudo-critical temperature of the NJL model increases for  $\mu_5 < \mu_5^* \lesssim 350$  MeV. Above this value it drops down, but at  $\mu_5 > \mu_5^*$ , NJL<sub>4</sub> model could not provide very reliable predictions. One can see that up to the values of  $\mu_5 = 300$  MeV there is a good qualitative and even quantitative agreement with the lattice results (see figure 4).



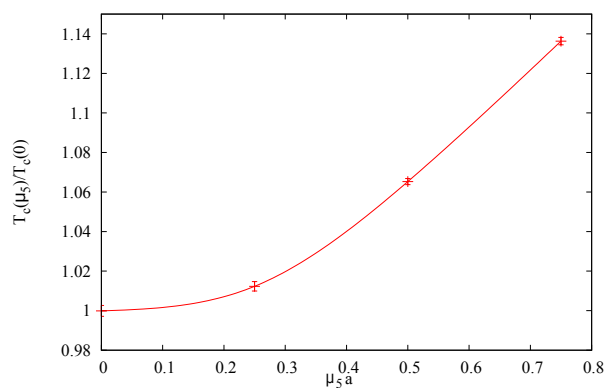
**Figure 1.** The  $(\nu, T)$ -phase portraits at  $\mu = \nu_5 = 0$  MeV. Results obtained in NJL model.



**Figure 2.** The  $(\nu, T)$ -phase portraits at  $\mu = \nu_5 = 0$  MeV. Results obtained in LQCD. Taken from *PoS LATTICE 2016 039*; *Phys. Rev. D* 97, 054514 (2018) [4, 5].



**Figure 3.** The  $(\nu_5(\mu_5), T)$ -phase portraits at  $\mu = \nu = 0$  MeV. Results obtained in NJL model.



**Figure 4.** The  $(\nu_5(\mu_5), T)$ -phase portraits at  $\mu = \nu = 0$  MeV. Results obtained in LQCD. Taken from *Phys. Rev. D* 93, 034509 (2016) [17].

We have two lattice simulation results,  $(\mu_I, T)$  and  $(\mu_5, T)$  phase diagrams (figure 2 and figure 4). These phase diagrams have been also obtained in the NJL model and the results are in good agreement with lattice QCD simulations. But in terms of NJL model we can consider the general case and we know that there is the duality between CSB and charged PC phenomena in the leading order of the large- $N_c$  approximation, but the duality in the case of the physical point is only approximate. Since the particular phase diagrams  $(\mu_I, T)$  and  $(\mu_5, T)$  in these two approaches agrees, one can conclude that the duality is observed in the lattice QCD simulations.

The question of catalysis of chiral symmetry breaking by chiral chemical potential (increase of *pseudo*-critical temperature) is a rather debated one and there are a number of papers that predicted that critical temperature decrease with chiral chemical potential [7, 8, 9, 10] as well there are a number of papers that supports the catalysis [14, 15, 16]. Then it has been shown that Lattice QCD results predict the catalysis. But lattice simulations are performed with

unphysically large value of pion mass so it does not give ultimate result either. Here it is shown that the catalysis can be established in terms of duality notion. Let us briefly explain why.  $(\mu_I, T)$ -phase diagram is well established one and the duality fails only in the region of small isospin and chiral chemical potentials (smaller than half of the pion mass). In the region of isospin chemical potential larger than half of the pion mass the critical temperature increases with  $\mu_I$ , so the critical temperature at the duality conjugated  $(\mu_5, T)$  phase diagram should increase with  $\mu_5$  as well.

#### 4. Summary

- The particular cross sections of the full phase portrait  $(\mu, \nu, \nu_5)$  are in qualitative accordance with the recent lattice simulations [3, 4, 5, 17]. So it has been established that lattice QCD results support the existence of the approximate duality in real QCD.
- The rise of pseudo-critical temperature with increase of chiral chemical potential  $\mu_5$  has been established in terms of duality notion. It gives additional argument in favour of this behaviour of the pseudo-critical temperature and it is of importance because, although the lattice results confirming this behaviour are conclusive, the pion mass that is used in these simulations is still quite high and well above the physical pion mass and our results are obtained at the physical point with the right physical value of the pion mass.

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