SENSITIVITY OF OSCILLATION EXPERIMENTS TO THE NEUTRINO MASS HIERARCHY

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The large value of θ_{13} recently discovered at reactor neutrino experiments has opened the door to determine the ordering of their mass eigenstates in the near future. However, since the neutrino mass ordering is a discrete parameter it is not clear whether the median sensitivity of a given experiment would coincide with the usual values reported in the literature. In this talk we present a summary of the different possibilities to determine the neutrino mass ordering in the near future, and we briefly discuss the statistical issues related to the significance of the signal for this measurement.

1 Introduction

Neutrino oscillations evidence the existence of non-zero neutrino masses. In order to fit the current solar, atmospheric and long baseline neutrino oscillation data, at least three neutrino mass eigenstates are needed [a], with masses m_1, m_2, m_3. These need to satisfy the values of the solar and atmospheric mass splittings: $\Delta m^2_{21} \equiv m^2_2 - m^2_1 \sim 7.5 \times 10^{-5}\,\mathrm{eV}^2$, and $\Delta m^2_{31} \equiv m^2_3 - m^2_1 \sim \pm 2.5 \times 10^{-3}\,\mathrm{eV}^2$, respectively. The solar mass splitting is taken to be positive by convention, while the atmospheric mass splitting can be either positive (if $m_3 > m_1$) or negative (if $m_3 < m_1$) given the current experimental data. In the former case neutrino masses are said to be normally ordered (NO) as opposed to the latter where the ordering would be inverted (IO).

The ordering of neutrino masses (*a.k.a.* the mass hierarchy) has important consequences in neutrino-less double beta decay searches, since the effective mass mediating the process would be a combination of the neutrino masses and the elements of the leptonic mixing matrix. Furthermore, an unknown mass ordering may affect our ability to discover CP violation in the leptonic sector at future neutrino oscillation facilities, if matter effects are relatively small but sizable enough to affect the neutrino oscillation probabilities [1]. Finally, the ordering of neutrino masses

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[a] We deliberately omit here the results from sterile neutrino searches since they are not relevant for the present discussion.

also has profound implications for the flavor puzzle, as well as phenomenological consequences for cosmology and in searches for the absolute scale of neutrino masses.

In order to quantify the sensitivity of future experiments to this parameter, one should note that the ordering of neutrino masses is clearly determined once the sign of Δm^2_{31} is measured. Besides, the current precision on Δm^2_{31} is approximately at the level of[2] 4% (at 1σ) and therefore the two allowed regions are well separated. In other words, the parameter to determine is therefore *discrete* and can take only two values, $+1$ or -1. As a consequence of this, Wilks' theorem[3] does not apply, and the resulting sensitivities may not coincide with the usual results reported in the literature, which are obtained in absence of statistical fluctuations and under the assumption that Wilks' theorem holds. In the present work[4] we address this issue in detail. We provide useful equations for the case where the test statistic is distributed according to a Gaussian. Then, we obtain the sensitivity for each experiment by performing a MC simulation, and we compare the results to those obtained within the Gaussian approximation. Finally, we compare the median sensitivities for the different facilities under consideration, as well as the probability that each of them will achieve a 3σ rejection of the wrong mass ordering.

2 The Gaussian Approximation

In the following we will consider a test statistics based on a log-likelihood ratio:

$$T = \min_{\theta \in \mathrm{IO}} \chi^2(\theta) - \min_{\theta \in \mathrm{NO}} \chi^2(\theta) \equiv \chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}}, \tag{1}$$

where θ is the set of neutrino oscillation parameters which are confined to a given mass ordering during minimization. Under the approximation that T is Gaussian-distributed,

$$T = \mathcal{N}(\pm T_0, 2\sqrt{T_0}), \tag{2}$$

where T_0 is the value of the test statistic in the absence of statistical fluctuations.

Let us take as null hypothesis $H_0 \equiv$ NO, *i.e.*, normal ordering for the neutrino masses. Under the Gaussian approximation, the type I error rate α follows from the above expression as:

$$\alpha = \frac{1}{2}\,\mathrm{erfc}\left(\frac{T_0^{\mathrm{NO}} - T_c^\alpha}{\sqrt{8T_0^{\mathrm{NO}}}}\right), \tag{3}$$

where T_c^α is the critical value of T associated to α. Therefore, if the experimental outcome is more extreme than T_c^α, then the NO hypothesis is rejected at $(1-\alpha)$ confidence level (CL).

An analogous expression can be derived for the type II error rate β:

$$\beta = \frac{1}{2}\,\mathrm{erfc}\left(\frac{T_0^{\mathrm{IO}} + T_c^\alpha}{\sqrt{8T_0^{\mathrm{IO}}}}\right) \approx \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{T_0}{2}} - \mathrm{erfc}^{-1}(2\alpha)\right), \tag{4}$$

where we have used $T = \mathcal{N}(-T_0^{\mathrm{IO}}, 2\sqrt{T_0^{\mathrm{IO}}})$ for the alternative hypothesis H_1. It should be mentioned that the type II error rate is related to the power of the test, $p \equiv 1-\beta$, which is the probability with which we can reject the null hypothesis (NO in this example) at the CL $(1-\alpha)$ if the alternative hypothesis (IO in this example) is true.

Let us now define in a precise way the sensitivity of the median experiment, since this is what is generally used in the literature as a figure of merit. It may be defined as the CL $(1-\alpha)$ at which a false hypothesis can be rejected with a probability of 50%, *i.e.*, with $p = 0.5$. This automatically implies that $\beta = 0.5$. After substituting $\beta = 0.5$ in Eq. 4, the following expression for the number of sigmas for the median experiment is easily obtained:

$$n = \sqrt{2}\,\mathrm{erfc}^{-1}\left[\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{T_0}{2}}\right)\right], \tag{5}$$

where a two-sided Gaussian has been used to convert α into the number of sigmas. This result is shown in the left panel of Fig. 1.

Up to now, we have considered the case of simple hypothesis testing, where the test statistic does not depend on the oscillation parameters. However, this may not always be the case. In the case of long-baseline experiments the value of the CP phase δ generally has a sizable impact on the sensitivity to the mass ordering. Moreover, both at long-baseline experiments and at atmospheric experiments the sensitivity to the mass ordering will sizeably depend on the value of θ_{23}. Therefore, in these situations we will be dealing with the more general case of composite hypothesis testing, where the test statistic depends on additional parameters, which we may generically denote as θ. In this case, one must ensure that the null hypothesis can be rejected for *all possible values* of θ at $(1-\alpha)$ CL. This implies that the critical value of T_c^α needs to be computed for all values of θ, keeping the less extreme result in order to compute the median sensitivity. In particular, we find that:

$$\alpha(\theta) \approx \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{T_0^{\rm IO}(\theta)}{2}} \tag{6}$$

is a useful expression for estimating the median sensitivity for composite hypotheses within the Gaussian approximation.

Finally, let us mention that even though the median experiment is well-defined through the condition $\beta = 0.5$, one may want to require a smaller type II error rate for a given experiment. It should be kept in mind that $p = 1-\beta$ corresponds to the probability of rejecting the null hypothesis if the alternative hypothesis is true, and it would be desirable to maximize this probability. For instance, one could request that the type II error rate β is at most equal to α. In this case, it would be automatically guaranteed that at least one of the two hypotheses can be rejected at $(1-\alpha)$ CL. We will refer to this as the "crossing sensitivity" in the following [4]. From Eqs. 3 and 4 it can be shown that, in this case, the type I error rate would be:

$$\alpha = \frac{1}{2}\,\mathrm{erfc}\left(\frac{T_0^{\rm NO}+T_0^{\rm IO}}{\sqrt{8T_0^{\rm NO}}+\sqrt{8T_0^{\rm IO}}}\right) \approx \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{T_0}{2}}\right) \qquad (T_c^{\rm NO}=T_c^{\rm IO})\,, \tag{7}$$

giving a smaller number of sigmas with respect to the case of the median experiment by roughly a factor of two. The number of sigmas corresponding to the crossing sensitivity is shown in the left panel of Fig. 1 as a function of T_0 (red lines). In the case of a composite hypothesis, the corresponding expression for α would be very similar [4] to Eq. 7.

3 Numerical results

In this section we show the numerical results as obtained from explicit MC simulations. Full simulation details can be found in [4]. We have considered three main possibilities to determine the neutrino mass ordering at neutrino oscillation experiments: (1) medium-baseline reactor experiments; (2) long-baseline experiments; (3) atmospheric neutrino experiments.

Reactor experiments at medium baselines. For reactor experiments with baselines around $\mathcal{O}(10-100)$ km, sizable interference arises between the solar and atmospheric oscillation amplitudes (if θ_{13} is relatively large), which is sensitive to the sign of the atmospheric mass splitting [5]. Two main experiments are currently being considered to determine the mass ordering with this method: JUNO [6,7] and RENO50 [8]. In this work, we have simulated the JUNO medium baseline reactor experiment as in [9], but using an experimental configuration based on [6,10,7]. In this case the test statistic presents little dependence on the oscillation parameters and therefore we are dealing with a simple hypothesis scenario. The sensitivity of the experiment mainly depends on the energy resolution of the detector [11]. We consider a Gaussian energy resolution function with $\sigma(E) = 0.03\times\sqrt{E}$, where E is the neutrino energy, but we have also studied how

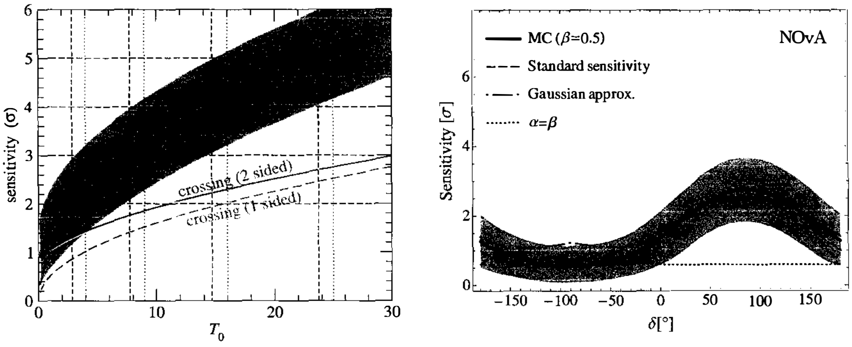

Figure 1 – Left panel: Number of sigmas at which the wrong ordering can be rejected, as a function of T_0, using the Gaussian approximation. The blue lines have been obtained for the median experiment ($\beta = 0.5$, see Eq. 5), while the red lines correspond to the "crossing sensitivity" ($\beta \lesssim \alpha$, see Eq. 7). Solid lines use a 2-sided Gaussian to convert α into number of sigmas, while dashed lines are based on a 1-sided test. The green (yellow) band shows the range of σ at which a false null hypothesis would be rejected in 68.27% and 95.45% of the experiments. Right panel: Median and crossing sensitivities for the NOνA experiment. Results are shown as a function of the true value of δ, for a true IO and $\theta_{23} = 40°$. The solid blue line shows the result from MC simulation after generating 10^5 realizations of the experiment for each value of δ (taken in steps of 10°), for $\beta = 0.5$. The meaning of the green and yellow bands is the same as in the left panel. The dashed and dot-dashed black lines show the results using the Gaussian approximation using a 1-sided and 2-sided Gaussian to convert α into $n\sigma$, respectively. The horizontal dotted line shows the number of sigmas corresponding to the crossing sensitivity (which is independent on δ).

the results vary when this is worsened to $0.035 \times \sqrt{E}$. We find that the distribution of the test statistic is in all cases Gaussian up to a very good approximation. Therefore, we conclude that Eq. 5 can be safely used to extract the median sensitivity from the Asimov data set for this facility.

Long-baseline neutrino experiments. In long-baseline neutrino oscillation experiments, the MSW[12,13,14] effect would produce a resonance in the (anti-)neutrino channel for a NO (IO). This is the method that would be exploited by the NOνA[15] and LBNE[16,17] experiments, among others. In this work we have simulated the NOνA experiment and two possible configurations for LBNE, with a 10 kt and a 34 kt detector. We find that the distribution of the test statistic for NOνA is clearly non-Gaussian. The distributions for LBNE are more similar to a Gaussian although clear deviations are also observed for this setup [4]. Moreover, the results for long-baseline experiments present a large dependence on both the atmospheric mixing angle and the CP-violating phase δ. Thus, a composite hypothesis test is needed in this case. We find that the resulting median sensitivity computed using a MC simulation is in rather good agreement with the expected median sensitivity as extracted from Eq. 5. This is shown in the right panel in Fig. 1 for NOνA, where the number of sigmas expected for the median experiment are shown as a function of the true value of δ.

Atmospheric neutrino experiments. The MSW effect can in principle be observed in the ν_μ and $\bar{\nu}_\mu$ disappearance channels as well, provided that θ_{13} is sufficiently large[18,19]. In this case good energy and angular resolutions are needed in order to avoid a washout of the effect. In principle, magnetization is not needed and therefore large water or ice Čerenkov detectors could be used. If magnetization is available, the sensitivity increases considerably and similar results can be achieved with a much smaller exposure. Two atmospheric neutrino experiments have also been considered in this work: the PINGU proposal[20], with a non-magnetized detector; and the ICAL magnetized iron detector at INO[21,22] (INO for short in the following). We find that the distributions of T are in both cases very close to a Gaussian, although the approximation is not as good as it was for the reactor experiments. Besides, the results for atmospheric experiments

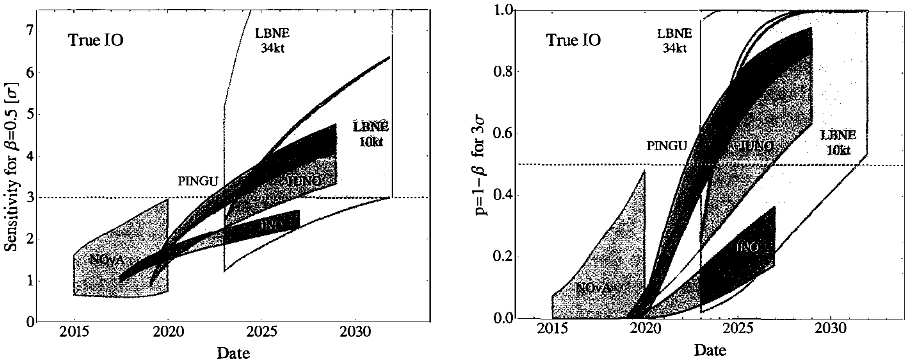

Figure 2 – Left panel: median sensitivity (in number of sigmas) for rejecting the NO assuming a true IO, for different facilities as a function of the date. Right panel: probability that the NO can be rejected at 3σ (99.73% CL), assuming true IO, for different facilities as a function of the date. The width of the bands correspond to different true values of the CP phase δ for NOνA and LBNE, different true values of θ_{23} between 40° and 50° for INO and PINGU, and energy resolution between $3\%\sqrt{1\text{ MeV}/E}$ and $3.5\%\sqrt{1\text{ MeV}/E}$ for JUNO. For the long baseline experiments, the bands with solid (dashed) contours correspond to a true value for θ_{23} of 40° (50°). In all cases, octant degeneracies are fully searched for.

present a large dependence with the atmospheric angle θ_{23} and therefore we have to deal with a composite hypothesis test.

Finally, in Fig. 2 we show a summary of the expected sensitivities for the experimental setups considered in this work. A true IO is assumed for both panels (the corresponding results for a true NO can be found in [4]). In order to keep the number of MC simulations down to a feasible level, we use the Gaussian approximation whenever it is reasonably justified. As already mentioned, this is indeed the case for PINGU, INO, and JUNO. Finally, since the largest deviations from the Gaussian case are observed for long baseline experiments, we have decided to use the results from the full MC simulation whenever possible. The results for the NOνA experiment are always obtained using MC simulations, while in the case of LBNE the results from a full MC are used whenever the number of simulations does not have to exceed 4×10^5 (per value of δ). This means that, in order to reach sensitivities above $\sim 4\sigma$ (for the median experiment), results from the full MC cannot be used.

For each experiment, we have determined the parameter which has the largest impact on the results, and we draw a band showing the range of sensitivities that should be expected in each case. It is important to stress that the meaning of each band may be different, depending on the particular experiment that is considered. In the case of long baseline experiments (NOνA, LBNE-10 and LBNE-34), the results depend on the value of the CP-violating phase δ. In this case, we do a composite hypothesis test and we draw the edges of the band using the values of true δ in the true ordering that give the worst and the best results for each setup. Besides, since the results also show some dependence with the value of θ_{23}, we show two results corresponding to values of θ_{23} in the first and second octant. In the case of PINGU and INO, the most relevant parameter is θ_{23}. Therefore, in this case we also do a composite hypothesis test, using θ_{23} as an extra parameter. Finally, the case of JUNO is somewhat different. In this case, the energy resolution is the parameter which is expected to have the greatest impact [11] on the results, while the dependence with the oscillation parameters is small. Thus, we perform a simple hypothesis test for this setup, and the width of the band shows in this case the variation on the results if the energy resolution is changed.

4 Conclusions

In this work, we have studied the sensitivity of future neutrino oscillation experiments to the ordering of neutrino masses. Since the neutrino mass ordering is a discrete parameter (which can be identified with the sign of Δm^2_{31}), Wilks' theorem does not apply and the median sensitivities need to be extracted from a full MC simulation. The sensitivity of a future experiment for a hypothesis test can be quantified by reporting two numbers: the CL $(1-\alpha)$ at which the null hypothesis can be rejected, which corresponds to a type I error rate α; and the probability p that the null hypothesis can be rejected at some CL, which is related to the type II error rate as $p = 1 - \beta$. We have derived useful formulas for these two quantities in the case where the test statistic is distributed as a Gaussian. Then, we have obtained the sensitivity to the mass ordering for several proposed neutrino oscillation experiments (reactor experiments at medium baselines, long-baseline neutrino beam and atmospheric neutrino experiments) from MC simulations, and we have compared the results to the median sensitivities obtained within the Gaussian approximation. We conclude that the agreement is in all cases rather good, even when the distribution of the test statistic presents large deviations from a Gaussian.

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