HARMONIC MOTION OF ELECTRON TRAJECTORY IN PLANAR UNDULATOR*

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Abstract

For planar undulator, the expression of electron trajectory including harmonic motion has been deduced. It were shown that the electrons oscillate at odd harmonics in the transverse direction, and at even harmonics in the axial direction; the amplitude of *n*th harmonic oscillation is proportional to the *n*th power of ratio of undulator deflection parameter to the electron energy.

The electrons produce synchrotron radiation as they passed through magnetic field. The pattern and characteristics of the radiation is dependent on the mode of the electron motion[1][2]. For a planar undulator, a sinusoidal periodic magnetic field in the direction perpendicular to the direction of the electron motion causes the electrons to oscillate transversely and thus radiate in the direction forward. In this paper we analyses the harmonic motion of electron in a planar undulator.

ANALYSI S

The relativistic electron moves in a planar undulator, it's transverse velocity component (normalized to the velocity of light) in the sinusoidal magnetic field is specified through the Lorentz force equation.

$$\beta_x = -\frac{K}{\gamma}\cos k_u z \tag{1}$$

Where K is undulator deflection parameter, it is dimensionless vector potential of the magnet field, k_u is the undulator field wave number , γ is the dimensionless energy of electron. In writing Eq. (1), we define the longitudinal axis as z, the vertical magnetic field direction as y and horizontal direction that electron oscillate in as x; we also have assumed that the relativistic electron with an initial zero transverse velocity component. The longitudinal component of the electron velocity is obtained from

$$\beta_z = 1-\frac{1}{2}\left(\frac{1}{\gamma^2}+\beta_x^2\right) = \bar{\beta}_z - \left(\frac{K}{2\gamma}\right)^2 \cos(2k_u z) \tag{2}$$

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where $\bar{\beta}_z$ is the average longitudinal velocity along the z-axis

$$\bar{\beta}_z = 1-\frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right),$$

it shows the reduced axial velocity due to the finite magnetic field (*K*). Beside direct current term the axial velocity includes an oscillating term, it is this oscillation term that produce harmonics in electron motion, and thus result in harmonic frequencies of radiation, as we will show in the following.

For the first order approximation, we take $z \approx \bar{\beta}_z ct = \bar{z}$ in the velocity expressions (Eqs.(1-2)), then the trajectory of the electron can be directly given by integral over *t*:

$$x = -\frac{K}{k_u\gamma}\sin(k_u\bar{\beta}_z ct) + x_0 \tag{3}$$

$$z = \bar{\beta}_z ct - \frac{K^2}{8k_u\gamma^2}\sin(2k_u\bar{\beta}_z ct) \tag{4}$$

Above equation show that the longitudinal motion of the electron has an axial oscillation term, which has a frequency twice that of the transverse oscillation In the frame of reference moving with electrons, the electron performs a "figure-of-eight" motion in the x'-z' plane, which can be decomposed into dipole oscillation in the x'-direction with fundamental frequency and dipole oscillation in the z'-direction with second harmonic frequency. The dipole radiation pattern is axisymmetric about x' and z', respectively. Back in the laboratory frame of reference, the radiation pattern is relativistically contracted into a narrow cone that is axisymmetric about z for fundamental and two narrow cones off z-axis for second harmonic (Fig.1).

For more rigorous analysis，we substitute Eq.(4) into Eq.(1) then get

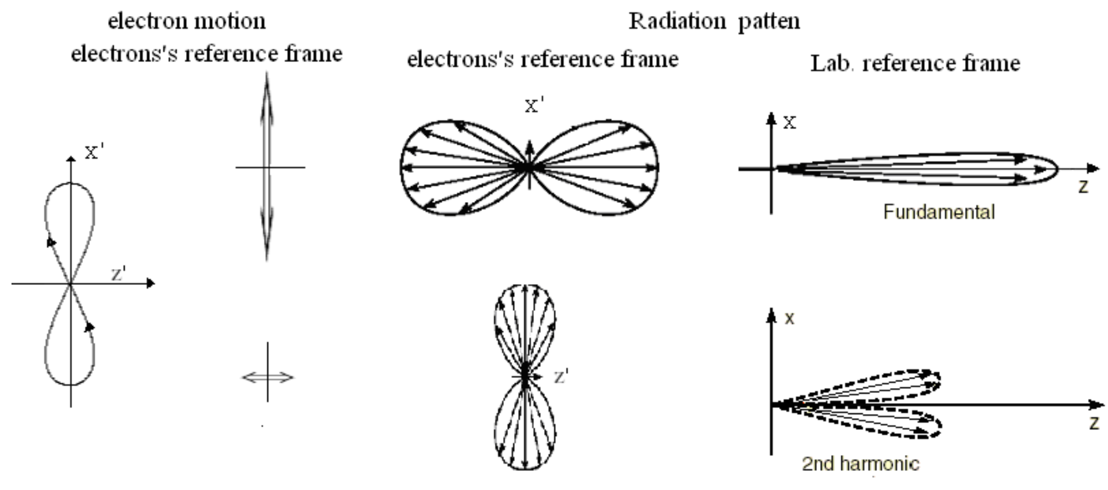

Figure: 1 The electron motion and radiation pattern.

$$\beta_x = -\frac{K}{\gamma}\mathrm{Re}\, e^{ik_u(\bar{z}-\frac{K^2}{8k_u\gamma^2}\sin(2k_u\bar{z}))}$$

$$= -\frac{K}{\gamma}\mathrm{Re}\, e^{ik_u\bar{z}}\sum_{m=-\infty}^{\infty} J_m(\frac{K^2}{8\gamma^2})e^{-i2mk_u\bar{z}} \quad (5)$$

$$= -\frac{K}{\gamma}\sum_{m=1}^{\infty}[J_m(\frac{K^2}{8\gamma^2})-(-1)^m J_{m-1}(\frac{K^2}{8\gamma^2})]*$$

$$*\cos[(2m-1)k_u\bar{z}]$$

Expanding the Bessel function in series and only retaining the lowest terms upto $(K/\gamma)^n$ (using the condition $(K/\gamma)^2/8 \ll 1$) for the nth harmonic oscillation, Eq.(5) becomes

$$\beta_x \approx -4\sum_{n=1,3,5,...}(\frac{K}{4\gamma})^n\frac{(-1)^{(n-1)/2}}{[(n-1)/2]!}\cos[nk_u\bar{z}] \quad (6)$$

Therefore the horizontal motion of electron is

$$x \approx -\frac{4}{k_u}\sum_{n=1,3,5,...}(\frac{K}{4\gamma})^n\frac{(-1)^{(n-1)/2}}{n[(n-1)/2]!}\sin[nk_u\bar{\beta}_z ct]+x_0 \quad (7)$$

Similarly substituting Eq.(4) back into Eq.(2) yields.

$$\beta_{II}-\bar{\beta}_{II} = \left(\frac{K}{2\gamma}\right)^2\mathrm{Re}\, e^{i2k_u(\bar{z}-\frac{K^2}{8k_u\gamma^2}\sin(2k_u\bar{z}))}$$

$$= -\left(\frac{K}{2\gamma}\right)^2\mathrm{Re}\, e^{i2k_u\bar{z}}\sum_{m=-\infty}^{\infty}J_m(\frac{K^2}{4\gamma^2})e^{-i2mk_u\bar{z}}$$

$$= -\left(\frac{K}{2\gamma}\right)^2\{J_1(\frac{K^2}{4\gamma^2})+\sum_{m=1}^{\infty}[J_m(\frac{K^2}{4\gamma^2})-$$

$$-(-1)^m J_{m-1}(\frac{K^2}{4\gamma^2})]\cos[2mk_u\bar{z}]\}$$

$$\approx -\frac{1}{2}\left(\frac{K}{2\gamma}\right)^4-\sum_{m=1}^{\infty}(\frac{K^2}{8\gamma^2})^m\frac{(-1)^{m-1}2}{(m-1)!}\cos[2mk_u\bar{z}]$$

$$\approx -\frac{1}{2}\left(\frac{K}{2\gamma}\right)^4+2\sum_{n=2,4,..}^{\infty}(\frac{K}{\sqrt{2}2\gamma})^n\frac{(-1)^{\frac{n}{2}}}{[(n-2)/2]!}\cos[nk_u\bar{z}]$$

Namely the longitudinal component of the electron velocity is

$$\beta_{II} \approx \bar{\beta}_{II}+2\sum_{n=2,4,..}^{\infty}(\frac{K}{4\gamma})^n\frac{(-2)^{\frac{n}{2}}}{[(n-2)/2]!}\cos[nk_u\bar{z}] \quad (8)$$

Accordingly

$$z \approx \bar{\beta}_z ct-\frac{1}{k_u}\sum_{n=2,4,..}^{\infty}(\frac{K}{4\gamma})^n\frac{(-2)^{n/2}}{(n/2)!}\sin[nk_u\bar{\beta}_z ct] \quad (9)$$

Equations (6-9) exhibit that the electrons oscillate at odd harmonics frequency in the transverse direction, and at even harmonics frequency in the axial direction; thus cause the odd harmonics radiations on-axis with horizontally polarization, and the even harmonics radiations off-axis with vertically polarization. The maximum amplitude of nth harmonic oscillation is proportional to the nth power of ratio of undulator deflection parameter to electron energy: $(K/\gamma)^n$. Therefore, the harmonics component is more significant for the cases of the larger undulator deflection parameter K and the lower electron energy; and the harmonics strength decrease rapidly as the harmonic number increases.

REMARK

It is noteworthy that the analysis here is made for an ideal pure sine-wave undulator field, it may assist in the fundamental understanding of undualtor radiation. In actual undulators, the actual magnetic field is non-sinusoidal, when expanded in Fourier series the field itself includes odd spatial harmonics due to the symmetry of the magnetic structure[3]. Hence the more complex situation must be considered for the electron motion.

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